

Conditional Probability within Statistical Distributions

1.

The average man's height can be modelled as a normal distribution with a mean of 70 inches and a standard deviation of 3 inches. A new ride at every mathematician's favourite theme park Pi-Land is called Westie's Woller-coaster and requires a minimum height of 5 foot (60 inches) to ride it. It also recommends, however that no one over 6 foot 4 inches rides it as it would be too uncomfortable.

Jenny is on the prowl for a new man. She is quite picky though and only likes to date men taller than herself and she is 6 foot tall! She sits next to a man on Westie's Woller-coaster and really hits it off with him. What is the probability she will be deeply disappointed when they stand up after the ride?

2.

A machine produces buttons with a mean diameter of 13mm and at a standard deviation of 1 mm. Any button produced that is under 11.5mm will immediately be discarded as being too small and anything larger than 15mm is discarded for being too big. All acceptable buttons are put in a big jar labelled "acceptable buttons".

A company, known purely as "Big Ol' Buttons" only wishes to trade in buttons wider than 14mm.

What proportion of the buttons from the "acceptable buttons" jar would "Big Ol' Buttons" be happy to trade with?

3.

Geoff loves a game of whack-a-mole. His reactions are good too and he reckons he can base any hit he gets in after a mole pops up as a normal distribution with a mean of 0.25 seconds and a standard deviation of 0.15.

When a mole pops up, he has 0.6 seconds to hit it, but to get the high score he needs to hit all moles within half a second. Given he hits all 20 moles in a round, find the probability he has achieved a high score.

4.

Bananas grow following a normal distribution with a mean mass of 118g and standard deviation of 15g. Only bananas weighing 100g or more are sold to companies.

A company known as 'Lightweight Bananas' only sell bananas with a mass lower than average.

- a) What would be the median mass of a banana sold by "Lightweight Bananas"?
- b) What is the probability that a banana sold by "Lightweight Bananas" has a mass less than 105g?

5.

Trevor has inherited his very own power plant! It was, however, built by his eccentric uncle Horatio and is susceptible to the occasional power surge. It's OK though, because he has five high voltage fuses which will save the system if the power surge exceeds 130,000 Volts - when a surge exceeds 130kV, one of these will take the hit instead of the plant. In order for the plant to keep functioning, at least three must stay intact though. When a surge happens the Volts produced can be modelled by a normal distribution with a mean of 120kV and standard deviation of 8 kV.

Trevor discovers that in one day there were five - yes five - surges. Given that at least one of the fuses was blown, find the probability the power plant stopped functioning.

6.

Cheryl is a sucker for a bargain and finds a second hand laptop which is only a year old from an online retailer known as 'Used but not bruised'. She knows it is good value too because she used to work at a company that produced them. She also knows that the average laptop of that kind had a mean lifespan of five years with a standard deviation of two years.

The laptop comes with its original two year guarantee. What is the probability the warranty will have expired by the time the laptop breaks?

'Used but not bruised' sell 156 of these laptops over the course of the year. How many would they be expected to be returned under this warranty?

7.

Jeffrey 'The Juggernaut' Johnson is competing on sports day in his favourite event, the javelin. Even though his average throw is 60m, he knows he can beat the school record of 75m – he does it once in every 20 throws when he practises. However, he only has one throw left in the qualifying round and is yet to make a legal throw. He needs to throw over 50m to qualify for the final and his throws can typically be modelled by a normal distribution. He decides to 'go big or go home' on his final throw and successfully makes a legal throw that gets him through qualifying.

What is the probability he goes into the final with a new school record?

8.

Henry is a dog breeder. This year, he feels he has a really good chance of winning the top prize on the agility course at Crufts with his prize pooch "Dash". Dash is seeded first for the competition and due to take to the course last. In order to win, Dash needs to finish the course in under 37 seconds, which fills Henry with confidence as Dash completes the course in training with a mean time of 34.5 seconds and a standard deviation of 1.5 seconds. However, for each obstacle Dash completes there is a 2 second penalty for making a mistake. There are 15 obstacles in total and Henry knows that, on average, Dash makes a mistake 5% of the time. If Dash were to make three or more mistakes, he would be immediately disqualified. What is the probability that Dash and Henry walk away with the first place medal?

9.

Kirsty is the managing director at F1 engine supplier “Engines, Engines, Engines”. As the F1 season approaches its seventh race of the season, all eight of the engines from “Engines, Engines, Engines” that were in race cars are still operating without fault. Each engine has an average lifespan of 2500 miles with a standard deviation of 300 miles. Going into the seventh race, the cars have already covered 2100 miles and are due to cover another 350 miles in this race weekend. Kirsty is worried that numerous engine failures in the same race will reflect badly on her company. What is the probability that multiple engines failing during the race.

During the race “Chase Swift” is running a close second behind the race leader “Leigh Ding” as they come up to the final pit stop. Leigh, as always, goes into the pits first but has a slow stop, opening the door for Chase to take the lead. In order to leave the pits in first, the changing of tyres needs to take less than 2.5 seconds. There are four teams all working simultaneously to change each tyre at the same time. Each team can change a tyre in a mean time of 2.3 seconds with a standard deviation of 0.2 seconds. What is the probability of Chase coming out ahead after the pit stop?

