Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

A Level Further Mathematics B (MEI)

Y420 Core Pure

SAMPLE MARK SCHEME

Duration: 2 hours 40 minutes

MAXIMUM MARK 144

This document consists of 24 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

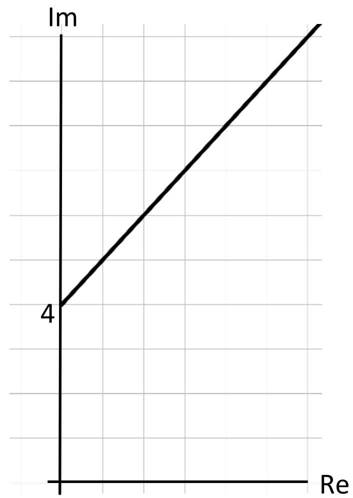
g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

k Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned on this occasion, but shows what a complete solution might look like

Question			Answer	Marks	AOs	Guidance	
1			$\begin{pmatrix}1\\2\\-1\end{pmatrix}.\begin{pmatrix}3\\1\\-2\end{pmatrix}=7$	**M1**	**1.1**		
			$\cos\theta=\dfrac{7}{\sqrt{6}\times\sqrt{14}}=0.7637...$	**M1**	**1.1**		
			$\theta=40.2°$ or 0.702 radians	**A1** **[3]**	**1.1**		
2	**(i)**			**B1**	**1.1a**	half line from 4i	Allow 4i included or excluded
				B1 **[2]**	**1.1**	direction $\frac{\pi}{4}$ above the positive x axis	
2	**(ii)**		$\|z-(6+4\text{i})\|=3\sqrt{2}$	**M1**	**1.1**	correct form	
				A1	**1.1**	centre correct	
				M1	**3.1a**	Attempt to find distance from centre to line	
				A1 **[4]**	**1.1**	radius correct	

Question			Answer	Marks	AOs	Guidance	
3	(i)		parallelogram vertices (0, 0), (2, 1), (5, 5), (3, 4)	M1 A1 [2]	1.1 1.1	3 correct coordinates calculated or drawn all correct	
3	(ii)	(*A*)	Solve $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ kx \end{pmatrix} = 5\begin{pmatrix} x \\ kx \end{pmatrix}$ Obtain $k = 1$ twice	M1 A1 [2]	1.1a 1.1		
3	(ii)	(*B*)	Hence $y = x$ is an invariant line	E1 [1]	2.2a		
3	(ii)	(*C*)	Draw $y = x$.	B1 [1]	1.1		
4			Conjugate root 1 – 2i [because $q \in \mathbb{R}$] $(1 + 2i) + (1 - 2i) + \gamma = 5$ $\gamma = 3$	B1 M1 A1	2.2a 1.1a 1.1	 or $(1 + 2i)(1 - 2i)\gamma = 15$	**Alternative solution** Conjugate root 1 – 2i [because $q \in \mathbb{R}$] **B1** $(z-1-2i)(z-1+2i)$ $= z^2 - 2z + 5$ $z^3 - 5z^2 + qz - 15$ $= (z^2 - 2z + 5)(z - 3)$ **M1A1** $q = 6 + 5 = 11$ **M1A1** The other roots are 3 and $(1-2i)$
			either $(1+2i)(1-2i)+3(1+2i)+3(1-2i) = q$ $q = 11$	M1 A1	3.1a 1.1	FT	
			or $3^3 - 5\times 3^2 + 3q - 15 = 0$ $q = 11$	M1 A1 [5]			

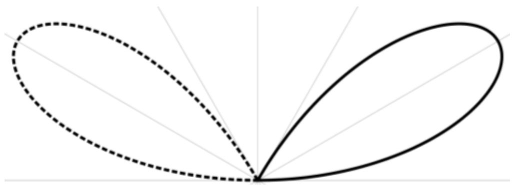
Question			Answer	Marks	AOs	Guidance	
5	(i)		$\frac{2}{(r+1)(r+3)} = \frac{A}{(r+1)} + \frac{B}{(r+3)}$ $2 = A(r+3) + B(r+1)$	M1	1.1	Correct form plus some progress	
			$A = 1, B = -1$ $\frac{2}{(r+1)(r+3)} = \frac{1}{(r+1)} - \frac{1}{(r+3)}$	A1	1.1		
				[2]			

