11A Standard Integrals

1. Find the value of $\int_{-3}^{-2} \left(\frac{1}{x}\right) \, dx$

2. Find the following integral:

$$\int \left(2cosx + \frac{3}{x} - \sqrt{x}\right)\, dx$$

3. Find the following integral:

$$\int \left(\frac{cosx}{sin^2x} - 2e^x\right)\, dx$$

4. Given that a is a positive constant and:

$$\int_a^{3a} \left(\frac{2x+1}{x}\right) dx = ln12$$

Find the exact value of a.

11B Basic Reverse Chain Rule

1. Find the following integral:

$$\int cos(2x + 3)\ dx$$

2. Find the following integral:

$$\int (e^{4x+1})\ dx$$

3. Find the following integral:

$$\int (sec^2 3x)\ dx$$

4. Find:

$$\int \left(\frac{1}{3x+2}\right) dx$$

5. Find:

$$\int (2x+3)^4 \, dx$$

11D Spotting Derivatives

1. Find:

$$\int \left(\frac{2x}{x^2 + 1}\right) dx$$

2. Find:

$$\int \left(\frac{cosx}{3 + 2sinx}\right) dx$$

3. Find:

$$\int (3\sin(x)cos(x))\ dx$$

4. Find:

$$\int (3cos^2(x)sin(x))\ dx$$

5. Find:

$$\int x(x^2+5)^3\ dx$$

6. Find:

$$\int 5\tan(x)\, sec^4(x)\ dx$$

7. Find $\int \frac{\sin 2x}{(3+\cos 2x)^3}\, dx$ (1f from textbook exercise)

11C Using Trigonometric Identities

1. Find the following integral:

$$\int (tan^2x)\ dx$$

2. Find:

$$\int (sin3xcos3x)\ dx$$

3. Find:

$$\int (secx + tanx)^2 \; dx$$

4. Show that:

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (sin^2x)\, dx = \frac{\pi}{48} + \frac{1-\sqrt{2}}{8}$$

11G Integration by Partial Fractions

1. Find:

$$\int \frac{x-5}{(x+1)(x-2)}\,dx$$

2. Find:

$$\int \frac{9x^2 - 3x + 2}{9x^2 - 4}\,dx$$

11F Integration by Parts

1. Find:

$$\int (x\cos x)\, dx$$

2. Find:

$$\int (x^2 lnx)\, dx$$

3. Find:

$$\int (x^2 e^x)\, dx$$

4. Evaluate:

$$\int_{1}^{2} lnx \; dx$$

11E Integration by Substitution

1. Find $\int \left(x\sqrt{2x+5}\right)\,dx$ using the following substitution:

$$u = 2x + 5$$

2. Find $\int \left(x\sqrt{2x+5}\right) dx$ using the following substitution:

$$u^2 = 2x + 5$$

3. Use the substitution $u = sinx + 1$ to find:

$$\int cosxsinx(1 + sinx)^3 \, dx$$

4. Prove that:

$$\int \left(\frac{1}{\sqrt{1-x^2}}\right) dx = arcsinx + c$$

5. Use integration by substitution to evaluate:

$$\int_0^2 x(x+1)^3 \, dx$$

6. Use integration by substitution to evaluate:

$$\int_0^{\frac{\pi}{2}} cosx\sqrt{1 + sinx}\, dx$$

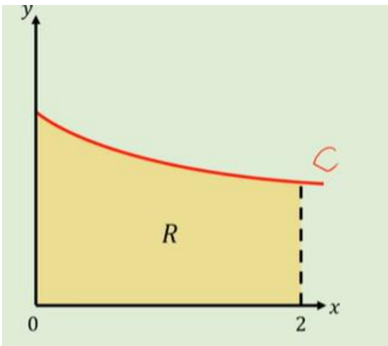
11H part 2 Integrating Parametric Equations

Note that this was initially left out of the textbook, so depending on your edition, you may not have practise questions. I will be using the old C4 notes and examples to talk through the theory.

1. The cure has parametric equations

$$x = t(1+t) \qquad y = \frac{t}{1+t}$$

Find the exact area of the region R, bounded by the curve C, the x-axis and the lines x=0 and x=2

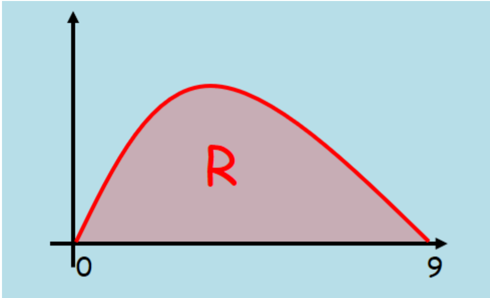


2. A curve has Parametric equations:

$$x = 5t^2 \qquad y = t^3$$

Work out:

$$\int_5^{20} y \, dx$$



3. The diagram shows a sketch of the curve with Parametric equations:

   The curve meets the x-axis at x = 0 and x = 9. The shaded region is bounded by the curve and the x-axis.

