

2A Modelling with Graphs

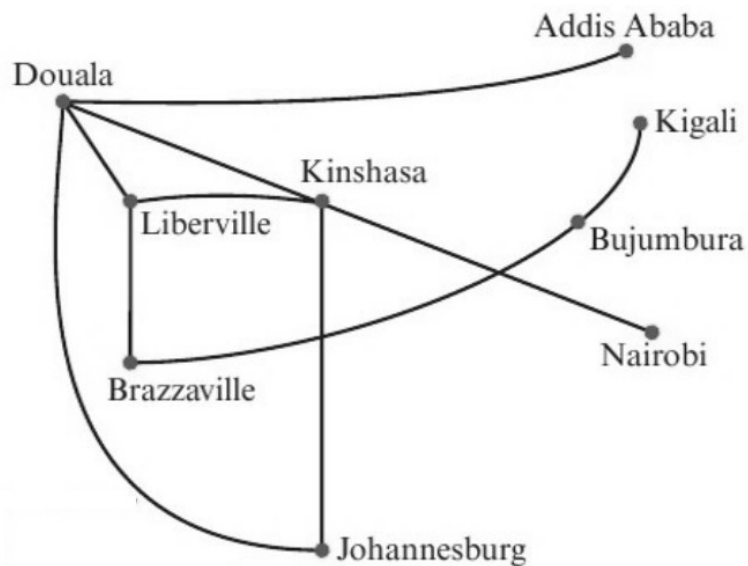
Graph:

Vertex (node):

Edge (arc):

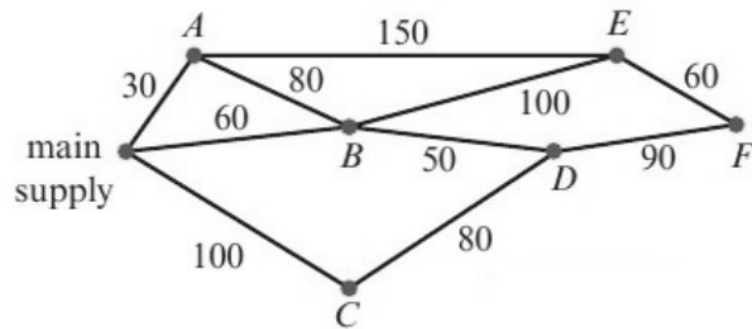
Weighted graph (network):

1. This graph shows the routes flown by an airline.



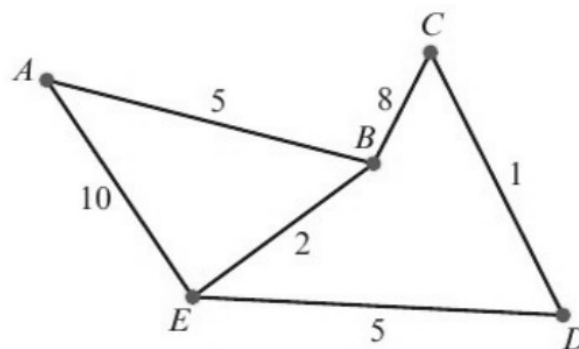
- Explain why this is a graph.
- Describe what is represented by i the vertices ii the edges.
- Describe two possible flight routes from Johannesburg to Liberville.

2. This network shows the lengths of pipes running between water standpipes on a camp site. The lengths on the edges are given in metres.



- a) Write down the length of the pipe connecting standpipe B to standpipe D.
- b) Write a list of the standpipes that are directly connected to standpipe E.

3. The network shows the times taken, in minutes, by a car to travel along some sections of road.



Calculate the minimum time needed to travel from A to D and give the route taken.

Route	Time needed (mins)

2B Graph Theory

Vertex set and edge set:

Subgraph:

Degree (valency / order) of a vertex:

Even degree / odd degree:

Walk:

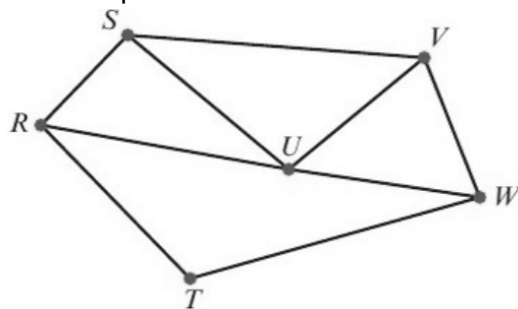
Path:

Trail:

Cycle:

Hamiltonian cycle:

1. For the graph below, give an example of:



a) a walk

b) a path

c) a trail

d) a cycle

e) a Hamiltonian cycle.

Connected vertices / connected graph:

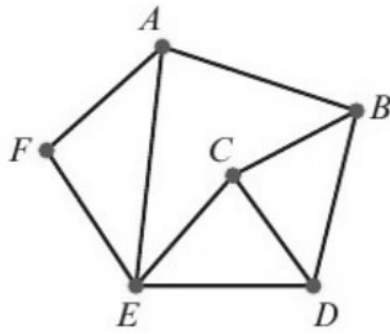
Loop:

Simple graph:

Directed graph (digraph):

Euler's handshaking lemma:

2. For the graph below:



a) Find the sum of the valencies of the vertices in this graph.