Question			Answer	Marks	AOs	Guidance	
5	(ii)		$\sum_{r=1}^{n}\frac{1}{(r+1)(r+3)}=\frac{1}{2}\left\{\begin{array}{l}\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\ldots\ldots \\ +\left(\frac{1}{n}-\frac{1}{n+2}\right)+\left(\frac{1}{n+1}-\frac{1}{n+3}\right)\end{array}\right\}$	**M1**	**3.1a**	Write out series to identify: Cancelling terms, denominators 4, 5, … n+1 Ignore absence of initial $\frac{1}{2}$.	
			$=\frac{1}{2}\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{n+2}-\frac{1}{n+3}\right)$	**A1**	**2.1**	Non-cancelling terms at the beginning. Ignore absence of initial $\frac{1}{2}$.	
				A1	**2.1**	Non-cancelling terms at the end: Ignore absence of initial $\frac{1}{2}$.	
			$\frac{3(n+2)(n+3)+2(n+2)(n+3)-6(n+3)-6(n+2)}{12(n+2)(n+3)}$ $=\frac{5n^2+25n+30-12n-30}{12(n+2)(n+3)}$	**M1**	**2.1**	Attempt at writing as single fraction, with product of their linear terms.	
			$=\frac{5n^2+13n}{12(n+2)(n+3)}$	**A1** **[5]**	**1.1**	Completely correct argument; each of numerator and denominator can be either factorised or multiplied out.	

Question			Answer	Marks	AOs	Guidance	
6	(i)		$xy = \cosh^2 t - \sinh^2 t$ Substituting $\cosh^2 t - \sinh^2 t = 1$ So $xy = 1$	M1 A1 [2]	1.1 2.1		
6	(ii)		Perimeter $= 2x + 2y$ $= 2(\cosh t + \sinh t) + 2(\cosh t - \sinh t)$ $= 4\cosh t$ The minimum value of $\cosh t$ is 1 So the minimum value of the perimeter is 4	M1 A1 B1 A1 [4]	1.1 1.1 3.1a 3.2a		
7	(i)		$\ln 1.5 = 0.5 - \frac{0.5^2}{2} + \frac{0.5^3}{3} - \ldots$ 0.4167	M1 A1 [2]	1.1 1.1	Substitute $x = 0.5$ into series for $\ln(1 + x)$ up to x^3 Obtain 0.4167	
7	(ii)	(*A*)	error 0.0112	A1 [1]	1.1	Compare ln 1.5 = 0.4055 to 4 d.p.	
7	(ii)	(*B*)	ln 3 would require $x = 2$, beyond the range of convergence of the series	E1 [1]	2.3		

Question			Answer	Marks	AOs	Guidance	
7	(iii)		$\ln\frac{1+x}{1-x}=\ln(1+x)-\ln(1-x)$ $\ln(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}$ $\ln\frac{1+x}{1-x}=2\left(x+\frac{x^3}{3}\right)$	**M1** **A1** **[2]**	**3.1a** **1.1**	Attempt at series for $\ln(1-x)$	
7	(iv)	(*A*)	$\frac{1+x}{1-x}=1.5$ so $x=0.2$. $\ln 1.5=0.4053$ Using $x=0.5$, $\ln 3=1.083$.	**M1** **A1** **B1** **[3]**	**2.2a** **1.1** **1.1**		
7	(iv)	(*B*)	(Much) better approximation to ln 1.5 ln3: Inside range of convergence	**E1** **E1** **[2]**	**2.3** **2.3**		

