**GCE AS Further Mathematics (8FM0) – Paper 1**

**Core Pure Mathematics**

**October 2020 student-friendly mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.**

**This document is intended for guidance only and may differ significantly from the final mark scheme published in December 2020.**

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| **Guidance on the use of codes within this document** |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.  A1 – accuracy mark. This mark is generally given for a correct answer following correct working.  B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.  Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

**Question 1 (Total 6 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark**  **AO** | **Notes** |
| (a) | = *k*(*k* + *k*) – 3(–3*k* + 16) – 1(–3*k* – 16) | M1  2.1 | This mark is given for finding the determinant of the matrix corresponding to the system of equations |
| det = 0 ⇒ *k*2 + 6*k* – 16 = 0  *k* = 2  (*k* = −8 is rejected) | A1  1.1b | This mark is given for setting the determinant = 0 and solving to find *k* = 2 (and rejecting *k* = −8) |
| (b) | 5*x* + 2*y* = 1  – 10*x* – 4*y* = –2  so 20*x* + 8*y* = 4 | M1  3.1a | This mark is given for complete method to eliminate *z* from the equations (mark also given for eliminating *x* or *y*) |
| A1  1.1b | This mark is given for finding a correct pair of equations in the same two variables |
| The two equations are linear multiples of each other, therefore the equations are consistent. | A1  2.4 | This mark is given for stating a correct conclusion |
| (c) | The three planes form a sheaf | B1  2.2a | This mark is given for a correct interpretation |

**Question 2 (Total 8 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | ⏐*z*1⏐ = √13 and arg *z*1 = tan−1 | B1  1.1b | This mark is given for correct exact values for ⏐*z*1⏐and arg *z*1 |
| *z*1 = √13 (cos 0.9828 + i sin 0.9828) | B1  1.1b | This mark is given for a correct exact value for *z*1 in the form *r* (cos *θ* + i sin *θ*) |
| (b) | | *z*1| = √13  | *z*1 *z*2| = | *z*1| × | *z*2| = 39√2  |*z*2| =  |*z*2| = 3√26 | M1  3.1a | This mark is given for a complete method to find the modulus of *z*2 |
| A1  1.1b | This mark is given for the correct answer only |
| arg(*z*1 *z*2) = arg *z*1 + arg *z*2  ⇒ arg *z*2 =  – tan−1 | M1  3.1a | This mark is given for a complete method to find the argument of *z*2 |
| A1  1.1b | This mark is given for the correct answer only |
| *z*2 = 3√26 (cos −0.1974 + i sin −0.1974) | M1  1.1b | This mark is given for writing *z*2 in the form *r* (cos *θ* + i sin *θ*) |
| *z*2 = 15 − 3i | A1  2.2a | This mark is given for deducing that *z*2 = 15 − 3i only |

**Question 3 (Total 5 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark**  **AO** | **Notes** |
|  | *x*2 + *y*2 = *r*2 | B1  1.2 | This mark is given for the correct equation of the circle |
| *V* = *π* | B1  2.1 | This mark is given for a correct expression for the volume (including limits) |
| *V* = | M1  1.1b | This mark is given for integrating to find an equation in the form *ax* + *bx*3 |
| M1  1.1b | This mark is given for the correct use of limits to find the volume |
| *V* = 2*r*3 – *r*3 = *π r*3 | A1  1.1b | This mark is given for a fully correct argument |

**Question 4 (Total 13 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark**  **AO** | **Notes** |
| (a) | = ,  = ,  = | M1  3.3 | This mark is given for finding any two vectors from ,  or |
| **r** = **a** + *λ***b** + *μ***c** =  **r** = **a** + *λ* + *μ* | M1  1.1b | This mark is given for applying the vector equation of the plane **r** = **a** + *λ***b** + *μ***c**, where **a** is any point on the plane and the vectors **b** and **c** are two from ,  or . |
| A1  1.1b | This mark is given for a correct equation for the plane. |
| (b)(i) | **.** and **.** | M1  1.1b | This mark is given for applying the dot product between vectors **b** and **c** and the vector |
| **.**  = 4 + 6 – 10 = 0  **.** = 4 – 4 + 0 = 0  Thus the lawn is perpendicular to the vector | A1  2.4 | This mark is given for showing that both dot products are equal to 0 and stating a correct conclusion |
| (b)(ii) | **.** = **.** | M1  1.1b | This mark is given for applying the fomula **r.n** = **a.n** where **n** = |
| *x* + 2*y* – 10*z* = 2 | A1  1.1b | This mark is given for a correct Cartesian equation of the plane |

