



Oxford Cambridge and RSA

# AS Level Further Mathematics A

Y531/01 Pure Core

**Monday 14 May 2018 – Afternoon**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

## INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

## INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 (i) Find a vector which is perpendicular to both  $\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$  and  $\begin{pmatrix} -3 \\ -6 \\ 4 \end{pmatrix}$ . [2]

(ii) The cartesian equation of a line is  $\frac{x}{2} = y - 3 = 2z + 4$ .

Express the equation of this line in vector form. [3]

2 **In this question you must show detailed reasoning.**

The cubic equation  $2x^3 + 3x^2 - 5x + 4 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ . By making an appropriate substitution, or otherwise, find a cubic equation with integer coefficients whose roots are  $\frac{1}{\alpha}$ ,  $\frac{1}{\beta}$  and  $\frac{1}{\gamma}$ . [3]

3 **In this question you must show detailed reasoning.**

The complex numbers  $z_1$  and  $z_2$  are given by  $z_1 = 2 - 3i$  and  $z_2 = a + 4i$  where  $a$  is a real number.

(i) Express  $z_1$  in modulus-argument form, giving the modulus in exact form and the argument correct to 3 significant figures. [3]

(ii) Find  $z_1 z_2$  in terms of  $a$ , writing your answer in the form  $c + id$ . [2]

(iii) The real and imaginary parts of a complex number on an Argand diagram are  $x$  and  $y$  respectively. Given that the point representing  $z_1 z_2$  lies on the line  $y = x$ , find the value of  $a$ . [2]

(iv) Given instead that  $z_1 z_2 = (z_1 z_2)^*$  find the value of  $a$ . [2]

4 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} 2 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 2 & a \end{pmatrix}$ .

(i) Show that  $\det \mathbf{A} = 6 - 3a$ . [2]

(ii) State the value of  $a$  for which  $\mathbf{A}$  is singular. [1]

(iii) Given that  $\mathbf{A}$  is non-singular find  $\mathbf{A}^{-1}$  in terms of  $a$ . [4]

**5 In this question you must show detailed reasoning.**

(i) Express  $(2 + 3i)^3$  in the form  $a + ib$ . [3]

(ii) Hence verify that  $2 + 3i$  is a root of the equation  $3z^3 - 8z^2 + 23z + 52 = 0$ . [3]

(iii) Express  $3z^3 - 8z^2 + 23z + 52$  as the product of a linear factor and a quadratic factor with real coefficients. [4]

**6** The matrices **A** and **B** are given by  $\mathbf{A} = \begin{pmatrix} t & 6 \\ t & -2 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 2t & 4 \\ t & -2 \end{pmatrix}$  where  $t$  is a constant.

(i) Show that  $|\mathbf{A}| = |\mathbf{B}|$ . [2]

(ii) Verify that  $|\mathbf{AB}| = |\mathbf{A}||\mathbf{B}|$ . [3]

(iii) Given that  $|\mathbf{AB}| = -1$  explain what this means about the constant  $t$ . [2]

**7** Prove by induction that  $2^{n+1} + 5 \times 9^n$  is divisible by 7 for all integers  $n \geq 1$ . [6]

**8** The  $2 \times 2$  matrix **A** represents a transformation T which has the following properties.

- The image of the point  $(0, 1)$  is the point  $(3, 4)$ .
- An object shape whose area is 7 is transformed to an image shape whose area is 35.
- T has a line of invariant points.

(i) Find a possible matrix for **A**. [8]

The transformation S is represented by the matrix **B** where  $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$ .

(ii) Find the equation of the line of invariant points of S. [2]

(iii) Show that any line of the form  $y = x + c$  is an invariant line of S. [3]

**END OF QUESTION PAPER**

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