

AS Level Further Mathematics B (MEI)

Y410 Core Pure

Sample Question Paper

Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question or part question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1 The complex number z_1 is $1 + i$ and the complex number z_2 has modulus 4 and argument $\frac{\pi}{3}$.

(i) Express z_2 in the form $a + bi$, giving a and b in exact form. [2]

(ii) Express $\frac{z_2}{z_1}$ in the form $c + di$, giving c and d in exact form. [2]

- 2 (i) Describe fully the transformation represented by the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. [2]

(ii) A triangle of area 5 square units undergoes the transformation represented by the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

Explaining your reasoning, find the area of the image of the triangle following this transformation. [2]

- 3 (i) Write down, in complex form, the equation of the locus represented by the circle in the Argand diagram shown in Fig. 3. [2]

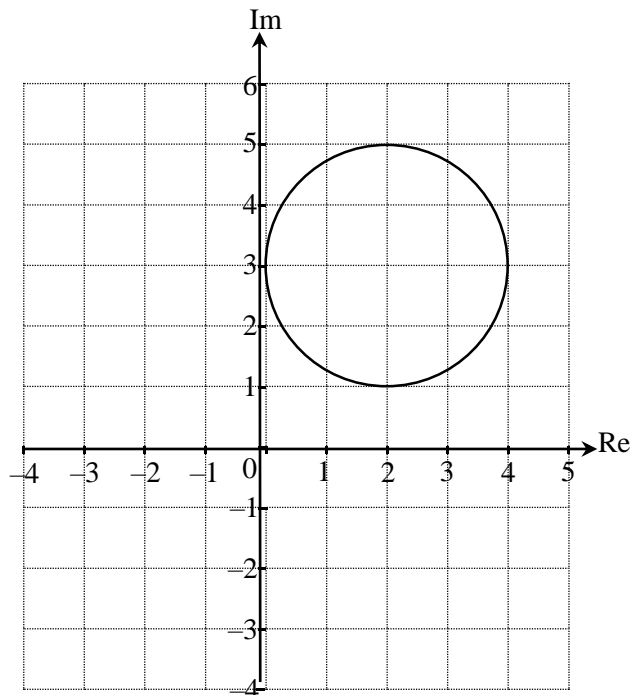


Fig. 3

- (ii) On the copy of Fig. 3 in the Printed Answer Booklet mark with a cross any point(s) on the circle for which $\arg(z - 2i) = \frac{\pi}{4}$. [2]

3

- 4 (i) Find the coordinates of the point where the following three planes intersect. Give your answers in terms of a .

$$\begin{aligned}x - 2y - z &= 6 \\3x + y + 5z &= -4 \\-4x + 2y - 3z &= a\end{aligned}$$

[4]

- (ii) Determine whether the intersection of the three planes could be on the z -axis.

[2]

- 5 The cubic equation $x^3 - 4x^2 + px + q = 0$ has roots α , $\frac{2}{\alpha}$ and $\alpha + \frac{2}{\alpha}$.

Find

- the values of the roots of the equation,
- the value of p .

[7]

- 6 (i) Show that, when $n = 5$, $\sum_{r=n+1}^{2n} r^2 = 330$.

[1]

- (ii) Find, in terms of n , a fully factorised expression for $\sum_{r=n+1}^{2n} r^2$.

[4]

- 7 The plane Π has equation $3x - 5y + z = 9$.

- (i) Show that Π contains

- the point $(4, 1, 2)$

and

- the vector $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

[4]

- (ii) Determine the equation of a plane which is perpendicular to Π and which passes through $(4, 1, 2)$. [3]

8 In this question you must show detailed reasoning.

(i) Explain why all cubic equations with real coefficients have at least one real root. [2]

(ii) Points representing the three roots of the equation $z^3 + 9z^2 + 27z + 35 = 0$ are plotted on an Argand diagram.

Find the exact area of the triangle which has these three points as its vertices. [7]

9 You are given that matrix $\mathbf{M} = \begin{pmatrix} -3 & 8 \\ -2 & 5 \end{pmatrix}$.

(i) Prove that, for all positive integers n , $\mathbf{M}^n = \begin{pmatrix} 1-4n & 8n \\ -2n & 1+4n \end{pmatrix}$. [6]

(ii) Determine the equation of the line of invariant points of the transformation represented by the matrix \mathbf{M} . [3]

It is claimed that the answer to part (ii) is also a line of invariant points of the transformation represented by the matrix \mathbf{M}^n , for any positive integer n .

(iii) Explain *geometrically* why this claim is true. [2]

(iv) Verify *algebraically* that this claim is true. [3]

END OF QUESTION PAPER

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