**Question 4 (Total 13 marks) continued**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark**  **AO** | **Notes** |
| (c) | **r** = **a** + *λ***d**  = | M1  3.3 | This mark is given for finding the vector  and using it as the direction vector in the formula **r** = **a** + *λ***d** |
| **r** =  + *λ* | A1  1.1b | This mark is given for a correct equation for **r** |
| (d) | The lawn will not be flat  The washing line will not be straight | B1  3.5b | This mark is given for a limitation of the model for the lawn or washing line stated |
| (e) | = 1.51 | M1  3.4 | This mark is given applying the distance formula using the point (2, 5, 2.75) and the normal vector |
| 1.71 m (or 171 cm) | A1  2.2b | This mark is given for a correct answer only |
| (f) | The distance to the lawn is longer than 1.5 m and so the model is not very accurate | B1  3.5a | This mark is given for a comparison of the answer to part (d) with 1.5 and an assessment of the model |

**Question 5 (Total 7 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | | | |
| (a) | Volume = *r* × (*r* + 1) × (*r* + 2) | B1  1.1b | | This mark is given for finding the correct volume of the block | | | |
| + 3*r*2 + 2*r*) | M1  3.1b | | This mark is given for a method to find the total volume of *n* blocks | | | |
| *V* = *n*2(*n* + 1)2 + 3 × *n*(*n* + 1)(2*n* + 1) + 2 × *n*(*n* + 1) | | | | | M1  2.1 | This mark is given for substituting standard formulae into the expression for the volume |
| *V* = *n*(*n* + 1)[*n*(*n* + 1) + 2(2*n* + 1) + 4] | M1  1.1b | | This mark is given for simplifying with a factor of *n*(*n* + 1) | | | |
| *V* = *n*(*n* + 1)(*n* + 2)(*n* + 3) | A1  1.1b | | This mark is given for a completely correct method to find the expression given | | | |
| (b) | *n*(*n* + 1)(*n* + 2)(*n* + 3) = *n*4 + 6*n*2 – 11710  3*n*4 + 18*n*3 – 11*n*2 – 6*n* – 46840 = 0 | | M1  1.1b | | This mark is given for a method to equate the two volumes to find determine the number of blocks | | |
| *n* = 10  There are 10 blocks | A1  3.2a | | This mark is given for the correct answer only | | | |

**Question 6 (Total 16 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark**  **AO** | **Notes** |
| (i)(a) | =  ⇒ 4 + *a*(*a* – 4) = 1 and 2*a* + *ab* = 0 | M1  3.1a | This mark is given for forming two equations, one involving *a* only and one involving *a* and *b* |
| *a*2 – 4*a* + 3 = 0  (*a* – 3)(*a* – 1) = 0 | M1  1.1b | This mark is given for solving a quadratic in *a* |
| *a* = 1, *a* = 3 | A1  1.1b | This mark is given for finding two correct values for *a* |
| 2 + *b* = 0 or 6 + 3*b* = 0 | M1  1.1b | This mark is given for substituting a value for *a* to find the a value for *b* |
| *b* = –2 | A1  1.1b | This mark is given for the correct answer only |
| (i)(b) |  | M1  3.1a | Extracts simultaneous equations using their matrix A with their smaller value of a. |
|  | M1  1.1b | Gathers terms from their two equations. |
|  | A1  2.1 | Shows the equations are consistent and deduces the correct line. Accept equivalent equations as long as both have been shown to be the same. |
| (ii)(a) | Area of triangle = × base × height  = × 2 × 3 = 3 | B1  1.1b | This mark is given for finding the correct area of the triangle *T* |
| (3*p* × *p*) – (–1 × 2*p*) =  = | M1  3.1a | This mark is given for a full method to find a value for *p* using the determinant of **P** |
| 3*p*2 + 2*p* ­ – 5 = 0 | A1  1.1b | This mark is given for forming a correct quadratic equation to be solved |
| *p* = 1  (*p* = – rejected) | A1  1.1b | This mark is given for the correct answer only |

**Question 6 (Total 16 marks) continued**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark**  **AO** | **Notes** |
| (ii)(b) | **Q** = | B1  1.1b | This mark is given for one correct row or column |
| B1  1.1b | This mark is given for a fully correct matrix **Q** |
| (ii)(c) | **R** = | M1  1.1b | This mark is given for multiplying the matrices **Q** and **P** in the correct order |
| **R** = | A1  1.1b | This mark is given for a fully correct matrix **R** |

**Question 7 (Total 6 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | |
|  | *z*2 = 2 – 3i | B1  1.1b | | This mark is given for stating the value of *z*2 | |
| *z*3 = *p* + 3i or *z*4 = *p* – 3i | M1  3.1a | | This mark is given for a method to find the third and fourth roots in the form *p*± 3i | |
| *z*3 = –4 + 3i or *z*4 = –4 – 3i | A1  1.1b | | This mark is given for finding the third and fourth roots (i.e. *p* = ­–4) | |
| *a* = −[(2 − 3i) + (2 + 3i) + (−4 + 3i) + (−4 − 3i)]  *b* = (2 − 3i)(2 + 3i) + (2 − 3i)(−4 + 3i)  + (2 − 3i)(−4 − 3i) + (2 + 3i)(−4 + 3i)  + (2 + 3i)(−4 − 3i) + (−4 − 3i)(−4 + 3i)  *c* = −[(2 − 3i)(2 + 3i)(−4 + 3i)  + (2 − 3i)(2 + 3i)(−4 − 3i)  + (2 − 3i)(−4 + 3i)(−4 − 3i)  + (2 + 3i)(−4 + 3i)(−4 − 3i)]  *d* = (2 − 3i)(2 + 3i)(−4 + 3i)(−4 − 3i) | | M1  3.1a | | This mark is given for a method to find the values of *a*, *b*, *c* and *d* |
| f(*z*) = *z*4 + 4*z*3 + 6*z*2 + 4*z* + 325    (*a* = 4, *b* = 6, *c* = 4, *d* = 325) | A1  1.1b | | This mark is given for at least two of the values of *a*, *b*, *c* and *d* correct | |
| A1  1.1b | | This mark is given for a fully correct answer | |