$$x = t^2 \qquad y = 2t(3 - t) \qquad t \geq 0$$

a) Find the value of t when:

   i) $x = 0$

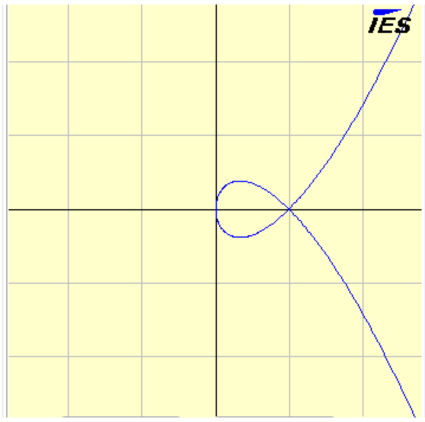
   ii) $x = 9$

b) Find the Area of R

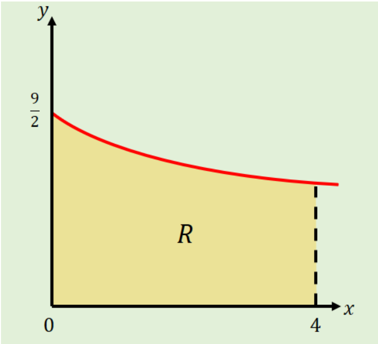
4. The diagram shows a sketch of the curve with Parametric equations:

$$x = 2t^2 \qquad y = t(4 - t^2)$$

Calculate the finite area inside the loop…



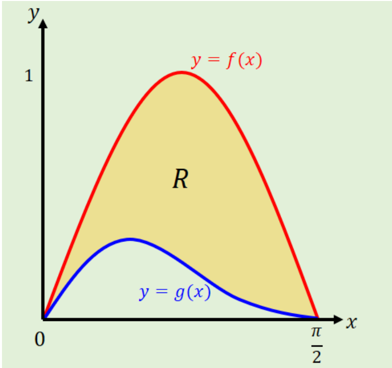
<u>11H Part 1 Integrating to Find Areas</u>



1. The diagram shows part of the curve:

$$y = \frac{9}{\sqrt{4+3x}}$$

The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 4$, and shown. Use integration to find the area of R.



2. The diagram shows part of the curves $y = f(x)$ and $y = g(x)$, where:

$$f(x) = sin2x$$

$$g(x) = sinxcos^2x$$

$$0 \leq x \leq \frac{\pi}{2}$$

The region R is bounded by the two curves. Use integration to find the area of R.

11I The Trapezium Rule

1. Using 4 strips, estimate the area under the curve:

$$y = \sqrt{2x + 3}$$

   Between the lines x = 0 and x = 2

2. Using 8 strips, estimate the area under the curve:

$$y = \sqrt{2x + 3}$$

Between the lines x = 0 and x = 2

3. Complete the table of values and use it to find an estimate for:

$$\int_0^{\frac{\pi}{3}} secx\, dx$$

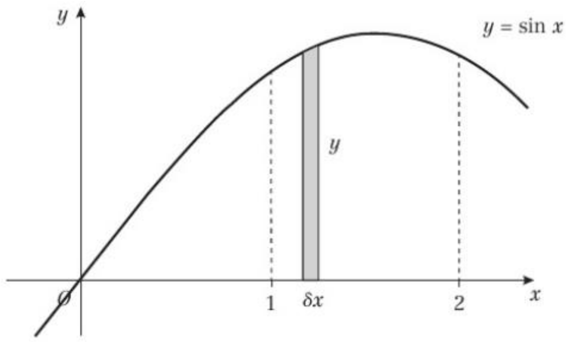
x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y					

4. Use the trapezium rule with 4 strips to find an approximation for:

$$\int_0^2 x\sin x \, dx$$

11L Integration as the Limit of a Sum

1. The diagram shows a sketch of the curve with equation $y = \sin x$.
   The area under the curve between $x = 1$ and $x = 2$ can be thought of as a thin series of strips of height y and width δx.
   Calculate $\lim_{\delta x \to 0} \sum_{x=1}^{2} \sin x \ \delta x$, giving your answer correct to 4 significant figures.



11J Separating the Variables

1. Find a general solution to the differential equation:

$$(1 + x^2)\frac{dy}{dx} = xtany$$

2. Find the **particular solution** of the differential equation:

$$\frac{dy}{dx} = \frac{-3(y-2)}{(2x+1)(x+2)}$$

given that x = 1 when y = 4

11K Modelling with Integration

1. The rate of increase of a population P of micro organisms at time t, in hours, is given by:

$$\frac{dP}{dt} = 3P, \quad t > 0$$

Initially, the population was of size 8.

a) Find a model for P in the form $P = Ae^{3t}$, stating the value of A

b) Find, to the nearest hundred, the size of the population at the time $t = 2$

c) Find the time at which the population will be 1000 times its starting value.

d) State one limitation of this model for large values of t

2. Water in a manufacturing plant is held in a large cylindrical tank of diameter 20m. Water flows out of the bottom of the tank through a tap at a rate proportional to the cube root of the volume (of the water).

a) Show that after t minutes after the tap is opened, $\frac{dh}{dt} = -k\sqrt[3]{h}$ for some constant k.

b) Show that the general solution to this differential equation may be written as $h = (P - Qt)^{\frac{3}{2}}$, where P and Q are constants

Initially, the height of the water is 27m. 10 minutes later, the height is 8m.

c) Find the values of the constants P and Q

d) Find the time in minutes when the water is at a depth of 1m