QQQ – Statistics Yr2 - Chapter 2 – Conditional Probability

Total Marks: 29

(29 = Platinum, 27= Gold, 24 = Silver, 21 = Bronze)

1.

A mechanic carried out a survey on the defects of cars he was servicing. He found that the probability of a car needing a new tyre is 0.33 and that a car needing a new tyre has a probability of 0.7 of needing tracking. A car not needing a new tyre has a probability of 0.04 of needing tracking.

(a) Draw a tree diagram to represent this information.

(3)

(b) Find the probability that a randomly chosen car has exactly one of the two defects, needing a new tyre or needing tracking.

(2)

The mechanic also finds that cars need new brake pads with probability 0.35 and that this is independent of needing new tyres or tracking. A car is chosen at random.

(c) Find the probability that the car has at least one of these three defects.

(2)

(d) What advice would you give to motorists?

(1)

(Total 8 marks)

2.

$$P(E) = 0.25,\ P(F) = 0.4 \text{ and } P(E \cap F) = 0.12$$

(a) Find $P(E'|F')$

(2)

(b) Explain, showing your working, whether or not E and F are statistically independent. Give reasons for your answer.

(2)

The event G has $P(G) = 0.15$.

The events E and G are mutually exclusive and the events F and G are independent.

(c) Draw a Venn diagram to illustrate the events E, F and G, giving the probabilities for each region.

(5)

(d) Find $P([F \cup G]')$

(2)

(Total 11 marks)

3.

The table below shows the number of gold, silver and bronze medals won by two teams in an athletics competition.

	Gold	**Silver**	**Bronze**
Team *A*	29	17	18
Team *C*	21	23	17

The events G, S and B are that a medal is gold, silver or bronze respectively. Let A be the event that team A won a medal and C team C won a medal. A medal winner is selected at random. Find

(a) $\text{P}(G)$,

(2)

(b) $\text{P}([A \cap S]')$.

(2)

(c) Explain, showing your working, whether or not events S and A are statistically independent. Give reasons for your answer.

(2)

(d) Determine whether or not events B and C are mutually exclusive. Give a reason for your answer.

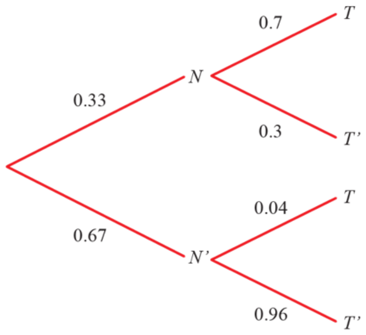
(2)

(e) Given that 30% of the gold medal winners are female, 60% of the silver medal winners are female and 40% of the bronze medal winners are female, find the probability that a randomly selected medal winner is female.

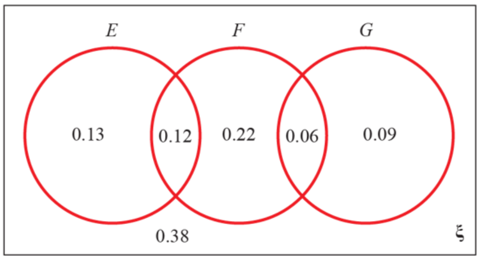
(2)

(Total 10 marks)

Solutions

2a	 Let *N*~ new tyre and *T*~ tracking	**B1**	2.5	3rd Draw and use tree diagrams with three branches and/or three levels.
	P(*N*) = 0.33 and P(*T*) = 0.67	**B1**	1.1b	
	0.7, 0.3, 0.04 and 0.96	**B1**	1.1b	
		(3)		
2b	P(exactly one defect) = $0.33 \times 0.3 + 0.67 \times 0.04$	**M1**	3.1b	5th Understand the language and notation of conditional probability.
	$= 0.1258$	**A1**	1.1b	
		(2)		
2c	1 − P(no defects) $=1 - 0.67 \times 0.96 \times 0.65$	**M1**	3.1b	5th Understand the language and notation of conditional probability.
	$= 0.5819\ldots$ awrt 0.582 (3 d.p.)	**A1**	1.1b	
		(2)		
2d	To have their cars checked regularly as there is over a 50 % chance they need new tyres, tracking or brake pads.	**B1**	3.2a	5th Understand the language and notation of conditional probability.
		(1)		

(8 marks)

5a	$\mathrm{P}(E' \mid F') = \dfrac{\mathrm{P}(E' \cap F')}{\mathrm{P}(F')}$ or $\dfrac{0.47}{0.6}$	**M1**	3.1a	4th Calculate probabilities using set notation.
	$= \dfrac{47}{60}$ or 0.783 (3 s.f.)	**A1**	1.1b	
		(2)		
5b	$\mathrm{P}(E) \times \mathrm{P}(F) = 0.25 \times 0.4 = 0.1 \neq \mathrm{P}(E \cap F) = 0.12$	**M1**	2.1	4th Understand and use the definition of independence in probability calculations.
	So, E and F are not statistically independent.	**A1**	2.4	
		(2)		
5c		**B1**	2.5	3rd Understand and use Venn diagrams for multiple events.
	Use of independence and all values in G correct.	**M1A1**	3.1a	
	All values correct.	**M1A1**	1.1b	
			1.1b	
			1.1b	
		(5)		
5d	$\mathrm{P}([F \cup G]') = 0.13 + 0.38$	**M1**	3.1a	4th Calculate probabilities using set notation.
	$= 0.51$	**A1**	1.1b	
		(2)		

(11 marks)

3a	$\frac{29+21}{29+21+17+23+18+17} = \frac{50}{125}$	**M1**	1.1b	2nd Calculate probabilities from relative frequency tables and real data.
	$= 0.4$	**A1**	1.1b	
		(2)		
3b	$\frac{125-17}{125} = \frac{108}{125}$	**M1**	3.1a	4th Understand set notation.
	$= 0.864$	**A1**	1.1b	
		(2)		
3c	$\text{P}(S \cap A) = \frac{17}{125} = 0.136 \neq \text{P}(S) \times \text{P}(A) = \frac{40}{125} \times \frac{64}{125} = 0.163\ldots$	**M1**	2.1	4th Understand and use the definition of independence in probability calculations.
	So, *S* and *A* are not statistically independent.	**A1**	2.4	
		(2)		
3d	*B* and *C* are not mutally exclusive	**B1**	2.2a	3rd Understand and use the definition of mutually exclusive in probability calculations.
	Being in team *C* does not exclude the possibility of winning a bronze medal	**B1**	2.4	
		(2)		
3e	$\frac{15+24+14}{125} = \frac{53}{125}$	**M1**	3.1b	5th Calculate conditional probabilities using formulae.
	$= 0.424$	**A1**	1.1b	
		(2)		

(10 marks)