

**1.** The transformation *P* is an enlargement, centre the origin, with scale factor *k*, where *k* > 0

The transformation *Q* is a rotation through angle *θ* degrees anticlockwise about the origin.

The transformation *P* followed by the transformation *Q* is represented by the matrix

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(*a*)Determine

(i) the value of *k*,

(ii) the smallest value of *θ*

**(4)**

A square *S* has vertices at the points with coordinates (0, 0), (*a*, –*a*), (2*a*, 0) and (*a*, *a*)

where *a* is a constant.

The square *S* is transformed to the square *S*′ by the transformation represented by **M**.

(*b*)Determine, in terms of *a*, the area of *S*′

**(2)**

**(Total for Question 1 is 6 marks)**

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**2.** (*a*)Use the Maclaurin series expansion for cos *x* to determine the series expansion of

in ascending powers of *x*, up to and including the term in *x*4

Give each term in simplest form.

**(2)**

(*b*)Use the answer to part (*a*)and calculus to find an approximation, to 5 decimal places, for



**(3)**

(*c*)Use the integration function on your calculator to evaluate



Give your answer to 5 decimal places.

**(1)**

(*d*)Assuming that the calculator answer in part (*c*)is accurate to 5 decimal places,

comment on the accuracy of the approximation found in part (*b*).

**(1)**

**(Total for Question 2 is 7 marks)**

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**3.** The cubic equation

*ax*3 + *bx*2 – 19*x* – *b* = 0

where *a* and *b* are constants, has roots *α*, *β* and *γ*

The cubic equation

*w*3 – 9*w*2 – 97*w* + *c* = 0

where *c* is a constant, has roots (4*α* – 1), (4*β* – 1) and (4*γ* – 1)

Without solving either cubic equation, determine the value of *a*, the value of *b* and the

value of *c*.

**(6)**

**(Total for Question 3 is 6 marks)**

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**4.** (i) **A** is a 2 by 2 matrix and **B** is a 2 by 3 matrix.

Giving a reason for your answer, explain whether it is possible to evaluate

(*a*)**AB**

(*b*)**A** + **B**

**(2)**

(ii) Given that



where *a*, *b* and *λ* are constants,

(*a*)determine

• the value of *λ*

• the value of *a*

• the value of *b*

(*b*)Hence deduce the inverse of the matrix 

**(3)**

(iii) Given that



determine the values of *θ* for which the matrix **M** is singular.

**(4)**

**(Total for Question 4 is 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**5.** (i) Evaluate the improper integral



**(3)**

(ii) The air temperature, *θ* °C, on a particular day in London is modelled by the equation



where *t* is the number of hours after midnight.

(*a*)Use calculus to show that the mean air temperature on this day is 8 °C,

according to the model.

**(3)**

Given that the actual mean air temperature recorded on this day was higher than 8 °C,

(*b*)explain how the model could be refined.

**(1)**

**(Total for Question 5 is 7 marks)**

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**6.** A tourist decides to do a bungee jump from a bridge over a river.

One end of an elastic rope is attached to the bridge and the other end of the elastic rope

is attached to the tourist.

The tourist jumps off the bridge.

At time *t* seconds after the tourist reaches their lowest point, their vertical displacement

is *x* metres above a fixed point 30 metres vertically above the river.

When *t* = 0

• *x* = –20

• the velocity of the tourist is 0 m s–1

• the acceleration of the tourist is 13.6 m s–2

In the subsequent motion, the elastic rope is assumed to remain taut so that the vertical

displacement of the tourist can be modelled by the differential equation

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where *k* is a positive constant.

(*a*)Determine the value of *k*

**(2)**

(*b*)Determine the particular solution to the differential equation.

**(7)**

(*c*)Hence find, according to the model, the vertical height of the tourist above the river

15 seconds after they have reached their lowest point.

**(2)**

(*d*)Give a limitation of the model.

**(1)**

**(Total for Question 6 is 12 marks)**

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**7.** The plane *Π* has equation



where *λ* and *μ* are scalar parameters.

(*a*)Show that vector 2**i** + 3**j** – 4**k** is perpendicular to *Π*.

**(2)**

(*b*)Hence find a Cartesian equation of *Π*.

**(2)**

The line *l* has equation



where *t* is a scalar parameter.

The point *A* lies on *l*.

Given that the shortest distance between *A* and *Π* is 

(*c*)determine the possible coordinates of *A*.

**(4)**

**(Total for Question 7 is 8 marks)**

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**8.** Two different colours of paint are being mixed together in a container.

The paint is stirred continuously so that each colour is instantly dispersed evenly

throughout the container.

Initially the container holds a mixture of 10 litres of red paint and 20 litres of blue paint.

The colour of the paint mixture is now altered by

• adding red paint to the container at a rate of 2 litres per second

• adding blue paint to the container at a rate of 1 litre per second

• pumping fully mixed paint from the container at a rate of 3 litres per second.

Let *r* litres be the amount of red paint in the container at time *t* seconds after the colour

of the paint mixture starts to be altered.

(*a*)Show that the amount of red paint in the container can be modelled by the

differential equation



where *α* is a positive constant to be determined.

**(2)**

(*b*)By solving the differential equation, determine how long it will take for the mixture

of paint in the container to consist of equal amounts of red paint and blue paint,

according to the model. Give your answer to the nearest second.

**(6)**

It actually takes 9 seconds for the mixture of paint in the container to consist of equal

amounts of red paint and blue paint.

(*c*)Use this information to evaluate the model, giving a reason for your answer.

**(1)**

**(Total for Question 8 is 9 marks)**

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**9.** (*a*)Use a hyperbolic substitution and calculus to show that



where *k* is an arbitrary constant.

**(6)**

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Figure 1 shows a sketch of part of the curve *C* with equation

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The finite region *R*, shown shaded in Figure 1, is bounded by the curve *C*, the *x*‑axis

and the line with equation *x* = 3

(*b*) Using algebraic integration and the result from part (*a*), show that the area of *R* is given by



**(5)**

**(Total for Question 9 is 11 marks)**

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**TOTAL FOR PAPER IS 75 MARKS**