

2A The Hungarian Algorithm

Allocation problem:

Cost matrix:

To reduce the cost matrix:

Optimal allocation:

1. The table shows the times taken, in minutes, by four workers to complete each task of a four-stage production process.

	Task W	Task X	Task Y	Task Z
Kris	12	23	15	40
Laura	14	21	17	20
Sam	13	22	20	30
Steve	14	24	13	10

The time taken for the process is the sum of the times of the four separate tasks. Reduce the cost matrix.

Reduce rows:

	Task W	Task X	Task Y	Task Z
Kris				
Laura				
Sam				
Steve				

Reduce columns:

	Task W	Task X	Task Y	Task Z
Kris				
Laura				
Sam				
Steve				

Optimal allocation and total time:

The Hungarian algorithm:

2. Barbara, Percival and Spike do garden maintenance. The table shows the times, in minutes, that they would take to do each task in Mrs Green's garden. Allocate the tasks so that the time taken to do the whole job is as small as possible.

	Dig vegetable patch	Weed flower beds	Cut lawn and hedges
Barbara	250	80	160
Percival	230	90	150
Spike	230	110	140

	Dig vegetable patch	Weed flower beds	Cut lawn and hedges
Barbara			
Percival			
Spike			

	Dig vegetable patch	Weed flower beds	Cut lawn and hedges
Barbara			
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Spike			

	Dig vegetable patch	Weed flower beds	Cut lawn and hedges
Barbara			
Percival			
Spike			

The short cut (augmenting the reduced cost matrix by e):

3. Five workers Ben, Ellie, Greg, Hyo and Toby are to be assigned to five tasks involved in making a soft toy. The table shows the times in minutes taken to complete each task.

	Cut	Sew	Fill	Finish	Pack
Ben	9	8	3	6	10
Ellie	5	5	7	5	5
Greg	10	9	3	9	10
Hyo	10	7	2	9	7
Toby	9	8	2	7	10

Reducing rows first, use the Hungarian algorithm to determine an allocation that minimises the total time.

	Cut	Sew	Fill	Finish	Pack
Ben					
Ellie					
Greg					
Hyo					
Toby					

	Cut	Sew	Fill	Finish	Pack
Ben					
Ellie					
Greg					
Hyo					
Toby					

	Cut	Sew	Fill	Finish	Pack
Ben					
Ellie					
Greg					
Hyo					
Toby					

	Cut	Sew	Fill	Finish	Pack
Ben					
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