**Question 8 (Total 6 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | When *n* = 1, 2*n* + 2 + 32*n* + 1 = 8 + 27 = 35  So the statement is true for *n* = 1 | B1  2.2a | This mark is given for showing that f(1) = 35 and concluding that it is divisible by 7 |
| Assume true for *n* = *k*,  so 2*k* + 2 + 32*k* + 1 is divisible by 7 | M1  2.4 | This mark is given for a statement that assumes the result is true for some value of *n* |
| f(*k* + 1) − f(*k*)  = 2*k* + 3 + 32*k* + 3 − (2*k* + 2 + 32*k* + 1) | M1  2.1 | This mark is given for a method to use f(*k* + 1) − f(*k*) |
| 2 × 2*k* + 2 + 9 × 32*k* + 1 − 2*k* + 2 − 32*k* + 1  ⇒ 2*k* + 2 + 8 × 32*k* + 1  = f(*k*) + 7 × 32*k* + 1  or 8f(*k*) − 7 × 2*k* + 2 | A1  1.1b | This mark is given for a correct expression for f(*k* + 1) − f(*k*) in terms of f(*k*) |
| f(*k* + 1) = 2f(*k*) + 7 × 32*k* + 1  or  f(*k* + 1) = 9f(*k*) − 7 × 2*k* + 2 | A1  1.1b | This mark is given for a correct expression for f(*k* + 1) in terms of f(*k*) |
| If true for *n* = *k* then true for *n* = *k* + 1 and as it is true for *n* = 1 the statement is true for all (positive integers) *n* | A1  2.4 | This mark is given for a fully correct conclusion supported by correct working |

**Question 9 (Total 6 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | |
| --- | --- | --- | --- | --- | --- |
| (a) | *αβγ* = – and *αβ* + *αγ* + *βγ*  = – | B1  3.1a | | This mark is given for correct values for the product and pair sum of the roots | |
| +  +  =  = – ÷ – | M1  1.1b | | This mark is given for a complete method to find the sum of  +  +  using substitution of values | |
| = 4 | A1  1.1b | | This mark is given for the correct answer only | |
| (b) | New product:  *α* + *β* + *γ* = –  +  +  =  = 1 ÷ – = –3  New pair sum:  +  +  =  = – ÷ – = 1 | M1  3.1a | | This mark is given for a correct method to find the value of the new pair sum and the value of the new product | |
| *x*3 – 4*x*2 + (new pair sum)*x* – (new product) = 0 | | M1  1.1b | | This mark is given for applying *x*3 – (answer to part(a))*x*2 + (new pair sum)*x* – (new product) = 0 |
| *x*3 – 4*x*2 + *x* + 3 = 0 | A1  1.1b | | This mark is given for the correct answer only | |

**Question 10 (Total 7 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
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|  | (*x* − 3)2 + (*y* − 5)2 = (2*r*)2  and  *y* = −*x* + 2 | B1  1.1b | This mark is given for correct equations for each loci of points |
| (*x* − 3)2 + (−*x* + 2 −5)2 = (2*r*)2  or  (−*y* + 2 − 3)2 + (*y* − 5)2 = (2*r*)2 | M1  3.1a | This mark is given for a complete method to find a quadratic in one variable |
| 2*x*2 + 18 − 4*r*2 = 0  or  2*y*2 − 8*y* + 26 − 4*r*2 = 0 | A1  1.1b | This mark is given for a fully correct quadratic equation |
| *b*2 − 4*ac* > 0 ⇒ 02 − 4(2)(18 − 4*r*2) > 0  or  *b*2 − 4*ac* > 0 ⇒ (−8)2 − 4(2)(26 − 4*r*2) > 0 | M1  3.1a | This mark is given for a complete method using *b*2 − 4*ac* > 0 to find a range of values for *r* |
| Maximum value for *r*  (2*r*)2 < 52 + (3− 2)2 | M1  3.1a | This mark is given for a realising there will be a upper limit for *r* and using Pythagoras theorem |
| < *r* < | A1  1.1b | This mark is given for one correct limit |
| A1  1.1b | This mark is given for a fully correct inequality |