

5A The Classical and Practical Travelling Salesman Problems

Walk:

Tour:

The travelling salesman problem:

Heuristic algorithm:

Upper and lower bounds ('trapping' the optimal solution):

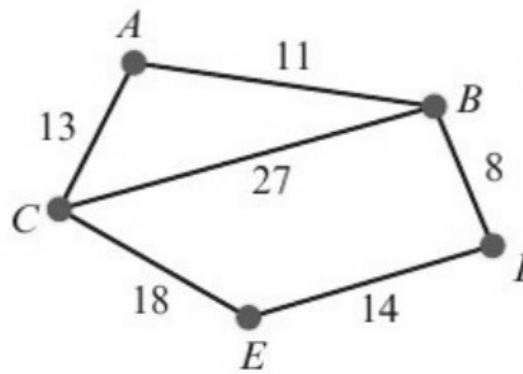
The classical problem:

The practical problem:

Triangle inequality:

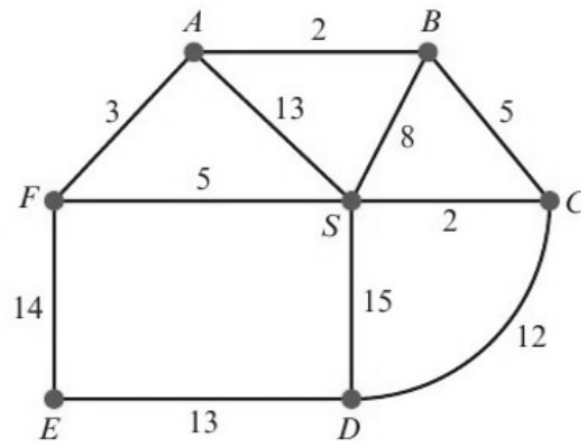
Table of least distances:

1. Create a table of least distances for the network below.



	A	B	C	D	E
A	-				
B		-			
C			-		
D				-	
E					-

2. The network below shows the distances, in km, between the central sorting office at S and six post offices A, B, C, D, E and F. The table shows a partially completed table of least distances.



	S	A	B	C	D	E	F
S	-			2			5
A		-	2		19	17	3
B		2	-	5	17		
C	2		5	-	12	21	7
D		19	17	12	-	13	
E		17		21	13	-	14
F	5	3		7		14	-

Complete the table of least distances for the network, stating your shortest route for each of the entries.

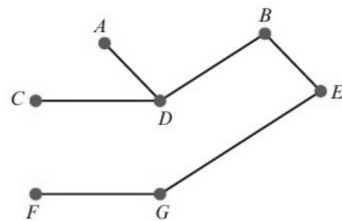
5B Using a Minimum Spanning Tree Method to Find an Upper Bound

Using a minimum spanning tree method to find an upper bound for the practical problem:

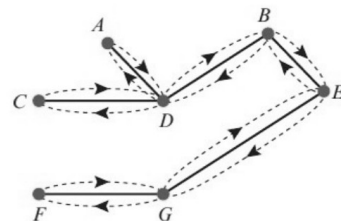
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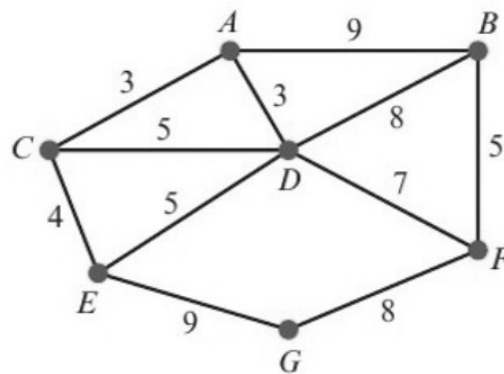
Minimum spanning tree



Initial upper bound

Initial upper bound:

1. For the network below:
 - a) Use Kruskal's algorithm to find a minimum spanning tree for the network below.
 - b) Hence find an initial upper bound for the travelling salesman problem.



2. The table of least distances for a network is given below. Find an initial upper bound for the travelling salesman problem for this network.

	A	B	C	D	E
A	-	11	13	19	31
B	11	-	24	8	22
C	13	24	-	32	18
D	19	8	32	-	14
E	31	22	18	14	-

Improving the upper bound (shortcuts):

3. Starting from the initial upper bound found in question 2, use a shortcut to reduce the upper bound to below 70.

4. The table shows the distances, in miles, between eight cities. A politician has to visit each city, starting and finishing at A. She wishes to minimise the total distance travelled.

