QQQ – Core Pure 2 - Chapter 5 – Polar Coordinates

Total Marks: 21

(21 = Platinum, 19 = Gold, 17 = Silver, 15 = Bronze)

1.

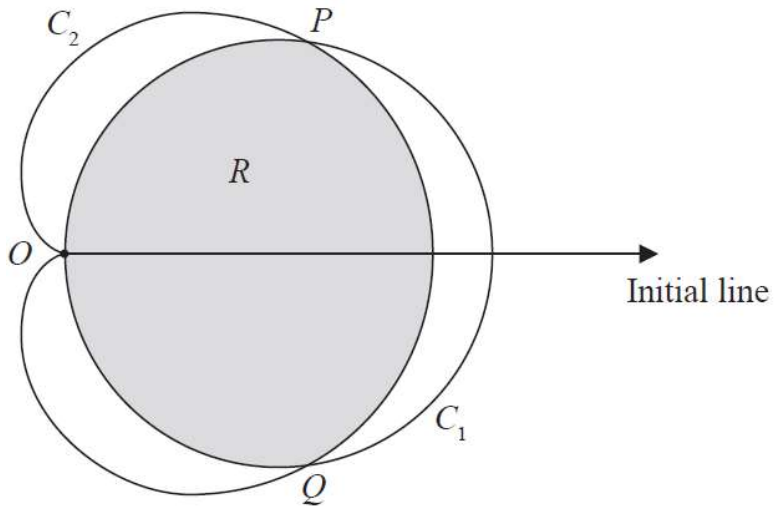

Figure 1

The curve C_1 with equation

$$r = 7\cos\theta, \quad -\frac{\pi}{2} < \theta \leqslant \frac{\pi}{2}$$

and the curve C_2 with equation

$$r = 3(1 + \cos\theta), \quad -\pi < \theta \leqslant \pi$$

are shown on Figure 1.

The curves C_1 and C_2 both pass through the pole and intersect at the point P and the point Q.

(a) Find the polar coordinates of P and the polar coordinates of Q.

(3)

The regions enclosed by the curve C_1 and the curve C_2 overlap, and the common region R is shaded in Figure 1.

(b) Find the area of R.

(7)

2.

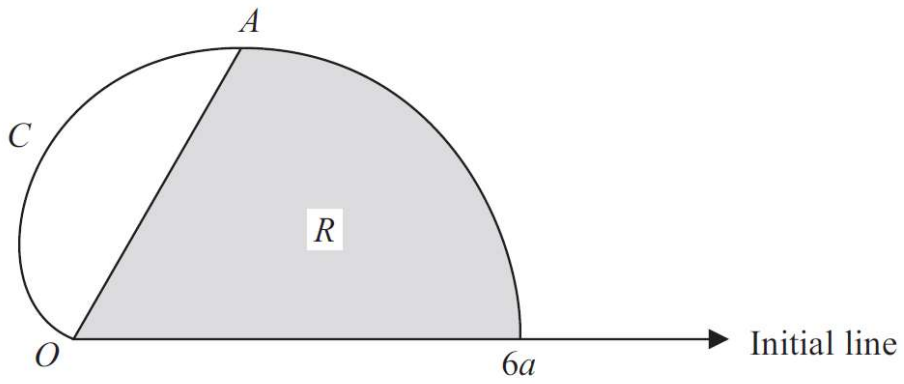

Figure 1

The curve C, shown in Figure 1, has polar equation

$$r = 3a\,(1 + \cos\theta), \qquad 0 \leqslant \theta < \pi$$

The tangent to C at the point A is parallel to the initial line.

(a) Find the polar coordinates of A.

(6)

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line OA.

(b) Use calculus to find the area of the shaded region R, giving your answer in the form $a^2\left(p\pi + q\sqrt{3}\right)$, where p and q are rational numbers.

(5)