Nested Binomial Distribution Problems

1. In a competition to find the ‘luckiest’ person, 200 participants are selected who believe they are innately fortunate. In order to qualify for the second round in the competition, a participant must correctly call a fair coin at least 15 times out of 20 tosses. What is the probability that at least three people qualify for the second round?

2. In a class of 20 pupils, the chances that any individual passes a test is 80%. In order for their teacher to receive a pay raise, at least 90% of a class must pass the majority of tests they sit in a year. If the class sit 7 tests in a year, what is the chance of the teacher receiving a pay raise?

3. A trampoline gymnast must score well in at least 5 out of their 6 routines to qualify for the final round. Each routine comprises of twenty-five elements and a routine scores well if they make no more than two mistakes. The probability of a gymnast making a mistake during an element is 5%. What is the probability they qualify for the final round?

4. Dave enjoys going to the arcade. In fact, he enjoys it so much he goes every day and plays 10 games each visit. He has worked out that he is able to win tickets on a game 40% of the time. In order to win enough tickets to take home a toy, he must win at least six games. Find the probability that he wins at least five toys in March.

5. A new antibiotic has a 70% chance of working effectively against a new strain of bacteria in a petri dish. A Biologist is able to fit 15 petri dishes in a tray and stack 20 trays in a cabinet. There are 10 cabinets in a storage facility. What is the probability that at least 8 of the cabinets contain more than 12 trays with at least 10 petri dishes that have effectively worked against the bacteria?

6. A good tennis player must win at least four out of seven points to win a game of ‘Sudden Death but Carry on Playing Tennis’. They need to win at least six out of 11 games to win a set and win at least three sets out of five to win a match. In the first round of a competition, their chance of winning any point is 60% (points start with a rally and both players agree when to start the point). What is the probability they get into the next round?

7. The tennis player makes in to the final, but as their opponent is also quite good, their chance of winning points drops to 51%. What is the probability that they win the championship?

Answers

1. X~B(20, 0.5) P(X>=15) = 0.0207 (3SF)
   Y~B(200, 0.0207…) P(Y>=3) = 0.785 (3SF)

2. X~B(20, 0.8) P(X>=18) = 0.206 (3SF)
   Y~B(7, 0.206…) P(Y>=4) = 0.0370 (3SF)

3. X~B(25, 0.05) P(X<=2) = 0.873 (3SF)
   Y~B(6, 0.873…) P(Y>=5) = 0.829 (3SF)

4. X~B(10, 0.4) P(X>=6) = 0.166 (3SF)
   Y~B(30, 0.166…) P(Y>=5) = 0.573 (3SF)

5. X~B(15, 0.7) P(X>=10) = 0.722 (3SF)
   Y~B(20, 0.722…) P(Y>=13) = 0.834 (3SF)
   Z~B(10, 0.834…) P(Z>=8) = 0.776 (3SF)

6. X~B(7, 0.6) P(X>=4) = 0.710 (3SF)
   Y~B(11,0.710…) P(Y>=6) = 0.933 (3SF)
   Z~B(5,0.933…) P(Z>=3) = 0.997 (3SF)

7. X~B(7,0.51) P(X>=4) = 0.522 (3SF)
   Y~B(11,0.522…) P(Y>=6) = 0.559 (3SF)
   Z~B(5,0.559…) P(Z>=3) = 0.610 (3SF)