OCR
Oxford Cambridge and RSA

AS Level Further Mathematics A

Y531/01 Pure Core

Monday 14 May 2018 – Afternoon

Time allowed: 1 hour 15 minutes

* 7 0 6 5 3 5 5 7 7 5 *

You must have:
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 **(i)** Find a vector which is perpendicular to both $\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -6 \\ 4 \end{pmatrix}$. **[2]**

(ii) The cartesian equation of a line is $\frac{x}{2} = y - 3 = 2z + 4$.

Express the equation of this line in vector form. **[3]**

2 **In this question you must show detailed reasoning.**
The cubic equation $2x^3 + 3x^2 - 5x + 4 = 0$ has roots α, β and γ. By making an appropriate substitution, or otherwise, find a cubic equation with integer coefficients whose roots are $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$. **[3]**

3 **In this question you must show detailed reasoning.**
The complex numbers z_1 and z_2 are given by $z_1 = 2 - 3\text{i}$ and $z_2 = a + 4\text{i}$ where a is a real number.

(i) Express z_1 in modulus-argument form, giving the modulus in exact form and the argument correct to 3 significant figures. **[3]**

(ii) Find $z_1 z_2$ in terms of a, writing your answer in the form $c + \text{i}d$. **[2]**

(iii) The real and imaginary parts of a complex number on an Argand diagram are x and y respectively. Given that the point representing $z_1 z_2$ lies on the line $y = x$, find the value of a. **[2]**

(iv) Given instead that $z_1 z_2 = (z_1 z_2)^*$ find the value of a. **[2]**

4 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 2 & a \end{pmatrix}$.

(i) Show that $\det\mathbf{A} = 6 - 3a$. **[2]**

(ii) State the value of a for which $\mathbf{A}$ is singular. **[1]**

(iii) Given that $\mathbf{A}$ is non-singular find $\mathbf{A}^{-1}$ in terms of a. **[4]**

5 **In this question you must show detailed reasoning.**

(i) Express $(2+3\mathrm{i})^3$ in the form $a + \mathrm{i}b$. **[3]**

(ii) Hence verify that $2+3\mathrm{i}$ is a root of the equation $3z^3-8z^2+23z+52=0$. **[3]**

(iii) Express $3z^3-8z^2+23z+52$ as the product of a linear factor and a quadratic factor with real coefficients. **[4]**

6 The matrices **A** and **B** are given by $\mathbf{A}=\begin{pmatrix} t & 6 \\ t & -2 \end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix} 2t & 4 \\ t & -2 \end{pmatrix}$ where t is a constant.

(i) Show that $|\mathbf{A}|=|\mathbf{B}|$. **[2]**

(ii) Verify that $|\mathbf{AB}|=|\mathbf{A}||\mathbf{B}|$. **[3]**

(iii) Given that $|\mathbf{AB}|=-1$ explain what this means about the constant t. **[2]**

7 Prove by induction that $2^{n+1}+5\times 9^n$ is divisible by 7 for all integers $n \geqslant 1$. **[6]**

8 The 2×2 matrix **A** represents a transformation T which has the following properties.

- The image of the point $(0, 1)$ is the point $(3, 4)$.
- An object shape whose area is 7 is transformed to an image shape whose area is 35.
- T has a line of invariant points.

(i) Find a possible matrix for **A**. **[8]**

The transformation S is represented by the matrix **B** where $\mathbf{B}=\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$.

(ii) Find the equation of the line of invariant points of S. **[2]**

(iii) Show that any line of the form $y=x+c$ is an invariant line of S. **[3]**

END OF QUESTION PAPER