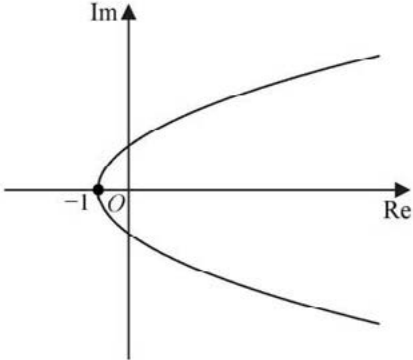


Paper 4A: Further Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1(i)	$602 = 3 \times 161 + 119$	M1	1.1b
	$161 = 119 + 42, 119 = 2 \times 42 + 35$	M1	1.1b
	$42 = 35 + 7, 35 = 5 \times 7, \text{ hcf} = 7$	A1	1.1b
		(3)	
(ii)	Number of codes under old system = $5 \times 4 \times 4 \times 3 \times 2$ (= 480)	B1	3.1b
	Number of codes under new system = $4 \times 3 \times 7 \times 6 \times 5$ (= 2520)	B1	3.1b
	Subtracts first answer from second	M1	1.1b
	Increase in number of codes is 2040	A1	1.1b
		(4)	
(7 marks)			
Notes:			
(i)			
M1: Attempts Euclid's algorithm – (there may be an arithmetic slip finding 119)			
M1: Uses Euclid's algorithm a further two times with 161 and “their 119” and then with “their 119” and “their 42”			
A1: This should be accurate with all the steps shown			
(ii)			
B1: Correctly interprets the problem and uses the five odd digits and four even digits to form a correct product			
B1: Interprets the new situation using the four even digits, then the seven digits which have not been used, to form a correct product			
M1: Subtracts one answer from the other			
A1: Correct answer			

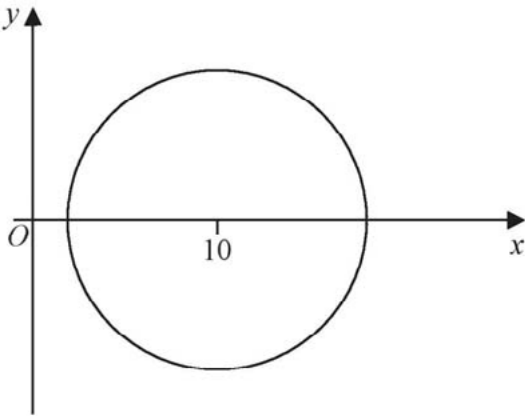
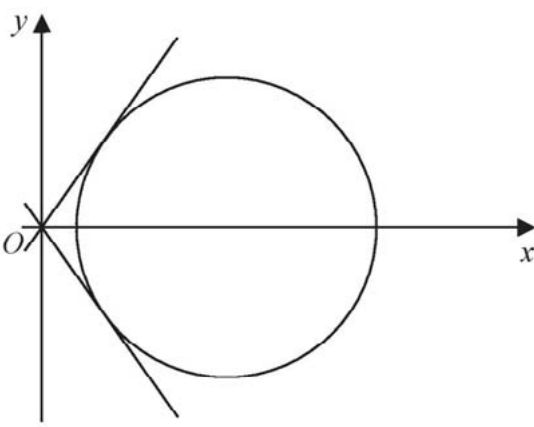
Question	Scheme	Marks	AOs
2(a)	Let $z = x + i$	M1	2.1
	$w = (x+i)^2 = (x^2 - 1) + 2xi$	A1	1.1b
	Let $w = u + iv$, then $u = (x^2 - 1)$ and $v = 2x$	M1	2.1
	$\Rightarrow v^2 = 4(u+1)$, which therefore represents a parabola	A1ft	2.2a
		(4)	
(b)	 M1: Sketches a parabola with symmetry about the real axis A1: Accurate sketch	M1	1.1b
		A1	1.1b
		(2)	
(6 marks)			
Notes:			
(a) M1: Translates the information that $\text{Im}(z) = 1$ into a cartesian form; e.g. $z = x + i$ A1: Obtains a correct expression for w M1: Separates the real and imaginary parts and equates to u and v respectively A1ft: Obtains a quadratic equation and states that their quadratic equation represents a parabola			
(b) M1: Sketches a parabola with symmetry about the real axis A1: Accurate sketch			

