**GCE A level Pure Mathematics (9MA0) – Paper 1**

**Pure Mathematics 1**

**October 2021 student-friendly mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.**

**This document is intended for guidance only and may differ significantly from the final mark scheme published in December 2021.**

|  |
| --- |
| **Guidance on the use of codes within this document** |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.  A1 – accuracy mark. This mark is generally given for a correct answer following correct working.  B1 – accuracy mark. This mark is usually given when working and the answer cannot easily be separated.  Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

**Question 1 (Total 3 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  |  | M1 | This mark is given for attempting  to set up an equation in *a* |
|  | M1 | This mark is given for solving *a* |
|  | A1 | This mark is given for finding the correct value of *a* |

**Question 2 (Total 4 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) |  | M1 | This mark is given for writing f(x) in the form  with |
|  | A1 | This mark is given for writing f(x) in the form  with and |
| (b)(i) |  | B1 | This mark is given for correct coordinates for P. |
| (b)(ii) |  | B1 | This mark is given for correct coordinates for Q. |

**Question 3 (Total 6 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) |  | M1 | This mark is given for applying the sequence formula for either *u2* or *u3*. |
|  | M1 | This mark is given for applying the sequence formula for both *u2* and *u3*, and substitute into *u1 + 2u2 + u3 = 0* to form an equation in *k*. |
|  | A1 | This mark is given for fully correct working to the answer |
| (b) |  | M1 | This mark is given for attempting to solve the quadratic equation. |
| *k* = 6 as *k* must be an integer | A1 | This mark is given for choosing *k* = 6 and giving a reason. |
| (c) |  | B1 | This mark is given for calculating the correct value of *u3* |

**Question 4 (Total 9 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | Differentiate  to obtain | M1 | This mark is given for attempting to differentiate  to obtain |
|  | A1 | This mark is given for differentiating f(*x*) correctly. |
|  | M1 | This mark is given for setting and obtain a cubic equation. |
|  | A1 | This mark is given for simplifying to show the equation required. |
| (b)(i) |  | M1 | This mark is given for attempting to calculate *x2* by substituting *x1* = 0.3 |
|  | A1 | This mark is given for finding *x2* |
| (b)(ii) |  | A1 | This mark is given for finding *x4* |
| (c) |  | M1 | This mark is given for substituting and into a suitable function and gets one of the values correct. |
| * there is a change of sign * is continuous * *α* = 0.341 to 3dp | A1 | This mark is given for both calculations correct with correct statements given. |

**Question 5 (Total 6 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) |  | B1 | This mark is given for using a correct and full method to show that the profit in year 3 will be £23 328. |
| (b) |  | M1 | This mark is given for setting up an inequality or an equation to solve this problem. |
|  | M1 | This mark is given for correct working in attempt to solve the inequality or the equation formed above. |
| Year 17 | A1 | This mark is given for interpreting their decimal value to give the correct year number. |
| (c) |  | M1 | This mark is given for using the correct sum formula to find the total profit for 20 years. |
| = (£) 915 000 | A1 | This mark is given for finding a correct amount. |

**Question 6 (Total 5 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) |  | M1 | This mark is given for addingand |
|  | A1 | This mark is given for finding the correct vector. |
| (b) |  | M1 | This mark is given for attempting to “square and add” at least 2 of the 3 sides. |
|  | M1 | This mark is given for applying a correct cosine rule. |
|  | A1 | This mark is given for correct complete solution to find cos *ABC*. |

**Question 7 (Total 9 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a)(i) |  | M1 | This mark is given for attempting to complete the square. |
|  | A1 | This mark is given for finding the correct coordinates of the centre. |
| (a)(ii) |  | M1 | This mark is given for correct method to find the radius. |
|  | A1 | This mark is given for finding the exact radius. |
| (b) |  | M1 | This mark is given for substituting  into the given equation and attempting to expand the brackets. |
|  | A1 | This mark is given for the correct 3 term quadratic equation after collecting all like terms. |
|  | M1 | This mark is given for recognising the use of and substitute *a*, *b* and *c* from the 3 term quadratic equation above. |
|  | M1 | This mark is given for solving the new quadratic equation in *k*. |
|  | A1 | This mark is given for correct simplified value of *k*. |

**Question 8 (Total 9 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) |  | B1 | This mark is given for the correct value of *A*. |
|  | M1 | This mark is given for using the model to set up a correct equation in *k*. |
|  | M1 | This mark is given for correct working to solve the equation and obtain a value for *k*. |
|  | A1 | This mark is given for finding the correct equation of the model. |
| (b) |  | M1 | This mark is given for differentiating N and substituting *t = 8*. |
| 420 | A1 | This mark is given for correct answer to 2sf. |
| (c) |  | M1 | This mark is given for using both models to set up an equation in *T* using their value of *k*. |
|  | M1 | This mark is given for using lns to obtain an equation in *T*. |
| hours | A1 | This mark is given for correct value of *T*. |

