3A Arithmetic Sequences Introduction

1. An arithmetic sequence is generated as follows:

   6, 20, 34, 48, 72…

a) Find the nth term

b) Find the first term in the sequence that exceeds 200

2. An arithmetic sequence is generated as follows:
    101, 94, 87, 80, 73…
a) Find the nth term

b) Find the first term in the sequence that is negative

3. A sequence is generated by the formula $u_n = a + (n - 1)d$, where a and b are constants to be found.

    Given that $u_3 = 5$ and $u_8 = 20$, find the values of the constants a and d.

3B Arithmetic Series

Proof:

1. Find the sum of the first 50 terms of the arithmetic series:

   $32 + 27 + 22 + 17 + 12 + \dots$

2. Find the smallest number of terms required for the sum of $4 + 9 + 14 + 19 + \ldots$ to exceed 2000.

3C Geometric Sequences Introduction

1. Find the nth and 10th terms of the following sequences…
a) 3, 6, 12, 24…

b) 40, -20, 10, -5…

2. The second term of a Geometric sequence is 4, and the 4th term is 8. Find the values of the common ratio and the first term

3. The numbers 3, x, and (x + 6) form the first three terms of a positive geometric sequence. Calculate the 15th term of the sequence

4. What is the first term to exceed 1 million in the sequence: 3, 6, 12, 24…

3D Geometric Series

Proof:

1. Find the sum of the following Geometric Series:
   $2 + 6 + 18 + 54...$ (for 10 terms)

2. Find the sum of the following Geometric Series:

$1024 - 512 + 256 - 128 + \ldots\ldots + 1$

3. Find the least value of n such that the sum of the following series exceeds 2,000,000:

$$1 + 2 + 4 + 8 \dots$$

3E Geometric Sum to Infinity

1. For the following series:

$$16 + 8 + 4 + 2 \ldots$$

a) Find the sum of the first 10 terms

b) Find the sum to infinity

2. The fourth term of a geometric series is 1.08 and the seventh is 0.23328.

a) Show that the series is convergent

b) Calculate the sum to infinity of the series

3. For a geometric series with first term a, and common ratio r, $S_4 = 15$ and $S_\infty = 16$.

a) Find the possible values of r

b) Given that all terms in the series are positive, find the value of a

3F Sigma Notation

1. Calculate the following:

$$\sum_{r=1}^{20}(4r+1)$$

2. Find the value of:

$$\sum_{r=1}^{12}(5 \times 3^{r-1})$$

3. Find the value of:

$$\sum_{r=6}^{15}(5 \times 3^{r-1})$$

3G Recurrence Relationships

1. Find the first four terms of the following sequences:

   a) $u_{n+1} = u_n + 4, \ u_1 = 7$

   b) $u_{n+1} = u_n + 4, \ u_1 = 5$

2. Find the first five terms generated by the following sequence:

$u_{n+1} = 2u_n + 3, \ u_1 = 2$

3. A sequence $a_1, \ a_2, \ a_3,...$ is defined by:

$$a_1 = p$$

$a_{n+1} = (a_n)^2 - 1, n \geq 1$

where $p < 0$

a) Show that $a_3 = p^4 - 2p^2$

b)   Given that $a_2 = 0$, find the value of p

c)   Find:

$$\sum_{r=1}^{200} a_r$$

d) Find the value of a_{199}

3H Nature of Sequences

1. For the following relationship, state whether the sequence is increasing, decreasing, or periodic:

   a) $u_{n+1} = u_n + 3, \ u_1 = 7$

   b) $u_{n+1} = (u_n)^2, \ u_1 = \frac{1}{2}$

   c) $u_n = \sin(90n)$

3I Sequences & Series in Context

1. Bruce starts a new company. He estimates that in Year 1 his profits will be £20000, and he predicts that his profits will increase by £5000 per year from that point on. He then models that once his annual profits reach £100000, they will then remain constant.

a) Calculate the profit for Bruce's business in the first 20 years

b) State a reason why this model might not be suitable

Bruce's financial advisor says that it is more likely that his profits would increase by 5% per year.

c) Using this model instead, calculate the profits that Bruce will make in the first 20 years.

2. A piece of A4 paper is folded in half repeatedly. The thickness of the sheet is 0.5mm.

a) Work out the thickness after 4 folds

b) Work out the thickness after 20 folds

c) State one reason why this might be an unrealistic model