

5C Dynamic Programming Problems in Table Form

Terminology for problems in table form

Stage:

State:

Action:

Destination:

Value:

1. A fruit grower can use his surplus crop to make into jam, pie filling or dried fruit. He can process up to five units of fruit and the expected profits, in hundreds of pounds, from making the various products are shown in the table. He will use all five units to make these products and wishes to maximise his income. Dynamic programming will be used to solve this problem.

Number of units of fruit	1	2	3	4	5
Jam	12	27	33	36	38
Pie filling	16	18	20	22	24
Dried fruit	8	16	24	32	40

- a) Define the terms Stage, State, Action, Destination and Value in this context.
- b) Determine the number of units he should allocate to each product in order to maximise his income.

Stage	State	Action	Destination	Value
1 Dried fruit	0	0	0	
	1	1	0	
	2	2	0	
	3	3	0	
	4	4	0	
	5	5	0	
2 Pie filling	0	0	0	
	1	1	0	
		0	1	
	2	2	0	

Stage	State	Action	Destination	Value
		1	1	
		0	2	
	3	3	0	
		2	1	
		1	2	
		0	3	
	4	4	0	
		3	1	
		2	2	
		1	3	
		0	4	
	5	5	0	
		4	1	
		3	2	
		2	3	
		1	4	
		0	5	
3 Jan	5	5	0	
		4	1	
		3	2	
		2	3	
		1	4	
		0	5	

2. A clockmaker makes grandfather clocks during the winter. The order book for clocks over the next five months is shown in the table below.

Month	October	November	December	January	February
Number of clocks ordered	2	3	5	3	1

The clocks are delivered to customers at the end of each month. He can make up to four clocks in any month, but if he needs to make more than two in any one month he will need to hire additional help at £500 per month. The overhead costs are £300 in any month in which work is done. He can put up to two clocks into storage at a cost of £100 per clock per month. There are no clocks in storage at the beginning of October and there should be no clocks in storage after the February delivery.

- a) Use dynamic programming to determine the production schedule that minimises the costs.

Stage:

State:

Action:

Destination:

Value:

Stage	State	Action	Destination	Value
Feb	1	0	0	

Stage	State	Action	Destination	Value
	0	1	0	
Jan	2	2	1	
		1	0	
	1	3	1	
		2	0	
	0	4	1	
		3	0	
Dec	2	4	1	
		3	0	
	1	4	0	
Nov	2	3	2	
		2	1	
	1	4	2	
		3	1	
	0	4	1	
Oct	0	4	2	
		3	1	
		2	0	

Month	October	November	December	January	February
Number of clocks made					

The clockmaker's helper needs some extra income in October, so the clockmaker decides he will definitely make four clocks this month.

- b) Given this information, use your table to determine the optimum production schedule for the remaining months, and write down the total production costs.

Month	October	November	December	January	February
Number of clocks made					

3. A builder has purchased a site and will build three houses A, B and C on it at the rate of one a year. The profit made on each house will depend on site accessibility, interest rates and cost of materials and labour. The builder's estimates of potential profits are shown in the table below, in thousands of pounds.

Already built	A	B	C
None	50	70	65
A	-	65	60
B	65	-	70
C	60	70	-

Already built	A	B	C
A and B	–	–	80
A and C	–	60	–
B and C	70	–	–

Dynamic programming is to be used to determine the order in which the houses are to be built so that the minimum profit in each year is to be maximised.

a) Explain the meaning of Stage, State and Action in this context.

b) Find the order in which the houses should be built and the value of the minimum estimated annual profit.

Stage	State	Action	Destination	Value
1	AB	C	ABC	
	AC	B	ABC	
	BC	A	ABC	
2	A	B	AB	
		C	AC	
	B	A	AB	
		C	BC	
	C	A	AC	
		B	BC	
3	None	A	A	
		B	B	
		C	C	

4. Toby has a fairground ride and will take it to three county fairs over the next three weeks. He will leave home at the start of week 1 and travel to the first fair, then go to the second fair for the second week and then go to the third fair for the final week before returning home. There are seven fairs during the next three weeks so there is a choice of fair each week. Toby must decide which one of the fairs he should go to. Table 1 gives the week in which each fair is held. Table 2 gives the expected profits, in hundreds of pounds, in going to each fair. Table 3 gives the travel costs in hundreds of pounds.

Table 1: Week	1	2	3
Fair	A, B	C, D, E	F, G

Table 2: Fair	A	B	C	D	E	F	G
Profit (£100s)	6	5	8	10	11	14	12

Table 3: Travel costs (£100s)	A	B	C	D	E	F	G
Home	3	2				8	7
A			3	2	5		
B			1	3	3		
C						5	5
D						4	2
E						4	1

Toby wishes to maximise his income. Use dynamic programming to determine the fairs he should go to and his total income.

Stage:

State:

Action:

Destination:

Stage	State	Action	Destination	Value
1	F	F-Home	Home	
	G	G-Home	Home	
2	C	CF	F	
		CG	G	
	D	DF	F	
		DG	G	
	E	EF	F	
		EG	G	
3	A	AC	C	
		AD	D	
		AE	E	
	B	BC	C	
		BD	D	
		BE	E	
4	Home	Home-A	A	
		Home-B	B	