**Question 9 (Total 11 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a)(i) |  | M1 | This mark is given for using a correct identity and appropriate strategy to find one of the values, *B* or *C*. |
| *B* = 1 and *C* = 2 | A1 | This mark is given for finding both values correctly. |
| (a)(ii) |  | M1 | This mark is given for using a correct method to establish an equation linking *A*, *B* and *C* and use the values of *B* and *C* to find *A*. |
| *A* = 0 | A1 | This mark is given for correct complete proof with no errors. |
| (b)(i) |  | M1 | This mark is given for writing as  and taking out a factor of 2−2 to form a correct expression |
|  | M1 | This mark is given for correctly expand  up to the term in |
|  | A1 | This mark is given for expand the binomial expression  correctly. |
|  | M1 | This mark is given for expanding  using correct method. |
|  | M1 | This mark is given for using correct method to find f(*x*) with the values of *B* and *C* found in part (a). |
|  | A1 | This mark is given for correct expression. |
| (b)(ii) |  | B1 | This mark is given for stating the correct range. |

**Question 10 (Total 8 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) |  | M1 | This mark is given for using correct double angle formulae for both  and |
|  | A1 | This mark is given for using correct double angle formulae for both  and  to give correct expressions in both numerator and denominator. |
|  | M1 | This mark is given for factorising the numerator and denominator in order to cancel (sin*θ* + cos*θ*) |
|  | A1 | This mark is given for correct proof with no errors. |
| (b) |  | M1 | This mark is given for rewriting the left-hand side of the equation to tan 2*x*. |
|  | A1 | This mark is given for using double angle formulae to obtain |
|  | A1  A1 | This mark is given for correct values (A1 for two correct values, A1 A1 for all correct values with no other incorrect values in the given range) |

**Question 11 (Total 8 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) |  | B1 | This mark is given for using the correct strip width *h*. |
|  | M1 | This mark is given for using the trapezium rule correctly. |
| = 2.41 | A1 | This mark is given for correct value of the area to 3 sf. |
| (b) |  | M1 | This mark is given for using integration by parts the correct way. |
| A1 | This mark is given for correct expression |
|  | M1 | This mark is given for using integration by parts again. |
|  | M1 | This mark is given for applying the limits 4 and 2 |
|  | A1 | This mark is given for the correct answer. |

**Question 12 (Total 9 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) |  | M1 | This mark is given for translating the problem into a suitable model, using  to establish *c* = 3. |
| when *x* = 90 | M1 | This mark is given for correct attempt to use one of the two other pieces of information to produce a linear model. |
| A1 | This mark is given for at least one correct equation connecting *a* and *b*. |
| M1 | This mark is given for correct attempt to use both pieces of information to establish the values of both *a* and *b*. |
|  | A1 | This mark is given for the correct equation. |
| (b)(i) |  | B1 | This mark is given for correct *H* with correct units. |
| (b)(ii) |  | M1 | This mark is given for attempting to solve for *x* using *H* = 0 |
|  | A1 | This mark is given for finding the correct value of *x*. |
| E.g.   * The ground is unlikely to be horizontal * The ball is not a particle so has dimensions/size * The ball is unlikely to travel in a vertical plane (as it will spin) * *H* is not likely to be a quadratic function in *x* | B1 | This mark is given for providing a suitable limitation of the model. |

**Question 13 (Total 3 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  |  | M1 | This mark is given for substituting the parametric forms into the Cartesian equation. |
|  | M1 | This mark is given for processing the expression into a single fraction. |
| = 4 | A1 | This mark is given for fully correct proof showing all key steps. |

**Question 14 (Total 3 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  |  | M1 | This mark is given for using a correct rule to find |
| A1 | This mark is given for correct derivative in terms of a single variable. |
|  | M1 | This mark is given for simplifying and collecting like terms to form a single fraction. |
|  | A1 | This mark is given for correct expression showing all key steps with no errors. |

**Question 15 (Total 6 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (i) |  | M1 | This mark is given for using *n* = 1, 2, 3 and 4 to prove the given result. |
|  | So if  then (*n* + 1)3 > 3*n* | A1 | This mark is given for completing the proof with no errors. |
| (ii) | Let *m* be odd | M1 | This mark is given for beginning the proof by negating the statement. |
|  | *m*3 + 5 = (2*p* + 1)3 + 5 | M1 | This mark is given for setting  and attempting to expand (2*p* ± 1)3 + 5 |
|  | = 8*p*3 + 12*p*2 + 6*p* + 6  This is even. | A1 | This mark is given for correct working reaches 8*p*3 + 12*p*2 + 6*p* + 6 and states even. |
|  | * 8*p*3 + 12*p*2 + 6*p* + 6= 2(4*p*3 + 6*p*2 + 3*p* + 3) therefore *m* is even being even * 'this is a contradiction if *m*3 + 5 is odd then *m* must be even' | A1 | This mark is given for completing the proof with reason and conclusion. |