Question	Scheme	Marks	AOs
3(a)	Finds the characteristic equation $(2-\lambda)^2(4-\lambda)-(4-\lambda)=0$	M1	2.1
	So $(4-\lambda)(\lambda^2-4\lambda+3)=0$ so $\lambda=4^*$	A1*	2.2a
	Solves quadratic equation to give	M1	1.1b
	$\lambda=1$ and $\lambda=3$	A1	1.1b
		(4)	
(b)	Uses a correct method to find an eigenvector	M1	1.1b
	Obtains a vector parallel to one of $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$	A1	1.1b
	Obtains two correct vectors	A1	1.1b
	Obtains all three correct vectors	A1	1.1b
		(4)	
(c)	Uses their three vectors to form a matrix	M1	1.2
	$\begin{pmatrix} 0 & 1 & 3 \\ 0 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ or other correct answer with columns in a different order	A1	1.1b
		(2)	
(10 marks)			
Notes:			
(a) M1: Attempts to find the characteristic equation (there may be one slip) A1*: Deduces that $\lambda=4$ is a solution by the method shown or by checking that $\lambda=4$ satisfies the characteristic equation M1: Solves their quadratic equation A1: Obtains the two correct answers as shown above			
(b) M1: Uses a correct method to find an eigenvector A1: Obtains one correct vector (may be a multiple of the given vectors) A1: Obtains two correct vectors (may be multiples of the given vectors) A1: Obtains all three correct vectors (may be multiples of the given vectors)			
(c) M1: Forms a matrix with their vectors as columns A1: $\begin{pmatrix} 0 & 1 & 3 \\ 0 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 & 3 \\ 1 & 0 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ or $\begin{pmatrix} 3 & 1 & 0 \\ -3 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ or other correct alternative			

Question	Scheme	Marks	AOs
4(i)	If we assume $ab = ba$; as $a^2b = ba$ then $ab = a^2b$	M1	2.1
	So $a^{-1}abb^{-1} = a^{-1}a^2bb^{-1}$	M1	2.1
	So $e = a$	A1	2.2a
	But this is a contradiction, as the elements e and a are distinct so $ab \neq ba$	A1	2.4
		(4)	
(ii)(a)	2 has order 4 and 4 has order 2	M1	1.1b
	7, 8 and 13 have order 4	A1	1.1b
	11 and 14 have order 2 and 1 has order 1	A1	1.1b
		(3)	
(ii)(b)	Finds the subgroup $\{1, 2, 4, 8\}$ or the subgroup $\{1, 7, 4, 13\}$	M1	1.1b
	Finds both and refers to them as cyclic groups, or gives generator 2 and generator 7	A1	2.4
	Finds $\{1, 4, 11, 14\}$	B1	2.2a
	States each element has order 2 or refers to it as Klein Group	B1	2.5
		(4)	
(ii)(c)	J has an element of order 8, (H does not) or J is a cyclic group (H is not) or other valid reason	M1	2.4
	They are not isomorphic	A1	2.2a
		(2)	
(13 marks)			

Question 4 notes:	
(i)	
M1:	Proof begins with assumption that $ab = ba$ and deduces that this implies $ab = a^2b$
M1:	A correct proof with working shown follows, and may be done in two stages
A1:	Concludes that assumption implies that $e = a$
A1:	Explains clearly that this is a contradiction, as the elements e and a are distinct so $ab \neq ba$
(ii)(a)	
M1:	Obtains two correct orders (usually the two in the scheme)
A1:	Finds another three correctly
A1:	Finds the final three so that all eight are correct
(ii)(b)	
M1:	Finds one of the cyclic subgroups
A1:	Finds both subgroups and explains that they are cyclic groups, or gives generators 2 and 7
B1:	Finds the non cyclic group
B1:	Uses correct terms that each element has order 2 or refers to it as Klein Group
(ii)(c)	
M1:	Clearly explains how J differs from H
A1:	Correct deduction