Question			Answer	Marks	AOs	Guidance	
8			Need a vector perpendicular to the plane	M1	2.4	Or similar language e.g. need vector perpendicular to [2 calculated vectors] or Vector product gives vector perpendicular to plane or Find normal vector using vector product	
			$\begin{pmatrix}1\\2\\2\end{pmatrix}\times\begin{pmatrix}0\\1\\3\end{pmatrix}=\begin{pmatrix}4\\-3\\1\end{pmatrix}$	B1	3.1a	Use vector product with any two vectors in the plane.	
			Plane has equation $4x-3y+z=d$	M1	1.1		
			Passes through $(1, 0, -1)$ so $4-1=d$	A1	1.1		
			Equation of plane is $4x-3y+z=3$.	A1	3.2a		
				[5]			

Question			Answer	Marks	AOs	Guidance	
9	(i)			**B2** **[2]**	**1.1** **2.5**	B1 for loop in first quadrant B1 for loop in second quadrant with indication (e.g. dotted line) of *r* negative	
9	(ii)		DR Area is $\frac{1}{2}\int_0^{\frac{\pi}{3}} a^2 \sin^2 3\theta \, \mathrm{d}\theta$ Use $\cos 6\theta = 1 - 2\sin^2 3\theta$ to obtain $\frac{1}{4}\int_0^{\frac{\pi}{3}} a^2 (1 - \cos 6\theta) \, \mathrm{d}\theta$ $= \frac{a^2}{4}\left[\theta - \frac{1}{6}\sin 6\theta\right]_0^{\frac{\pi}{3}}$ $= \frac{\pi a^2}{12}$	**M1*** **M1*** **A1** **A1 FT** **A1** **[5]**	**2.1** **3.1a** **1.1** **1.1** **1.1**	Must be seen Must be seen Must be seen dep*	

Question			Answer	Marks	AOs	Guidance	
10	(i)		$\frac{dy}{dx}+\frac{3}{x}y=\frac{1}{x^2}$ integrating factor is exp(3 ln x) integrating factor is $\exp(\ln(x^3)) = x^3$ $x^3\frac{dy}{dx}+3x^2y=x$ $\frac{d(x^3y)}{dx}=x$ $x^3y=\frac{1}{2}x^2+c$ $x=1, y=1 \Rightarrow c=\frac{1}{2}$ $y=\frac{1+x^2}{2x^3}$	**M1** **A1** **M1** **A1A1** **B1** **A1** **[7]**	1.1 1.1 2.1 1.1 1.1 1.1 2.5	Write equation in correct form (may be implied) and attempt to find integrating factor Multiply by integrating factor **A1** each side of equation Use condition Expressing y as a function of x (can be in part **(ii)**)	
10	(ii)		$y=\frac{1}{2x^3}+\frac{1}{2x}$ Explain that y is the sum of two decreasing functions and hence decreasing	**B1** **E1** **[2]**	2.2a 2.4	Write y as sum of two fractions OR differentiate OR show that the derivative is negative hence decreasing	

Question			Answer	Marks	AOs	Guidance	
11	(i)		E.g. n = 2 gives $\frac{1}{2}=a-\frac{b}{2}$ and n = 3 gives $\frac{5}{6}=a-\frac{b}{6}$	M1	3.1a	Two values of $n \geq 2$ to give two simultaneous equations	
			Verify (or solve) $a=b=1$	A1 [2]	2.2a		
11	(ii)		Result holds for $n=2$ $\left(\frac{1}{2}=\frac{1}{2}\right)$	B1	1.1		
			Assume $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{k-1}{k!}=1-\frac{1}{k!}$	E1	2.1	Assume true for $n = k$	
			$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{k-1}{k!}+\frac{k}{(k+1)!}$ $=1-\frac{1}{k!}+\frac{k}{(k+1)!}$	M1A1	2.1 1.1	Add correct next term to both sides	
			RHS is $1-\frac{1}{k!}+\frac{k}{(k+1)!}=1-\left(\frac{(k+1)-k}{(k+1)!}\right)$	M1	1.1	1 minus fraction with correct denominator on RHS	
			which is $1-\left(\frac{1}{(k+1)!}\right)$	A1	2.2a	Showing that this is the given result with k + 1 replacing k	
			The result is true for n = 2. If true for $n = k$ it is also true for k + 1 hence true for n = 2, 3, 4, …	E1 [7]	2.5	Complete argument	

