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**1.** Find

**

writing each term in simplest form.

**(4)**

**(Total for Question 1 is 4 marks)**

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**2. In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

f(*x*) = 4*x*3 + 5*x*2 – 10*x* + 4*a x* ∈ ℝ

where *a* is a positive constant.

Given (*x* – *a*) is a factor of f(*x*),

(*a*) show that

*a*(4*a*2 + 5*a* – 6) = 0

**(2)**

(*b*)Hence

(i) find the value of *a*

(ii) use algebra to find the exact solutions of the equation

f(*x*) = 3

**(4)**

**(Total for Question 2 is 6 marks)**

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**3.** Relative to a fixed origin *O*

• the point *A* has position vector 5**i** + 3**j** + 2**k**

• the point *B* has position vector 2**i** + 4**j** + *a***k**

where *a* is a positive integer.

(*a*) Show that = 

**(1)**

(*b*)Find the smallest value of *a* for which



**(2)**

**(Total for Question 3 is 3 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**4. In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The curve *C* has equation *y* = f(*x*) where *x* ∈ ℝ

Given that

• f ′(*x*) = 2*x* + cos *x*

• the curve has a stationary point with *x* coordinate *α*

*• α* is small

(*a*) use the small angle approximation for cos *x* to estimate the value of *α* to

3 decimal places.

**(3)**

The point *P*(0, 3) lies on *C*

(*b*)Find the equation of the tangent to the curve at *P*, giving your answer in the

form *y* = *mx* + *c*, where *m* and *c* are constants to be found.

**(2)**

**(Total for Question 4 is 5 marks)**

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**5.** A continuous curve has equation *y* = f(*x*).

The table shows corresponding values of *x* and *y* for this curve, where *a* and *b*

are constants.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *x* | 3 | 3.2 | 3.4 | 3.6 | 3.8 | 4 |
| *y* | *a* | 16.8 | *b* | 20.2 | 18.7 | 13.5 |

The trapezium rule is used, with all the *y* values in the table, to find an approximate area

under the curve between *x* = 3 and *x* = 4

Given that this area is 17.59

(*a*) show that *a* + 2*b* = 51

**(3)**

Given also that the sum of all the *y* values in the table is 97.2

(*b*)find the value of *a* and the value of *b*

**(3)**

**(Total for Question 5 is 6 marks)**

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**6.** *a* = log2 *x b* = log2(*x* + 8)

Express in terms of *a* and/or *b*

(*a*) log2 

**(1)**

(*b*)log2(*x*2 + 8*x*)

**(2)**

(*c*)log2 

Give your answer in simplest form.

**(3)**

**(Total for Question 6 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**7.** The function f is defined by

f(*x*) = 3 +  *x* ∈ ℝ *x* > 2

(*a*) State the range of f

**(1)**

(*b*)Find f –1

**(3)**

The function g is defined by

g(*x*) =  *x* ∈ ℝ *x* ≠ 3

(*c*)Find gf(6)

**(2)**

(*d*)Find the exact value of the constant *a* for which

f(*a*2 + 2) = g(*a*)

**(2)**

**(Total for Question 7 is 8 marks)**

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**8.**

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Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles *ABO* and *GFO* joined to a sector *OCDEO*

of a circle, centre *O*, where

• angle *COE* = 2.3 radians

• arc length *CDE* = 27.6 m

• *AOG* is a straight line of length 15 m

(*a*) Show that *OC* = 12 m.

**(2)**

(*b*)Show that the size of angle *AOB* is 0.421 radians correct to 3 decimal places.

**(2)**

Given that the total length of the front of the stage, *BCDEF*, is 35 m,

(*c*)find the total area of the stage, giving your answer to the nearest square metre.

**(6)**

**(Total for Question 8 is 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**9.** The first three terms of a geometric sequence are

3*k* + 4 12 – 3*k k* + 16

where *k* is a constant.

(*a*) Show that *k* satisfies the equation

3*k*2 – 62*k* + 40 = 0

**(2)**

Given that the sequence converges,

(*b*)(i) find the value of *k*, giving a reason for your answer,

(ii) find the value of *S*∞

**(5)**

**(Total for Question 9 is 7 marks)**

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**10.** A circle *C* has equation

*x*2 + *y*2 + 6*kx* – 2*ky* + 7 = 0

where *k* is a constant.