Vertex	Valency
A	
B	
C	
D	
E	
F	
Total	

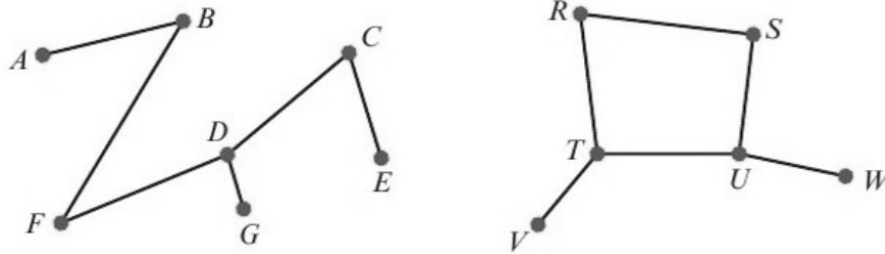
b) Verify that your answer to part a is twice the number of edges in the graph.

3. A graph has 5 nodes and 8 edges. The valencies of the nodes are x , $x - 1$, $x + 1$, $2x - 1$ and $x - 1$. Find the value of x .

2C Special Types of Graph

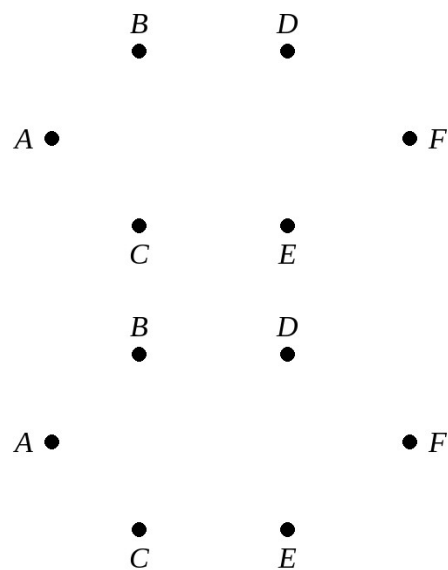
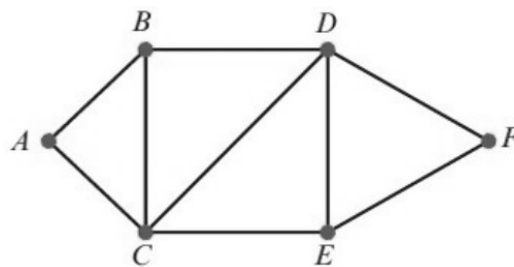
Tree:

1. Explain why the first graph is a tree but the second graph is not.



Spanning tree:

2. Draw two possible spanning trees for this graph.



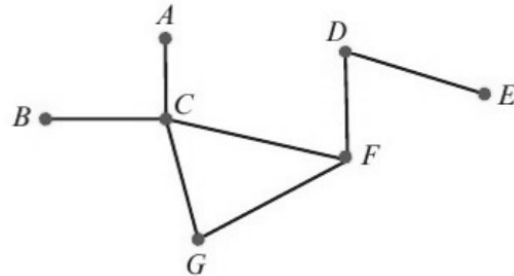
Complete graph:

Notation K_n :

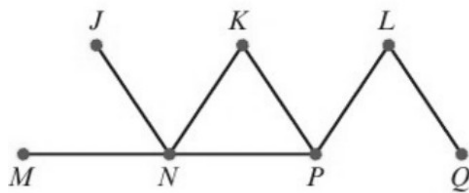
3. Draw the complete graphs K_3 , K_4 and K_6 .

Isomorphic graphs:

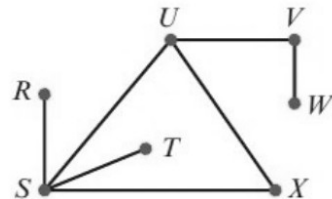
4. Show that this graph



is isomorphic to each of these graphs by pairing equivalent vertices.



and

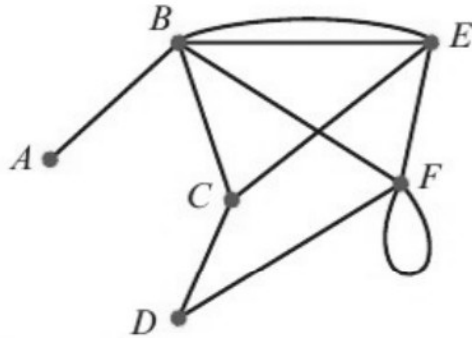


First graph	A	B	C	D	E	F	G
Second graph							
Third graph							

2D Representing Graphs and Networks Using Matrices

Adjacency matrix:

1. Use an adjacency matrix to represent this graph.

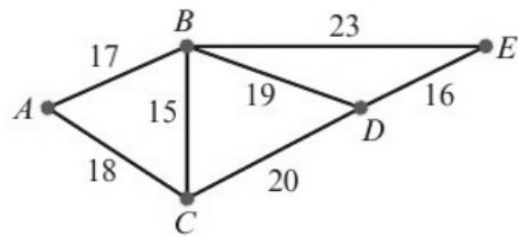


	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						

What does the 2 in row B, column E mean? What about the 2 in row F, column F?