10.

Colin the Clown absolutely loves his job, but there is one element that keeps holding him back. As a globophobe, he has an extreme and crippling fear of balloons popping. This is a shame because his greatest talent is making balloon animals – he’s a natural. He has been booked to appear at his nephew Charlie’s 6th birthday party tomorrow and needs to blow up 40 balloons in preparation. Each balloon needs at least 5 litres of air in it to be operable, but pops when an average of 6 litres is in it, with a standard deviation of 0.45 litres. Colin is very good at judging when enough air is in the balloon to be useable, but his doctor has warned him that if more than one balloon were to pop, it could be bad for his health and lead to a heart attack. Kevin is Colin’s roommate and is sure he heard one balloon burst, but is unsure if there were more than that (he was making popcorn at the time). What is the probability that he needs to call an ambulance?

Answers

1.

$$X \sim N(70, 3^2)$$

$$\frac{P(60 < X < 72)}{P(60 < X < 76)} = \frac{0.7470784113 \dots}{0.97682 \dots} = 0.765 \dots (3SF)$$

2.

$$X \sim N(13, 1^2)$$

$$\frac{P(14 < X < 15)}{P(11.5 < X < 15)} = \frac{0.1359 \dots}{0.91 \dots} = 0.149 (3SF)$$

3.

$$X \sim N(0.25, 0.15^2)$$

$$\frac{P(0 < X < 0.5)}{P(0 < X < 0.6)} = \frac{0.90 \dots}{0.94 \dots} = 0.9597 \dots (1 \text{ hit})$$

$$0.9597 \dots^{20} = 0.439 \dots (3SF)$$

4.

$$X \sim N(118, 15^2)$$

$$P(100 < X < 118) = 0.3849 \dots$$

$$\frac{0.3849}{2} = 0.19 \dots$$

$$1 - 0.19 \dots = 0.307 \dots$$

$$P(X < x) = 0.307 \dots$$

$$x = 110.457 \dots$$

$$\frac{P(100 < X < 105)}{P(100 < X < 118)} = \frac{0.07799266692 \dots}{0.2849303298 \dots} = 0.2026 \dots$$

5.

$$X \sim N(120, 8^2)$$

$$P(X > 130) = 0.1056 \dots$$

$$Y \sim B(5, 0.1056 \dots)$$

$$\frac{P(Y \geq 3)}{P(Y \geq 1)} = \frac{1 - P(Y \leq 2)}{1 - P(Y \leq 0)} = \frac{1 - 0.572187 \dots}{1 - 0.989997 \dots} = \frac{0.01000 \dots}{0.42781 \dots} = 0.0234 \text{ (3SF)}$$

6.

$$X \sim N(5, 2^2)$$

$$P(X > 2) = 0.9331927987 \dots$$

$$P(X > 1) = 0.977249868 \dots$$

$$\frac{P(X > 2)}{P(X > 1)} = 0.9549172932 \dots$$

$$1 - 0.9549172932 = 0.04508270681 \dots$$

$$0.04508270681 \times 156 = 7.032902262$$

Therefore they would expect 7 to be returned.

7.

Finding sigma

$$X \sim N(60, \sigma^2)$$

$$P(X > 75) = 0.05$$

$$P(Z < z) = 0.95, z = 1.644853667 \dots$$

$$z = \frac{x - \mu}{\sigma}$$

$$\sigma = \frac{175 - 160}{1.64485 \dots} = 9.119352256 \dots$$

$$P(X > 75) | P(X > 50) = \frac{P(X > 75)}{P(X > 50)} = \frac{0.05}{0.8635851018 \dots} = 0.05789717343 \dots$$

8.

$$X \sim B(15, 0.05)$$

$$Y \sim N(34.5, 1.5^2)$$

$$P(X = 0) \times P(Y < 37)$$

$$0.04632912302 \times 0.9522096477 = 0.4411503791$$

$$P(X = 1) \times P(Y < 35)$$

$$0.36575623 \times 0.63055865 = 0.2306307609$$

$$P(X = 2) \times P(Y < 33)$$

$$0.13475229 \times 0.15865525 = 0.02137915988$$

$$\text{Total probability} = 0.6931602999$$

9.

$$X \sim N(2600, 300^2)$$

$$\frac{P(2100 < X < 2450)}{P(X > 2100)} = \frac{0.26074718 \dots}{0.95220964 \dots} = 0.27383379 \dots$$

$$Y \sim B(8, 0.27383379 \dots)$$

$$P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - 0.31057344 \dots = 0.6894265584 \dots$$

$$Z \sim N(2.3, 0.2^2)$$

$$P(Z < 2.5) = 0.84134474 \dots$$

$$0.84134474^4 = 0.501067 \dots$$

10.

$$X \sim N(6, 0.45^2)$$

$$P(X < 5) = 0.01313414577 \dots$$

$$Y \sim B(40, 0.01313414588 \dots)$$

$$P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - 0.9029945162 = 0.09700548383 \dots$$

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - 0.5892843429 = 0.4107156571 \dots$$

$$\frac{P(Y \geq 2)}{P(Y \geq 1)} = 0.2361864763 \dots$$