Question			Answer	Marks	AOs	Guidance	
12	(i)		DR $\tan y = x$ so $\sec^2 y \frac{dy}{dx} = 1$ $\left(1+\tan^2 y\right)\frac{dy}{dx} = 1$ $\left(1+x^2\right)\frac{dy}{dx} = 1$ $\frac{dy}{dx} = \frac{1}{\left(1+\tan^2 y\right)}$ AG	**M1** **A1** **A1** **[3]**	**1.1** **1.2** **2.1**	Must be seen Evidence of $\sec^2 y = 1+\tan^2 y$ must be seen Use $x = \tan y$ to obtain given answer	
12	(ii)		DR $\frac{1}{1-(-1)}\left[\arctan x\right]_{-1}^{1}$ $\frac{\pi}{4}$	**M1** **A1** **A1** **[3]**	**1.1** **2.1** **1.1**	M1 for arctan must be seen **A0** for a decimal answer	

Question			Answer	Marks	AOs	Guidance	
12	(iii)		DR				
			$\pi\int_{-1}^{1}\frac{1}{\left(1+x^2\right)^2}\,\mathrm{d}x$	**M1**	**1.2**	Must be seen	
			$\pi\int_{-1}^{1}\frac{1}{(1+\tan^2 u)^2}\sec^2 u\,\mathrm{d}u$	**M1**	**3.1a**	Substitute $x=\tan u$, $\mathrm{d}x=\sec^2 u\,\mathrm{d}u$ One intermediate step required	
			$\pi\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\cos^2 u\,\mathrm{d}u$	**A1** **A1**	**2.1** **1.1**	Integrand New limits	
			$\pi\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{2}\left(1+\cos 2u\right)\mathrm{d}u$	**M1**	**3.1a**	Must be seen	
			$\frac{1}{2}\left(u+\frac{1}{2}\sin 2u\right)$	**A1**	**2.1**	Integrate	
			$\frac{\pi}{4}\left(\pi+2\right)$	**A1**	**1.1**	oe, but **A0** for a decimal answer	
				[7]			

Question			Answer	Marks	AOs	Guidance	
13	(i)		$\det\mathbf{M} = k(6+6)-(4-3)-5(4+3)$ Simplify to $12(k-3)$ AG	M1 A1 [2]	1.1 1.1	Answer given so method must be clear	
13	(ii)		$\begin{pmatrix} x^2 \\ y^2 \\ z^2 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ \frac{-1}{12} & \frac{1}{4} & \frac{1}{6} \\ \frac{7}{12} & \frac{-3}{4} & \frac{5}{6} \end{pmatrix} \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix}$ $= \begin{pmatrix} -6 \\ 0 \\ -6 \end{pmatrix}$ One of $x = \sqrt{6}\mathrm{i},\ y = 0,\ z = -\sqrt{6}\mathrm{i}$ or $x = -\sqrt{6}\mathrm{i},\ y = 0,\ z = \sqrt{6}\mathrm{i}$	M1 A1 A1 [3]	3.1a 1.1 3.2a	Use of inverse matrix with $k = 4$. BC A0 if any solution given as final answer with $x = z$.	
13	(iii)	(*A*)	$3\times2+0-5\times1=1$ $2\times2+3\times0-3\times1=1$ $-2+2\times0+2\times1=0$	B1 [1]	1.1	Convincing substitution of the point into all three equations	
13	(iii)	(*B*)	The coefficients are the matrix **M** with $k=3$ and determinant of **M** is zero when $k=3$ Hence no *unique* solution The planes are distinct, so the planes form a sheaf	B1 B1 E1 E1 [4]	2.4 2.4 2.1 2.2a	This may be implied	