Question	Scheme	Marks	AOs
5(a)	$\frac{dy}{dx} = -\sinh 2x$	B1	2.1
	So $S = \int \sqrt{1 + \sinh^2 2x} dx$	M1	2.1
	$\therefore S = \int \cosh 2x dx$	A1	1.1b
	$= \left[\frac{1}{2} \sinh 2x \right]_{-\ln a}^{\ln a} \text{ or } \left[\sinh 2x \right]_0^{\ln a}$	M1	2.1
	$= \sinh 2 \ln a = \frac{1}{2} [e^{2 \ln a} - e^{-2 \ln a}] = \frac{1}{2} \left(a^2 - \frac{1}{a^2} \right) \quad (\text{so } k = \frac{1}{2})$	A1	1.1b
		(5)	
(b)	$\frac{1}{2} \left(a^2 - \frac{1}{a^2} \right) = 2 \text{ so } a^4 - 4a^2 - 1 = 0$	M1	1.1b
	$a^2 = 2 + \sqrt{5} \quad (\text{and } a = 2.06 \text{ (approx.)})$	M1	1.1b
	When $x = \ln a, y = 0$ so $A = \frac{1}{2} \cosh(2 \ln a)$	M1	3.4
	Height $= A - 0.5 = \text{awrt } 0.62\text{m}$	A1	1.1b
		(4)	
(c)	The width of the base $= 2 \ln a = 1.4\text{m}$	B1	3.4
		(1)	
(d)	A parabola of the form $y = 0.62 - 1.19 x^2$, or other symmetric curve with its equation e.g. $0.62 \cos(2.2x)$	M1A1	3.3 3.3
		(2)	
(12 marks)			
Notes:			
(a)			
B1: Starts explanation by finding the correct derivative			
M1: Uses their derivative in the formula for arc length			
A1: Uses suitable identity to simplify the integrand and to obtain the expression in scheme			
M1: Integrates and uses appropriate limits to find the required arc length			
A1: Uses the definition of sinh to complete the proof and identifies the value for k			
(b)			
M1: Uses the formula obtained from the model and the length of the arch to create a quartic equation			
M1: Continues to use this model to obtain a quadratic and to obtain values for a			
M1: Attempts to find a value for A in order to find h			
A1: Finds a value for the height correct to 2sf (or accept exact answer)			
(c)			
B1: Finds width to 2 sf i.e. 1.4m			
(d)			
M1: Chooses or describes an even function with maximum point on the y axis			
A1: Gives suitable equation passing through $(0, 0.62)$ and $(0.7, 0)$ and $(-0.7, 0)$			

Question	Scheme	Marks	AOs
6(a)	$(x+6)^2 + y^2 = 4[(x-6) + y^2]$	M1	2.1
	$x^2 + y^2 - 20x + 36 = 0$ which is the equation of a circle*	A1*	2.2a
		(2)	
(b)		M1	1.1b
		A1	1.1b
		(2)	
(c)	Let $a = c + id$ and $a^* = c - id$ then $(c + id)(x - iy) + (c - id)(x + iy) = 0$	M1	3.1a
	So $y = -\frac{c}{d}x$	A1	1.1b
	 The gradients of the tangents (from geometry) are $\pm \frac{4}{3}$	B1	3.1a
	So $-\frac{c}{d} = \pm \frac{4}{3}$ and $\frac{d}{c} = \mp \frac{3}{4}$	M1	3.1a
	So $\tan \theta = \pm \frac{3}{4}$	A1	1.1b
		(5)	

Question 6 notes:	
(a)	
M1:	Obtains an equation in terms of x and y using the given information
A1*:	Expands and simplifies the algebra, collecting terms and obtains a circle equation correctly, deducing that this is a circle
(b)	
M1:	Draws a circle with centre at $(10, 0)$
A1:	(Radius is 8) so circle does not cross the y axis
(c)	
M1:	Attempts to convert line equation into a cartesian form
A1:	Obtains a simplified line equation
B1:	Uses geometry to deduce the gradients of the tangents
M1:	Understands the connection between $\arg a$ and the gradient of the tangents and uses this connection
A1:	Correct answers