Distance matrix:

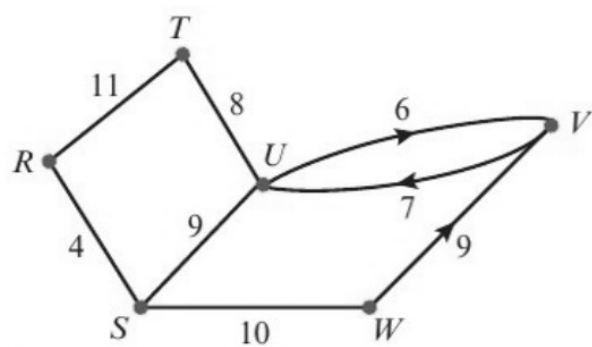
2. Use a distance matrix to represent this network.



	A	B	C	D	E
A					
B					
C					
D					
E					

What do you notice about the matrix?

3. Use a distance matrix to represent this directed network.



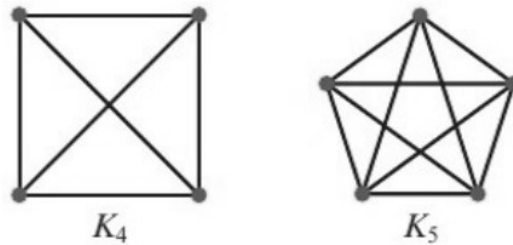
	R	S	T	U	V	W
R						
S						
T						
U						
V						
W						

What do you notice about the matrix this time?

2E The Planarity Algorithm

(A level only)

Planar graph:

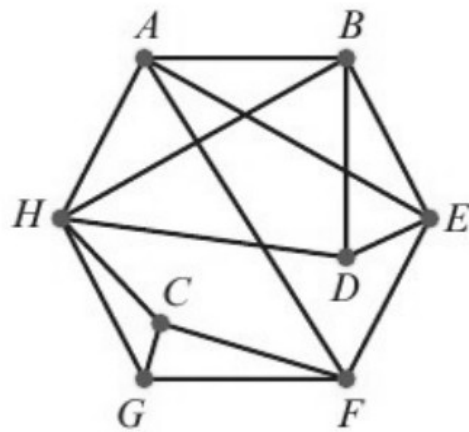


Which of K_4 and K_5 is planar? Redraw it with no edges crossing.

The planarity algorithm (steps):

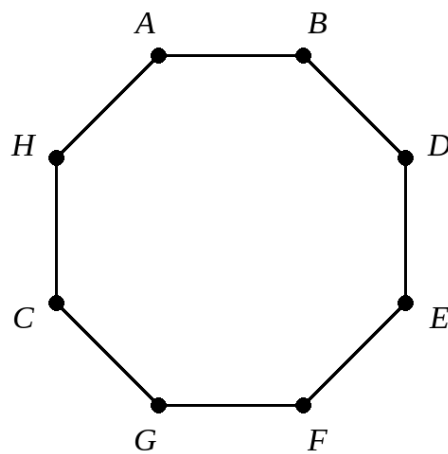
- 1 Identify a Hamiltonian cycle.
- 2 Draw a polygon ...
- 3 Draw edges inside the polygon ...
- 4 Make a list ...
- 5 Choose any unlabelled edge ...
- 6 Look at any unlabelled edges that cross the edge(s) just labelled:

1. Use the planarity algorithm to determine whether this graph is planar.



Step 1:

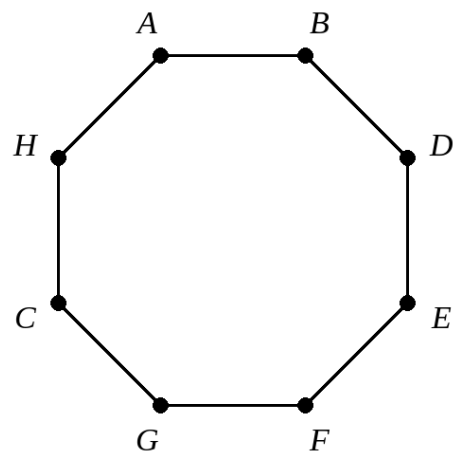
Steps 2, 3:



Step 4:

Steps 5 and 6:

Draw the graph with no edges crossing:



2. An electronics company attaches components A, B, C, D, E and F to a circuit board. These components are to be connected by wires that are not allowed to cross. The table gives information about the connections.

Component	Connected to
<i>A</i>	<i>B, D, E, F</i>
<i>B</i>	<i>C, D, E, F</i>
<i>C</i>	<i>D, E</i>
<i>D</i>	<i>E, F</i>
<i>E</i>	<i>F</i>

Using the Hamiltonian cycle ABCDEFA, apply the planarity algorithm to determine whether the connections can be made without any wires crossing.

