QQQ – Core Pure Yr1 - Chapter 2 – Complex Numbers & Argand Diagrams

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 16 = Bronze)

1.

Given that $z = 4\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$ and $w = 1 - \mathrm{i}\sqrt{3}$, find

(*a*) $\left|\frac{z}{w}\right|$,

(3)

(*b*) $\arg\left(\frac{z}{w}\right)$, in radians as a multiple of π.

(3)

(*c*) On an Argand diagram, plot points A, B, C and D representing the complex numbers z, w, $\left(\frac{z}{w}\right)$ and 4, respectively.

(3)

(*d*) Show that $\angle AOC = \angle DOB$.

(2)

(*e*) Find the area of triangle AOC.

(2)

2.

(a) Shade on an Argand diagram the set of points

$$\{z \in \mathbb{C} : |z - 1 - \mathrm{i}| \leqslant 3\} \cap \left\{z \in \mathbb{C} : \frac{\pi}{4} \leqslant \arg(z - 2) \leqslant \frac{3\pi}{4}\right\}$$

(5)

The complex number w satisfies

$$|w - 1 - \mathrm{i}| = 3 \text{ and } \arg(w - 2) = \frac{\pi}{4}$$

(b) Find, in simplest form, the exact value of $|w|^2$

(4)

1.

8 (a) $\left|\frac{z}{w}\right| = \frac{|z|}{|w|}; = \frac{4}{2} = 2$ M1M1A1 (3)

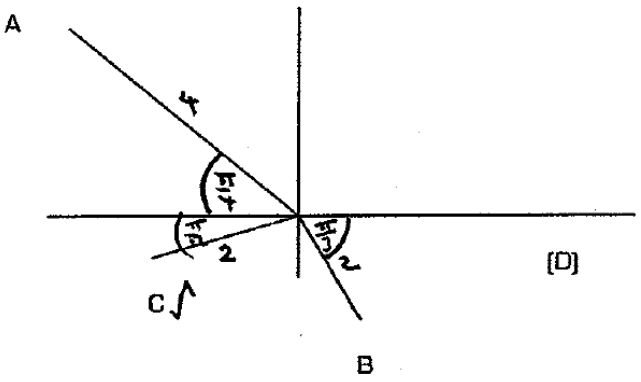
[M1 for correct modulus, M1 division of moduli]

(b) $\arg\left(\frac{z}{w}\right) = \arg z - \arg w$ M1

$= \frac{3\pi}{4} - \left(-\frac{\pi}{3}\right) = \left(\frac{13\pi}{12}\right); \quad -\frac{11\pi}{12}$ M1A1 (3)

[Second M1 for one correct arg]

8 (c)



(3)

For A B1
For B B1
For C B1√

(d) $\angle DOB = \frac{\pi}{3}$ or $60°$ B1

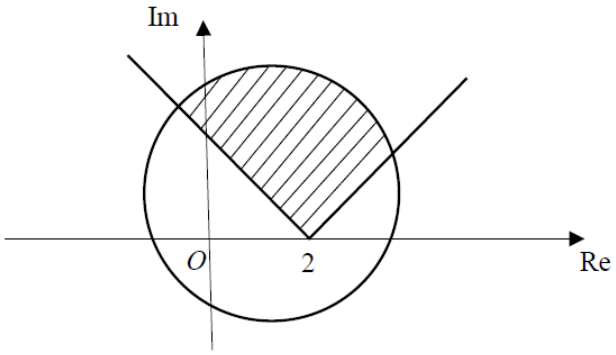
Correct method for $\angle AOC$ M1

$\angle AOC = \frac{\pi}{4} + \frac{\pi}{12} = \frac{\pi}{3}$ (cso) A1 (3)

(e) Area Δ AOC $= \frac{1}{2}$ x"4"x"2"x sin"$\frac{\pi}{3}$" $= 2\sqrt{3}$ (3.46 or better) M1A1 (2)

[14]

2.

3(a)		M1	1.1b
		M1	1.1b
		A1	2.2a
		M1	3.1a
		A1	1.1b
		(5)	
(b)	$(x-1)^2+(y-1)^2=9,\ \ y=x-2\Rightarrow x=...,$ or $y=...$	M1	3.1a
	$x=2+\frac{\sqrt{14}}{2}, y=\frac{\sqrt{14}}{2}$	A1	1.1b
	$\lvert w\rvert^2=\left(2+\frac{\sqrt{14}}{2}\right)^2+\left(\frac{\sqrt{14}}{2}\right)^2$	M1	1.1b
	$=11+2\sqrt{14}$	A1	1.1b
		(4)	

(9 marks)