Question	Scheme	Marks	AOs
7(a)	$I_n = \int_0^{\frac{\pi}{2}} \sin x \sin^{n-1} x \, dx$	M1	2.1
	$= \left[-\cos x \sin^{n-1} x \right]_0^{\frac{\pi}{2}} - (-) \int_0^{\frac{\pi}{2}} \cos^2 x (n-1) \sin^{n-2} x \, dx$	A1	1.1b
	Obtains $= 0 - (-) \int_0^{\frac{\pi}{2}} (1 - \sin^2 x) (n-1) \sin^{n-2} x \, dx$	M1	1.1b
	So $I_n = (n-1)I_{n-2} - (n-1)I_n$ and hence $nI_n = (n-1)I_{n-2}$ *	A1*	2.1
		(4)	
(b)	uses $I_n = \frac{(n-1)}{n} I_{n-2}$ to give $I_{10} = \frac{9}{10} I_8$ or $I_2 = \frac{1}{2} I_0$	M1	3.1b
	So $I_{10} = \frac{9 \times 7 \times 5 \times 3 \times 1}{10 \times 8 \times 6 \times 4 \times 2} I_0$ x	M1	2.1
	$I_0 = \frac{\pi}{2}$	B1	1.1b
	Required area is $2(I_2 - I_{10}) =$ or $8 \times \frac{1}{4} (I_2 - I_{10}) =$	M1	3.1b
	$= 2 \left(\frac{\pi}{4} - \frac{63\pi}{512} \right) = \frac{65\pi}{256} \text{ m}^2$	A1	1.1b
		(5)	
(9 marks)			
Notes:			
(a) M1: Splits the integrand into the product shown and begins process of integration by parts (there may be sign errors) A1: Correct work M1: Uses limits on the first term and expresses $\cos^2$ term in terms of $\sin^2$ A1*: Completes the proof collecting I_n terms correctly with all stages shown			
(b) M1: Attempts to find I_{10} and/or I_2 M1: Finds I_{10} in terms of I_0 B1: Finds I_0 correctly M1: States the expression needed to find the required area A1: Completes the calculation to give this exact answer			

Question	Scheme	Marks	AOs
8(a)	$u_1 = 1$ as there is only one way to go up one step	B1	2.4
	$u_2 = 2$ as there are two ways: one step then one step or two steps	B1	2.4
	If first move is one step then can climb the other $(n-1)$ steps in u_{n-1} ways. If first move is two steps can climb the other $(n-2)$ steps in u_{n-2} ways so $u_n = u_{n-1} + u_{n-2}$	B1	2.4
		(3)	
(b)	Sequence begins 1, 2, 3, 5, 8, 13, 21, 34,... so 34 ways of climbing 8 steps	B1	1.1b
		(1)	
(c)	To find general term use $u_n = u_{n-1} + u_{n-2}$ gives $\lambda^2 = \lambda + 1$	M1	2.1
	This has roots $\frac{1 \pm \sqrt{5}}{2}$	A1	1.1b
	So general form is $A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$	M1	2.2a
	Uses initial conditions to find A and B reaching two equations in A and B	M1	1.1b
	Obtains $A = \left(\frac{1+\sqrt{5}}{2\sqrt{5}}\right)$ and $B = -\left(\frac{1-\sqrt{5}}{2\sqrt{5}}\right)$ and so when $n = 400$ obtains $\frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2}\right)^{401} - \left(\frac{1-\sqrt{5}}{2}\right)^{401} \right]^*$	A1*	1.1b
		(5)	
(9 marks)			
Notes:			
(a)			
B1: Need to see explanation for $u_1 = 1$			
B1: Need to see explanation for $u_2 = 2$ with the two ways spelled out			
B1: Need to see the first move can be one step or can be two steps and clear explanation of the iterative expression as in the scheme			
(b)			
B1: The answer is enough for this mark			
(c)			
M1: Obtains this characteristic equation			
A1: Solves quadratic – giving exact answers			
M1: Obtains a general form			
M1: Use initial conditions to obtains two equations which should be $A(1 + \sqrt{5}) + B(1 - \sqrt{5}) = 2$ o.e. and $A(3 + \sqrt{5}) + B(3 - \sqrt{5}) = 4$ but allow slips here			
A1*: Must see exact correct values for A and B and conclusion given for $n = 400$			