Question			Answer	Marks	AOs	Guidance	
13	(iv)		$12(k-3)=6$ … … or –6 $k = 2\frac{1}{2}$ or $k=3\frac{1}{2}$	M1 M1 A1 [3]	1.1 3.1a 1.1	or statement relating determinant to volume scale factor	
14	(i)	(*A*)	$(\cos\theta + \text{i}\sin\theta)^n = (\text{e}^{\text{i}\theta})^n$ $= \text{e}^{\text{i}n\theta} = \cos n\theta + \text{i}\sin n\theta$ AG	M1 A1 [2]	1.1 2.1	Answer given so working must be convincing	
14	(i)	(*B*)	$\text{e}^{-\text{i}\theta} = \cos\theta - \text{i}\sin\theta$ AG Hence obtain given result	M1 A1 [2]	1.1 2.1	Answer given so working must be convincing	
14	(ii)		$\cos 6\theta = \text{Re}\left((c + \text{i}s)^6\right)$ $= c^6 - 15c^4s^2 + 15c^2s^4 - s^6$ $= c^6 - 15c^4(1-c^2) + 15c^2(1-c^2)^2 - (1-c^2)^3$ $= 32c^6 - 48c^4 + 18c^2 - 1$	M1 A1 M1A1 A1A1 [6]	3.1a 2.1 2.11.1 2.2a 1.1	**A1** any two terms correct, **A2** all four *a*, *b*, *c*, *d* implicit or explicit	

Question			Answer	Marks	AOs	Guidance	
14	(iii)		$\cos^6\theta = \left(\frac{1}{2}\right)^6\left(z+\frac{1}{z}\right)^6$ where $z=\mathrm{e}^{\mathrm{i}\theta}$	**M1**	**3.1a**	set up expansion	
			$=\frac{1}{64}\left(z^6+\frac{1}{z^6}+6\left(z^4+\frac{1}{z^4}\right)+15\left(z^2+\frac{1}{z^2}\right)+20\right)$	**A1** **M1**	**2.1** **1.1**	result of expansion collecting terms	
			$=\frac{1}{32}\cos6\theta+\frac{6}{32}\cos4\theta+\frac{15}{32}\cos2\theta+\frac{5}{16}$	**A2**	**2.2a 1.1**	**A1** for any two coefficients correct, **A2** all four *P*, *Q*, *R*, *S* implicit or explicit	
				[5]			
14	(iv)		Use result in part **(iii)**: $\cos^6\frac{\pi}{12}=\frac{1}{32}\cos\frac{\pi}{2}+\frac{6}{32}\cos\frac{\pi}{3}+\frac{15}{32}\cos\frac{\pi}{6}+\frac{5}{16}$	**M1**	**3.1a**		
			$=\frac{1}{32}\times0+\frac{6}{32}\times\frac{1}{2}+\frac{15}{32}\frac{\sqrt{3}}{2}+\frac{5}{16}$	**A1**	**1.1**	Putting correct trig values into their **(iii)**	
			$\cos\frac{\pi}{12}=\left(\frac{26+15\sqrt{3}}{64}\right)^{\frac{1}{6}}$ AG	**A1**	**2.1**		
				[3]			

Question			Answer	Marks	AOs	Guidance	
15			DR				
			Let $u = \text{arsinh}\,2x$, $\dfrac{dv}{dx} = 1$	**M1**	**3.1a**	Use parts 1 and arsinh $2x$	
			$\Rightarrow v = x$				
			derivative of arsinh $2x$ is $\dfrac{2}{\sqrt{1+4x^2}}$	**M1**	**1.1**		
			$\left[x\,\text{arsinh}\,2x\right]_0^{\frac{2}{3}} - \int_0^{\frac{2}{3}} \dfrac{2x}{\sqrt{1+4x^2}}\,dx$	**A1A1**	**1.1 1.1**	**A1** for each term must be seen	
			$\left[x\,\text{arsinh}\,2x - \dfrac{1}{2}\sqrt{1+4x^2}\right]_0^{\frac{2}{3}}$	**A1**	**2.1**		
			$\dfrac{2}{3}\text{arsinh}\dfrac{4}{3} - 0 - \dfrac{1}{2}\times\dfrac{5}{3} + \dfrac{1}{2}$	**A1**	**1.1**		
			$\dfrac{2}{3}\ln\left(\dfrac{4}{3} + \sqrt{1+\dfrac{16}{9}}\right) - \dfrac{1}{2}\times\dfrac{5}{3} + \dfrac{1}{2}$	**M1** **A1**	**1.1** **2.1**	converting to logarithms AG so must be convincing	
			$= \dfrac{2}{3}\ln 3 - \dfrac{1}{3}$ AG				
				[8]			