(*a*) Find in terms of *k*,

(i) the coordinates of the centre of *C*

(ii) the radius of *C*

**(3)**

The line with equation *y* = 2*x* – 1 intersects *C* at 2 distinct points.

(*b*)Find the range of possible values of *k*.

**(6)**

**(Total for Question 10 is 9 marks)**

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**11.**

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The value, *V* pounds, of a mobile phone, *t* months after it was bought, is modelled by

*V* = *abt*

where *a* and *b* are constants.

Figure 2 shows the linear relationship between log10 *V* and *t*.

The line passes through the points (0, 3) and (10, 2.79) as shown.

Using these points,

(*a*) find the initial value of the phone,

**(2)**

(*b*)find a complete equation for *V* in terms of *t*, giving the exact value of *a* and giving

the value of *b* to 3 significant figures.

**(3)**

Exactly 2 years after it was bought, the value of the phone was £320

(*c*)Use this information to evaluate the reliability of the model.

**(2)**

**(Total for Question 11 is 7 marks)**

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**12.** *y* = sin *x*

where *x* is measured in radians.

Use differentiation from first principles to show that

= cos *x*

You may

• use without proof the formula for sin(*A* ± *B*)

• assume that as *h* → 0,  and 

**(5)**

**(Total for Question 12 is 5 marks)**

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**13.** On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

• the maximum vertical height of a carriage above the ground is 60 m

• a carriage starts a circuit at a vertical height of 2 m above the ground

• the ground is horizontal

The vertical height, *H* m, of a carriage above the ground, *t* seconds after the carriage

starts the first circuit, is modelled by the equation

*H* = *a* – *b*(*t* – 20)2

where *a* and *b* are positive constants.

(*a*) Find a complete equation for the model.

**(3)**

(*b*)Use the model to determine the height of the carriage above the ground when *t* = 40

**(1)**

In an alternative model, the vertical height, *H* m, of a carriage above the ground,

*t* seconds after the carriage starts the first circuit, is given by

*H* = 29 cos(9*t* + *α*)° + *β* 0 ≤ *α* < 360°

where *α* and *β* are constants.

(*c*)Find a complete equation for the alternative model.

**(2)**

Given that the carriage moves continuously for 2 minutes,

(*d*)give a reason why the alternative model would be more appropriate.

**(1)**

**(Total for Question 13 is 7 marks)**

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**14.** Prove, using algebra, that

(*n* + 1)3 – *n*3

is odd for all *n* ∈ ℕ

**(4)**

**(Total for Question 14 is 4 marks)**

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**15.** A curve has equation *y* = f(*x*), where



(*a*) Show that



where *A* and *B* are constants to be found.

**(5)**

(*b*)Hence show that the *x* coordinates of the turning points of the curve are solutions of

the equation



**(2)**

The equation has two positive roots *α* and *β* where *β* > *α*

A student uses the iteration formula



in an attempt to find approximations for *α* and *β*

Diagram 1 shows a plot of part of the curve with equation *y* =  and part of the

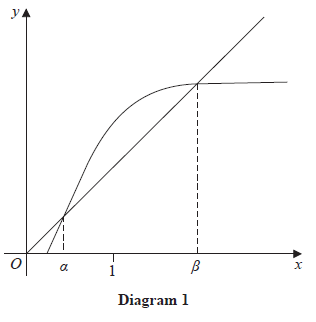
line with equation *y* = *x*

Using Diagram 1 below

(*c*)draw a staircase diagram to show that the iteration formula starting with *x*1 = 1 can

be used to find an approximation for *β*

**(1)**



Use the iteration formula with *x*1 = 1, to find, to 3 decimal places,

(*d*)(i) the value of *x*2

(ii) the value of *β*

**(3)**

Using a suitable interval and a suitable function that should be stated

(*e*)show that *α* = 0.432 to 3 decimal places.

**(2)**

**(Total for Question 15 is 13 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**