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**1.** A student was asked to answer the following:

For the complex numbers *z*1 = 3 – 3i and *z*2 =  + i , find the value of

The student’s attempt is shown below.

*Diagram

Description automatically generated*

The student made errors in line 1 and line 3

Correct the error that the student made in

(*a*)(i) line 1

(ii) line 3

**(2)**

(*b*)Write down the correct value of arg **

**(1)**

**(Total for Question 1 is 3 marks)**

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**2. In this question you must show all stages of your working.**

A college offers only three courses: Construction, Design and Hospitality.

Each student enrols on just one of these courses.

In 2019, there was a total of 1110 students at this college.

There were 370 more students enrolled on Construction than Hospitality.

In 2020 the number of students enrolled on

• Construction **increased** by 1.25%

• Design **increased** by 2.5%

• Hospitality **decreased** by 2%

In 2020, the total number of students at the college increased by 0.27% to

2 significant figures.

(*a*)(i) Define, for each course, a variable for the number of students enrolled on that

course in 2019.

(ii) Using your variables from part (a) (i), write down **three** equations that model

this situation.

**(4)**

(*b*)By forming and solving a matrix equation, determine how many students were

enrolled on each of the three courses in 2019.

**(4)**

**(Total for Question 2 is 8 marks)**

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**3. M** =  where *a* is a constant

(*a*)Prove by mathematical induction that, for *n* ∈ ℕ

**M***n* = 

**(6)**

Triangle *T* has vertices *A*, *B* and *C*.

Triangle *T* is transformed to triangle *T*′ by the transformation represented by **M***n*

where *n* ∈ ℕ

Given that

• triangle *T* has an area of 5 cm2

• triangle *T*′ has an area of 1215 cm2

• vertex *A*(2, –2) is transformed to vertex *A*′(123, –2)

(*b*)determine

(i) the value of *n*

(ii) the value of *a*

**(5)**

**(Total for Question 3 is 11 marks)**

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**4.** (i) Given that

*z*1 = 6 and *z*2 = 

show that

*z*1 + *z*2 = 12

**(3)**

(ii) Given that

arg(*z* – 5) = 

determine the least value of |*z*| as *z* varies.

**(3)**

**(Total for Question 4 is 6 marks)**

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**5.** (*a*)Given that

*y* = arcsin *x* –1 ≤ *x* ≤ 1

show that

=

**(3)**

(*b*)f(*x*) = arcsin (e*x*) *x* ≤ 0

Prove that f(*x*) has no stationary points.

**(3)**

**(Total for Question 5 is 6 marks)**

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**6.** The cubic equation

4*x*3 + *px*2 – 14*x* + *q* = 0

where *p* and *q* are real positive constants, has roots *α*, *β* and *γ*

Given that *α*2 + *β*2 + *γ*2 = 16

(*a*)show that *p* = 12

**(3)**

Given that 

(*b*)determine the value of *q*

**(3)**

Without solving the cubic equation,

(*c*)determine the value of (*α* – 1)( *β* – 1)(*γ* – 1)

**(4)**

**(Total for Question 6 is 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**7.**

Diagram

Description automatically generated

Figure 1 shows a sketch of the curve *C* with equation

*r* = 1 + tan *θ* 0 ≤ *θ* < 

Figure 1 also shows the tangent to *C* at the point *A*.

This tangent is perpendicular to the initial line.

(*a*)Use differentiation to prove that the polar coordinates of *A* are 

**(4)**

The finite region *R*, shown shaded in Figure 1, is bounded by *C*, the tangent at *A* and the

initial line.

(*b*)Use calculus to show that the exact area of *R* is (1 – ln 2)

**(6)**

**(Total for Question 7 is 10 marks)**

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**8.** Two birds are flying towards their nest, which is in a tree.

Relative to a fixed origin, the flight path of each bird is modelled by a straight line.

In the model, the equation for the flight path of the first bird is

**r**1 = 

and the equation for the flight path of the second bird is

**r**2 = 

where *λ* and *μ* are scalar parameters and *a* is a constant.

In the model, the angle between the birds’ flight paths is 120°

(*a*)Determine the value of *a*.

**(4)**

(*b*)Verify that, according to the model, there is a common point on the flight paths of

the two birds and find the coordinates of this common point.

**(5)**

The position of the nest is modelled as being at this common point.

The tree containing the nest is in a park.

The ground level of the park is modelled by the plane with equation

2*x* – 3*y* + *z* = 2

(*c*)Hence determine the shortest distance from the nest to the ground level of the park.

**(3)**

(*d*)By considering the model, comment on whether your answer to part (*c*) is reliable,

giving a reason for your answer.

**(1)**

**(Total for Question 8 is 13 marks)**

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**9.** *y* = cosh*n x n ≥* 5

(*a*)(i) Show that

= *n*2 cosh*n x* – *n*(*n* – 1)cosh*n* – 2 *x*

**(4)**

(ii) Determine an expression for 

**(2)**

(*b*)Hence determine the first three non‑zero terms of the Maclaurin series for *y*, giving

each coefficient in simplest form.

**(2)**

**(Total for Question 9 is 8 marks)**

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**TOTAL FOR PAPER IS 75 MARKS**