QQQ – Core Pure Yr1 - Chapter 2 – Complex Numbers & Argand Diagrams

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 16 = Bronze)

1.

Given that $z = 4\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$ and $w = 1 - \mathrm{i}\sqrt{3}$, find

(*a*) $\left|\frac{z}{w}\right|$,

(3)

(*b*) $\arg\left(\frac{z}{w}\right)$, in radians as a multiple of π.

(3)

(*c*) On an Argand diagram, plot points A, B, C and D representing the complex numbers z, w, $\left(\frac{z}{w}\right)$ and 4, respectively.

(3)

(*d*) Show that $\angle AOC = \angle DOB$.

(2)

(*e*) Find the area of triangle AOC.

(2)

2.

(a) Shade on an Argand diagram the set of points

$$\{z \in \mathbb{C} : |z - 1 - \mathrm{i}| \leqslant 3\} \cap \left\{z \in \mathbb{C} : \frac{\pi}{4} \leqslant \arg(z - 2) \leqslant \frac{3\pi}{4}\right\}$$

(5)

The complex number w satisfies

$$|w - 1 - \mathrm{i}| = 3 \text{ and } \arg(w - 2) = \frac{\pi}{4}$$

(b) Find, in simplest form, the exact value of $|w|^2$

(4)