	A	B	C	D	E	F	G	H
A	-	47	84	382	120	172	299	144
B	47	-	121	402	155	193	319	165
C	84	121	-	456	200	246	373	218
D	382	402	456	-	413	220	155	289
E	120	155	200	413	-	204	286	131
F	172	193	246	220	204	-	144	70
G	299	319	373	155	286	144	-	160
H	144	165	218	289	131	70	160	-

- Find a minimum spanning tree for this network.
- Hence find an upper bound for this problem.
- Use shortcuts to reduce this upper bound to a value below 1300 miles.

Selecting shortcuts: try the shortcuts BD, CB + AF + FD and CD for question 4. Which gives the best upper bound? (Aim to make the upper bound as ... as possible.)

5C Using a Minimum Spanning Tree Method to Find a Lower Bound

Using a minimum spanning tree method to find a lower bound for the classical problem:

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Residual minimum spanning tree (RMST):

Lower bound for the practical problem:

1. The table of least distances for a network is shown.

	A	B	C	D	E
A	-	11	13	19	31
B	11	-	24	8	22
C	13	24	-	32	18
D	19	8	32	-	14
E	31	22	18	14	-

a) By deleting vertex A, find a lower bound to the travelling salesman problem for this network.

	B	C	D	E
B	-	24	8	22
C	24	-	32	18
D	8	32	-	14
E	22	18	14	-

- b) State whether this lower bound represents an optimal solution. Give a reason for your answer.

2. The table below is the table of distances from question 4 in 5B.

	A	B	C	D	E	F	G	H
A	-	47	84	382	120	172	299	144
B	47	-	121	402	155	193	319	165
C	84	121	-	456	200	246	373	218
D	382	402	456	-	413	220	155	289
E	120	155	200	413	-	204	286	131
F	172	193	246	220	204	-	144	70
G	299	319	373	155	286	144	-	160
H	144	165	218	289	131	70	160	-

- a) By deleting vertices A then G, find two lower bounds to the travelling salesman problem for the network above.

Deleting A:

	B	C	D	E	F	G	H
B	-	121	402	155	193	319	165
C	121	-	456	200	246	373	218
D	402	456	-	413	220	155	289
E	155	200	413	-	204	286	131
F	193	246	220	204	-	144	70
G	319	373	155	286	144	-	160
H	165	218	289	131	70	160	-

Deleting G:

	A	B	C	D	E	F	H
A	-	47	84	382	120	172	144
B	47	-	121	402	155	193	165
C	84	121	-	456	200	246	218
D	382	402	456	-	413	220	289
E	120	155	200	413	-	204	131
F	172	193	246	220	204	-	70
H	144	165	218	289	131	70	-

b) Select the better lower bound of the two found in part a, giving a reason for your answer.

c) Taking your answer to b and using the better upper bound, 1237 miles, found in 5B, write down the smallest interval that must contain the length of the optimal route.

Choosing inequalities ($<$ or $\leq$):

5D Using the Nearest Neighbour Algorithm to Find an Upper Bound

The nearest neighbour algorithm:

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Difference between the nearest neighbour algorithm and Prim's algorithm:

1. Apply the nearest neighbour algorithm, using A then B then C as starting vertices, to find an upper bound to the travelling salesman problem.

	A	B	C	D	E
A	-	8	7	29	13
B	8	-	9	24	14
C	7	9	-	23	6
D	29	24	23	-	21
E	13	14	6	21	-

2. This is the table of distances from question 2 in 5C. We found that the optimal solution lay in the interval $971 \text{ miles} < \text{optimal solution} \leq 1237 \text{ miles}$.

	A	B	C	D	E	F	G	H
A	-	47	84	382	120	172	299	144
B	47	-	121	402	155	193	319	165
C	84	121	-	456	200	246	373	218
D	382	402	456	-	413	220	155	289
E	120	155	200	413	-	204	286	131
F	172	193	246	220	204	-	144	70
G	299	319	373	155	286	144	-	160
H	144	165	218	289	131	70	160	-

- a) Use the nearest neighbour algorithm, using A then B as starting vertices, to find upper bounds for the travelling salesman problem.
- b) Review the interval containing the optimal solution and amend it if necessary, giving a reason for your answer.