QQQ – Statistics Yr2 - Chapter 1 – More complex numbers

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 17 = Bronze)

1. A complex number z has modulus 1 and argument θ.

(a) Show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta, \qquad n \in \mathbb{Z}^+$$

(2)

(b) Hence, show that

$$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$$

(5)

2. In an Argand diagram, the points A, B and C are the vertices of an equilateral triangle with its centre at the origin. The point A represents the complex number $6 + 2\mathrm{i}$.

(a) Find the complex numbers represented by the points B and C, giving your answers in the form $x + \mathrm{i}y$, where x and y are real and exact.

(6)

The points D, E and F are the midpoints of the sides of triangle ABC.

(b) Find the exact area of triangle DEF.

(3)

3. The infinite series C and S are defined by

$$\mathrm{C} = \cos\theta + \frac{1}{2}\cos 5\theta + \frac{1}{4}\cos 9\theta + \frac{1}{8}\cos 13\theta + \ldots$$

$$\mathrm{S} = \sin\theta + \frac{1}{2}\sin 5\theta + \frac{1}{4}\sin 9\theta + \frac{1}{8}\sin 13\theta + \ldots$$

Given that the series C and S are both convergent,

(a) show that

$$\mathrm{C} + \mathrm{iS} = \frac{2\mathrm{e}^{\mathrm{i}\theta}}{2 - \mathrm{e}^{4\mathrm{i}\theta}}$$

(4)

(b) Hence show that

$$\mathrm{S} = \frac{4\sin\theta + 2\sin 3\theta}{5 - 4\cos 4\theta}$$

(4)