Question			Answer	Marks	AOs	Guidance	
16	(i)	(*A*)	$T=\frac{2\pi}{\omega}$ with $T=2$ $\omega=\pi$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-\pi^2x$	M1 A1 B1 [3]	3.1b 1.1 3.3		
16	(i)	(*B*)	$x=a\cos\pi t+b\sin\pi t$	B1 [1]	1.1	or e.g. $x=A\sin(\pi t+\varepsilon)$	
16	(ii)	(*A*)	Auxiliary equation $\lambda^2+2k\lambda+(k^2+9)=0$ Roots $\lambda=-k\pm3\mathrm{i}$ Hence $x=\mathrm{e}^{-kt}(A\cos3t+B\sin3t)$	M1 A1 A1 [3]	1.1a 1.1 2.2a		
16	(ii)	(*B*)	Motion is damped [SHM] … … and period is a bit more than 2s	E1 E1 [2]	3.5b 3.5b	must be comments about motion	
16	(iii)		$\mathrm{e}^{-\frac{2\pi}{3}k}=0.98$ $k=0.009646$	M1 B1 A1 [3]	3.1b 3.3 1.1	Attempt – may use incorrect period for correct period	

Question			Answer	Marks	AOs	Guidance	
16	(iv)		Starting with $x = e^{-kt}(A\cos 3t + B\sin 3t)$, with k as in **(iii)**. $t = 0, x = 0$ gives $A = 0$ $\frac{dx}{dt} = e^{-kt}(3B\cos 3t - kB\sin 3t)$ $t = 0, \frac{dx}{dt} = 12$ gives $B = 4$ $x = 4e^{-kt}\sin 3t$ with k as in **(iii)**	 **B1** **M1** **A1** **B1** **[4]**	 **1.1a** **1.1** **2.4** **3.3**	Must see reasoning for where B comes from.	
16	(v)		Without damping ie $k = 0$ the amplitude is 4 In fact there is slight damping, so amplitude slightly less than 4.	**E1** **E1** **[2]**	**3.4** **3.5a**		

Question	AO1	AO2	AO3(PS)	AO3(M)	Totals
1	3	0	0	0	3
2i	2	0	0	0	2
2ii	3	0	1	0	4
3i	2	0	0	0	2
3iiA	2	0	0	0	2
3iiB	0	1	0	0	1
3iiC	1	0	0	0	1
4	3	1	1	0	5
5i	2	0	0	0	2
5ii	1	3	1	0	5
6i	1	1	0	0	2
6ii	2	0	2	0	4
7i	2	0	0	0	2
7iiA	1	0	0	0	1
7iiB	0	1	0	0	1
7iii	1	0	1	0	2
7ivA	2	1	0	0	3
7ivB	0	2	0	0	2
8	2	1	2	0	5
9i	1	1	0	0	2
9ii	3	1	1	0	5
10i	5	2	0	0	7
10ii	0	2	0	0	2
11i	0	1	1	0	2
11ii	3	4	0	0	7
12i	2	1	0	0	3
12ii	2	1	0	0	3
12iii	3	2	2	0	7
13i	2	0	0	0	2
13ii	1	0	2	0	3
13iiiA	1	0	0	0	1
13iiiB	0	4	0	0	4
13iv	2	0	1	0	3
14iA	1	1	0	0	2
14iB	1	1	0	0	2
14ii	2	3	1	0	6
14iii	2	2	1	0	5
14iv	1	1	1	0	3
15	5	2	1	0	8

Question	AO1	AO2	AO3(PS)	AO3(M)	Totals
16iA	1	0	1	1	3
16iB	1	0	0	0	1
16iiA	2	1	0	0	3
16iiB	0	0	0	2	2
16iii	1	0	1	1	3
16iv	2	1	0	1	4
16v	0	0	0	2	2
Totals	74	42	21	7	144

Summary of Updates

Date	Version	Change
October 2019	2	Amendments to the front cover rubric instructions to candidates