QQQ – Statistics Yr2 - Chapter 3 – The Normal Distribution

Total Marks: 35

(35 = Platinum, 32= Gold, 28 = Silver, 25 = Bronze)

1.

The heights of a population of men are normally distributed with mean μ cm and standard deviation σ cm. It is known that 20% of the men are taller than 180 cm and 5% are shorter than 170 cm.

(a) Sketch a diagram to show the distribution of heights represented by this information.

(3)

(b) Find the value of μ and σ.

(7)

(c) Three men are selected at random, find the probability that they are all taller than 175 cm.

(2)

(Total 12 marks)

2.

A company sells seeds and claims that 55% of its pea seeds germinate.

(a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.

(1)

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

(b) Assuming that the company's claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate.

(3)

(c) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.

(1)

A random sample of 240 pea seeds was planted and 150 of these seeds germinated.

(d) Assuming that the company's claim is correct, use a normal approximation to find the probability that at least 150 pea seeds germinate.

(3)

(e) Using your answer to part (d), comment on whether or not the proportion of the company's pea seeds that germinate is different from the company's claim of 55%

(1)

3.

The lifetime, L hours, of a battery has a normal distribution with mean 18 hours and standard deviation 4 hours.

Alice's calculator requires 4 batteries and will stop working when any one battery reaches the end of its lifetime.

(a) Find the probability that a randomly selected battery will last for longer than 16 hours.

(1)

At the start of her exams Alice put 4 new batteries in her calculator.
She has used her calculator for 16 hours, but has another 4 hours of exams to sit.

(b) Find the probability that her calculator will not stop working for Alice's remaining exams.

(5)

Alice only has 2 new batteries so, after the first 16 hours of her exams, although her calculator is still working, she randomly selects 2 of the batteries from her calculator and replaces these with the 2 new batteries.

(c) Show that the probability that her calculator will not stop working for the remainder of her exams is 0.199 to 3 significant figures.

(3)

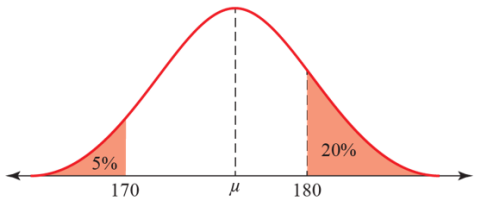
After her exams, Alice believed that the lifetime of the batteries was more than 18 hours. She took a random sample of 20 of these batteries and found that their mean lifetime was 19.2 hours.

(d) Stating your hypotheses clearly and using a 5% level of significance, test Alice's belief.

(5)

Solutions

1

3a	 bell shaped	**B1**	1.2	5th Understand the basic features of the normal distribution including parameters, shape and notation.
	170, 180 on axis	**B1**	1.1b	
	5% and 20%	**B1**	1.1b	
		(3)		
3b	$P(X < 170) = 0.05$	**M1**	3.3	7th Find unknown means and/or standard deviations for normal distributions.
	$\frac{170 - \mu}{\sigma} = -1.6449$	**B1**	3.4	
	$\mu = 170 + 1.6449\sigma$	**B1**	1.1b	
	$P(X > 180) = 0.2$	**B1**	3.4	
	$\mu = 180 - 0.8416\sigma$	**M1**	1.1b	
	Solving simultaneously gives:	**A1**	1.1b	
	$\mu = 176.615\ldots$ (awrt 176.6) and $\sigma = 4.021\ldots$(awrt 4.02)	**A1**	1.1b	
		(7)		
3c	P(All three are taller than 175 cm) = $0.656\ldots^3$	**M1**	1.1b	5th Understand informally the link to probability distributions.
	= 0.282… (using calculator) awrt 0.282	**A1**	1.1b	
		(2)		

(12 marks)

2.

5 (a)	The seeds would be destroyed in the process so they would have none to sell	B1	2.4
		(1)	
(b)	[S = no. of seeds out of 24 that germinate, $S \sim B(24, 0.55)$]		
	T = no. of trays with at least 15 germinating. $T \sim B(10, p)$	M1	3.3
	$p = P(S \geqslant 15) = 0.299126...$	A1	1.1b
	So $P(T \geqslant 5) = 0.1487\ldots$ awrt **<u>0.149</u>**	A1	1.1b
		(3)	
(c)	n is large and p close to 0.5	B1	1.2
		(1)	

(d)	$X \sim N(132, 59.4)$	B1	3.4
	$P(X \geqslant 149.5) = P\left(Z \geqslant \frac{149.5-132}{\sqrt{59.4}}\right)$	M1	1.1b
	$= 0.01158\ldots$ awrt **0.0116**	A1cso	1.1b
		(3)	
(e)	e.g The probability is very small therefore there is evidence that the company's claim is incorrect.	B1	2.2b
		(1)	

(9 marks)

3.

Qu 5	**Scheme**	**Marks**	**AO**
(a)	$P(L > 16) = 0.69146\ldots$ **awrt 0.691**	B1 (1)	1.1b
(b)	$P(L > 20 \mid L > 16) = \frac{P(L>20)}{P(L>16)}$	M1	3.1b
	$= \frac{0.308537...}{(a)}$ or $\frac{1-(a)}{(a)}, = 0.44621\ldots$	A1ft, A1	1.1b 1.1b
	For calc to work require $(0.44621...)^4 = 0.03964\ldots$ **awrt 0.0396**	dM1 A1 (5)	2.1 1.1b
(c)	Require: $[P(L>4)]^2 \times [P(L>20 \mid L>16)]^2$	M1	1.1a
	$= (0.99976...)^2 \times ("0.44621...")^2$	A1ft	1.1b
	$= 0.19901\ldots$ **awrt 0.199 (*)**	A1cso* (3)	1.1b
(d)	$H_0 : \mu = 18 \quad H_1 : \mu > 18$	B1	2.5
	$\bar{L} \sim N\left(18, \left(\frac{4}{\sqrt{20}}\right)^2\right)$	M1	3.3
	$P(\bar{L} > 19.2) = P(Z > 1.3416\ldots) = 0.089856\ldots$	A1	3.4
	$(0.0899 > 5\%)$ or $(19.2 < 19.5)$ or $1.34 < 1.6449$ so not significant	A1	1.1b
	Insufficient evidence to support Alice's claim (or belief)	A1 (5)	3.5a
		(14 marks)	