

**1.** Given that





(*a*)write down the exact value of

(i) | *z*1*z*2 |

(ii) arg (*z*1*z*2)

**(2)**

Given that *w* = *z*1*z*2 and that arg (*wn*) = 0 , where *n* ∈ ℤ+

(*b*)determine

(i) the smallest positive value of *n*

(ii) the corresponding value of | *wn* |

**(3)**

**(Total for Question 1 is 5 marks)**

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**2. **

The matrix **A** represents the linear transformation *M*.

Prove that, for the linear transformation *M*, there are no invariant lines.

**(5)**

**(Total for Question 2 is 5 marks)**

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**3.** f (*x*) = arcsin *x* −1 ≤ *x* ≤ 1

(*a*)Determine the first two non-zero terms, in ascending powers of *x*, of the Maclaurin

series for f (*x*) , giving each coefficient in its simplest form.

**(4)**

(*b*)Substitute *x* = into the answer to part (*a*)and hence find an approximate value for *π*

Give your answer in the form ** where *p* and *q* are integers to be determined.

**(2)**

**(Total for Question 3 is 6 marks)**

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**4.** In this question you may assume the results for



(*a*)Show that the sum of the cubes of the first *n* positive odd numbers is

*n*2 (2*n*2 − 1)

**(5)**

The sum of the cubes of 10 consecutive positive odd numbers is 99 800

(*b*)Use the answer to part (*a*)to determine the smallest of these 10 consecutive positive

odd numbers.

**(4)**

**(Total for Question 4 is 9 marks)**

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**5.** The curve *C* has equation

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(*a*)Show that *C* has no stationary points.

**(3)**

The normal to *C*, at the point where *x* = 1 , crosses the *x*-axis at the point *A* and crosses

the *y*-axis at the point *B*.

Given that *O* is the origin,

(*b*)show that the area of the triangle *OAB* is  where *p*, *q* and *r*

are integers to be determined.

**(5)**

**(Total for Question 5 is 8 marks)**

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**6.** The curve *C* has equation

*r* = *a*(*p* + 2 cos *θ*) 0 ≤ *θ* < 2*π*

where *a* and *p* are positive constants and *p* > 2

There are exactly four points on *C* where the tangent is perpendicular to the initial line.

(*a*)Show that the range of possible values for *p* is

2 < *p* < 4

**(5)**

(*b*)Sketch the curve with equation

*r* = *a*(3 + 2 cos *θ*) 0 ≤ *θ* < 2*π* where *a* > 0

**(1)**

John digs a hole in his garden in order to make a pond.

The pond has a uniform horizontal cross section that is modelled by the curve with equation

*r* = 20(3 + 2 cos *θ*) 0 ≤ *θ* < 2*π*

where *r* is measured in centimetres.

The depth of the pond is 90 centimetres.

Water flows through a hosepipe into the pond at a rate of 50 litres per minute.

Given that the pond is initially empty,

(*c*)determine how long it will take to completely fill the pond with water using the

hosepipe, according to the model. Give your answer to the nearest minute.

**(7)**

(*d*)State a limitation of the model.

**(1)**

**(Total for Question 6 is 14 marks)**

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**7. Solutions based entirely on graphical or numerical methods are not acceptable.**



Figure 1 shows a sketch of part of the curve with equation

*y* = arsinh *x x* ≥ 0

and the straight line with equation *y* = *β*

The line and the curve intersect at the point with coordinates (*α*, *β*)

Given that *β* = 

(*a*) show that 

**(3)**

The finite region *R*, shown shaded in Figure 1, is bounded by the curve with equation

*y* = arsinh *x* , the *y*-axis and the line with equation *y* = *β*

The region *R* is rotated through 2*π* radians about the *y*-axis.

(*b*)Use calculus to find the exact value of the volume of the solid generated.

**(6)**

**(Total for Question 7 is 9 marks)**

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**8.** (i) The point *P* is one vertex of a regular pentagon in an Argand diagram.

The centre of the pentagon is at the origin.

Given that *P* represents the complex number 6 + 6i , determine the complex

numbers that represent the other vertices of the pentagon, giving your answers in the

form *r*ei*θ*

**(5)**

(ii) (*a*)On a single Argand diagram, shade the region, *R*, that satisfies both

| *z* − 2i | ≤ 2 and 

**(2)**

(*b*)Determine the exact area of *R*, giving your answer in simplest form.

**(4)**

**(Total for Question 8 is 11 marks)**

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**9.** (*a*)Given that | *z* | < 1 , write down the sum of the infinite series

1 + *z* + *z*2 + *z*3 + …

**(1)**

(*b*)Given that *z* = 

(i) use the answer to part (a), and de Moivre’s theorem or otherwise, to prove that



**(5)**

(ii) show that the sum of the infinite series 1 + *z* + *z*2 + *z*3 + … cannot be purely

imaginary, giving a reason for your answer.

**(2)**

**(Total for Question 9 is 8 marks)**

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**TOTAL FOR PAPER IS 75 MARKS**