

7B Solving First-Order Recurrence Relations

The order of a recurrence relation:

A first-order linear recurrence relation can be written in the form:

It is homogeneous when:

Back substitution:

1. Find a closed form for the sequence $a_n = 5a_{n-1}$, $n > 0$; $a_0 = 1$.

The solution to the first-order homogeneous recurrence relation $u_n = au_{n-1}$ is:

2. Solve the recurrence relation $a_n = 2a_{n-1}$, $n \geq 1$, with $a_0 = 3$.

General solution and particular solution:

3. Solve the recurrence relation $a_n = -3a_{n-1}$, $n \geq 1$, with $a_1 = 6$.

4. Find a solution to the recurrence relation $u_n = u_{n-1} + n$, $n \geq 1$, with $u_0 = 0$.

The solution to $u_n = u_{n-1} + g(n)$ is:

5. Solve the following recurrence relations.

a) $u_n = u_{n-1} + 2n + 1, n \geq 1$, with $u_0 = 7$

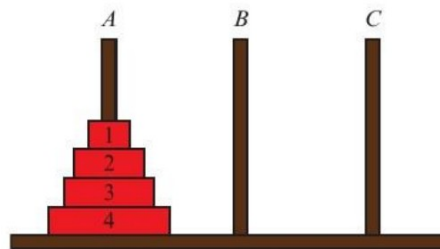
b) $u_n = u_{n-1} + 5^n, n \in \mathbb{N}$, with $u_1 = 3$

Solving $u_n = au_{n-1} + g(n)$: complementary function (C.F.) and particular solution (P.S.):

	Form of particular solution
p with $a \neq 1$	
$pn + q$ with $a \neq 1$	
kp^n with $p \neq a$	
ka^n	
p with $a = 1$	
$pn + q$ with $a = 1$	

6. Solve the recurrence relation $u_n = 3u_{n-1} + 2n, n \in \mathbb{Z}^+$, with $u_1 = 3$.

7. The Tower of Hanoi puzzle involves transferring a pile of different sized disks from one peg to another using an intermediate peg. The rules are: only one disk at a time can be moved; a disk can only be moved if it is the top disk on a pile; a larger disk can never be placed on a smaller one.



- a) Find the minimum number of moves needed to transfer two disks from one peg to another.
- b) Show that three disks can be transferred from one peg to another in 7 moves.

	Disk	From	To
Move 1			
Move 2			
Move 3			
Move 4			
Move 5			
Move 6			
Move 7			

- c) Explain why the minimum number of moves, d_n , needed to transfer n disks from one peg to another satisfies the recurrence relation $d_n = 2d_{n-1} + 1$, with $d_1 = 1$.

- d) Solve this recurrence relation for d_n .

- e) Hence determine the minimum number of moves needed to transfer 15 disks from one peg to another.