

7B The Simplex Method

In chapter 6 we used graphs to help visualise where each vertex would be with two variables. This is much harder with three variables and dang near impossible with four or more. The simplex method:

- Starts at one vertex
- Moves between vertices in sequence, increasing the objective function and one variable at a time, until it is optimal
- Slack variables are key

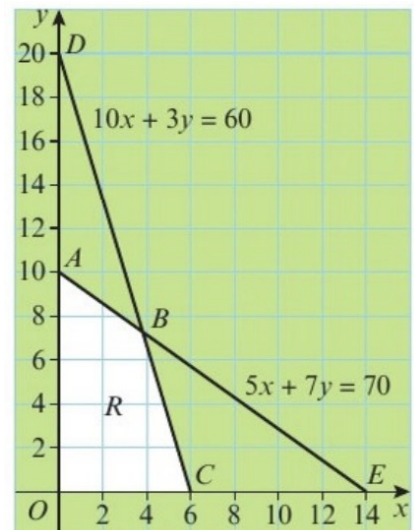
1.

- a) Explain the significance of the slack variables in the graphical representation of the linear programming problem below.

Maximise

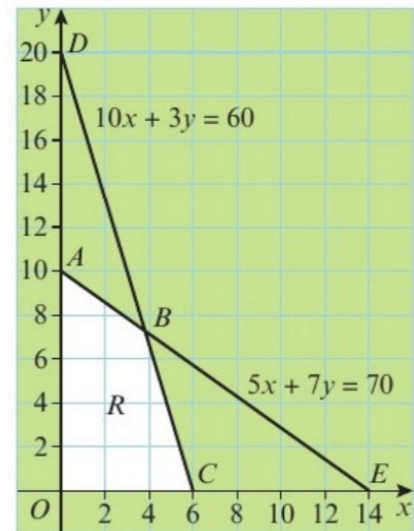
Subject to:

The feasible region, R, is shown.



At each vertex, two of the four vertices are zero.

- b) Explain, in detail, how the algebraic simplex algorithm is used to solve the above problem, relating each stage to the given graph.



2. Show, in detail, the application of the algebraic simplex algorithm to the linear programming problem below.

Maximise

Subject to :

3. Solve the linear programming problem in Example 1 using simplex tableaux (plural of tableau).

Maximise

Subject to:

Basic variable	x	y	r	s	Value

Basic variable	x	y	r	s	Value

Basic variable	x	y	r	s	Value

Basic variable	x	y	r	s	Value

Basic variable	x	y	r	s	Value

Basic variable	x	y	r	s	Value

Basic variable	x	y	r	s	Value

Always start at $x=0$, $y=0$ and follow the steps:

- 1** Draw the tableaux.
You need a basic variable column on the left, one column for each variable (including the slack variables) and a value column. You need one row for each constraint and the bottom row for the objective function.
- 2** Create the initial tableau.
Enter the coefficients of the variables in the appropriate column and row.
- 3** Look along the objective row for the most negative entry: this indicates the pivot column.
- 4** Calculate the θ values, for each of the constraint rows, where
 $\theta = (\text{the term in the value column}) \div (\text{the term in the pivot column})$
- 5** Select the row with the smallest, positive θ value to become the pivot row.
- 6** The element in the pivot row and pivot column is the pivot.
- 7** Divide the row found in step **5** by the pivot, and change the basic variable at the start of the row to the variable at the top of the pivot column. This is now the pivot row.
- 8** Use the pivot row to eliminate the pivot's variable from the other rows.
This means that the pivot column now contains one 1 and zeros.
- 9** Repeat steps **3** to **8** until there are no more negative numbers in the objective row.
- 10** The tableau is now optimal and the non-zero values can be read off using the basic variable column and value column.

4. Minimise

Subject to:

Basic variable	x	y	r	s	Value

Basic variable	x	y	r	s	Value

Basic variable	x	y	r	s	Value

- 5.
- a) Use simplex tableaux to solve the linear programming problem below (from Example 2).

Maximise

Subject to:

Basic variable						Value

Basic variable						Value

Basic variable						Value

Basic variable						Value

b) Verify your solution using the original problem.

6.

a) Use the simplex tableau method to solve the following linear programming problem.

Maximise

Subject to:

Basic variable							Value	

Basic variable							Value	

Basic variable							Value	

Basic variable							Value	

Basic variable							Value	

Basic variable							Value	

b) State the values of the objective function and every variable at the optimal point.

c) Write down the equations given by your optimal tableau.

d) Use the profit equation you wrote down in part c to explain why your final tableau is optimal.

