## AQA

Please write clearly in block capitals.

Centre number $\square$ Candidate number


Surname
Forename(s)
Candidate signature $\qquad$

## Level 2 Certificate FURTHER MATHEMATICS

## Paper 2 Calculator

Monday 17 June 2019

## Materials

For this paper you must have:

- a calculator
- mathematical instruments.


## Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.


## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must not be used.

Afternoon
Time allowed: 2 hours

| For Examiner's Use |  |
| :---: | :---: |
| Pages | Mark |
| 3 |  |
| $4-5$ |  |
| $6-7$ |  |
| $8-9$ |  |
| $10-11$ |  |
| $12-13$ |  |
| $14-15$ |  |
| $16-17$ |  |
| $18-19$ |  |
| $20-21$ |  |
| $22-23$ |  |
| $24-25$ |  |
| $26-27$ |  |
| $28-29$ |  |
| 30 |  |
| TOTAL |  |

## Formulae Sheet

Volume of sphere $=\frac{4}{3} \pi r^{3}$
Surface area of sphere $=4 \pi r^{2}$

Volume of cone $=\frac{1}{3} \pi r^{2} h$
Curved surface area of cone $=\pi r l$


In any triangle $A B C$
Area of triangle $=\frac{1}{2} a b \sin C$


Sine rule $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
Cosine rule $a^{2}=b^{2}+c^{2}-2 b c \cos A$

$$
\cos A=\frac{b^{2}+c^{2}-a^{2}}{2 b c}
$$

## The Quadratic Equation

The solutions of $a x^{2}+b x+c=0$, where $a \neq 0$, are given by $\quad x=\frac{-b \pm \sqrt{\left(b^{2}-4 a c\right)}}{2 a}$

Trigonometric Identities
$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \quad \sin ^{2} \theta+\cos ^{2} \theta \equiv 1$
$\qquad$
$a=$ $b=$ $\qquad$

1 (b) $\quad\left(\begin{array}{cc}m & -1 \\ 1 & 1\end{array}\right)\left(\begin{array}{cc}2 & 2 \\ -2 & -1\end{array}\right)=\mathbf{I} \quad$ where I is the identity matrix.
Work out the value of $m$.

2 Here is a sketch of quadrilateral $P Q R S$.
$M$ is the midpoint of $P S$.


Use gradients to show that $M R$ is parallel to $P Q$.
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$\qquad$
$\qquad$
$\qquad$
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$\qquad$
$\qquad$
$\qquad$
$\qquad$
$3-2<a<0$ and $-1<b<1$

Tick the correct box for each statement.

|  | Always true | Sometimes true | Never true |
| :--- | :--- | ---: | ---: |
| $a^{2}<0$ | $\square$ | $\square$ | $\square$ |
| $-1<b^{3}<1$ | $\square$ | $\square$ |  |
| $\frac{b}{a}<0$ | $\square$ | $\square$ | $\square$ |
| $a-b>0$ | $\square$ | $\square$ |  |

## Turn over for the next question

$4 \quad P$ is a point on a curve.
The curve has gradient function $\frac{x^{5}-17}{10}$
The tangent to the curve at $P$ is parallel to the line $\quad 3 x-2 y=9$
Work out the $x$-coordinate of $P$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Answer $\qquad$

5 (a) Write $\sqrt[4]{a \times a^{-9}}$ as an integer power of $a$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Answer $\qquad$

5 (b) Simplify fully $\frac{\left(4 c d^{2}\right)^{3}}{2 c d^{4}}$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Answer $\qquad$

## Turn over for the next question

$6 \quad$ Here is a sketch of the curve $y=(2 x+3)(x-2)$
The curve intersects the $x$-axis at $A$ and $B$.


6 (a) Complete the coordinates of $A$ and $B$.
A ( $\qquad$ , 0 )
B( $\qquad$ , 0 )

6 (b) Write down the range of values for $x$ for which $\quad(2 x+3)(x-2)<0$
$\qquad$
$\qquad$
$\qquad$

Answer $\qquad$

7 (a) On the grid, sketch a graph for which the rate of change of $y$ with respect to $x$ is always zero.


7 (b) On the grid, sketch a graph for which
the rate of change of $y$ with respect to $x$ is always a positive constant.


8 (a) A linear sequence has first term $7+12 \sqrt{5}$
The term-to-term rule is

$$
\text { add } 9-2 \sqrt{5}
$$

One term of the sequence is an integer.
Work out the value of this integer.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Answer $\qquad$

8 (b) The $n$th term of a different sequence is $\frac{3 n^{2}-1}{n^{2}+1}$
Work out the sum of the first three terms.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Answer $\qquad$

$9 \quad$ Factorise fully $\quad(p+6)^{11}-(p+6)^{10}$

## Answer

$\qquad$
$\qquad$
$\qquad$

10 (a) $\mathrm{f}(x)=x^{3}-2$
The domain of $\mathrm{f}(x)$ is $\quad x \leqslant 3$
Work out the range of $\mathrm{f}(x)$.
$\qquad$
$\qquad$

Answer $\qquad$

10 (b) $\mathrm{g}(x)=5-x^{2}$
The domain of $\mathrm{g}(x)$ is $-2 \leqslant x \leqslant 1$
Work out the range of $\mathrm{g}(x)$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Answer $\qquad$

11 Here is a sketch of a quadratic curve which has a maximum point at $(-2,5)$


What is the equation of the normal to the curve at the maximum point? Circle your answer.

$$
x=-2 \quad y=5 \quad x=5 \quad y=-2
$$

Turn over for the next question

12 The diagram shows a solid hemisphere.
The diameter is $12 a \mathrm{~cm}$
The volume is $486 \pi \mathrm{~cm}^{3}$

Work out the value of $a$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Answer $\qquad$


| 13 | Simplify fully $\quad \frac{x-x^{3}}{2 x+2 x^{2}}$ |  |
| :--- | :--- | :--- |
|  | You must show your working. | [4 marks] |

Answer $\qquad$

Turn over for the next question

14 Here is a triangle.


Not drawn
accurately

Use the cosine rule to work out the ratio $b^{2}: a^{2}$
Use the cosine rule to wor out the raio $b^{2}: a^{2}$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
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$\qquad$

Answer $\qquad$ : $\qquad$

15 Rearrange $m=\frac{2 p+1}{p}+\frac{p+5}{3 p} \quad$ to make $p$ the subject.

Answer $\qquad$

16 The curve $y=2 \sqrt{x-a}+5$ passes through the point (1, 8)
Work out the value of $a$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Answer $\qquad$

17 Show that $(x+1)(x+3)(x+4)-x\left(x^{2}+7 x+11\right)$ can be written in the form $\quad(x+a)(x+b) \quad$ where $a$ and $b$ are positive integers.
[5 marks]
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18 Solve $\quad 4(x-5)^{2}=k^{2} \quad$ where $k$ is a constant.
Give your answers in their simplest form in terms of $k$.
Do not write outside the box
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Answer $\qquad$

## Turn over for the next question

$19 \quad A B C$ is a right-angled triangle.
$A C D$ is an isosceles triangle.
All dimensions are in centimetres.


Not drawn accurately

19 (a) Show that $A C=5 x$
$\qquad$
$\qquad$
$\qquad$

19 (b) Work out an expression, in $\mathrm{cm}^{2}$, for the area of quadrilateral $A B C D$. Give your answer in the form $p x^{2} \quad$ where $p$ is an integer.
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$\qquad$

Answer
$\mathrm{cm}^{2}$

Turn over for the next question
$20 \quad A, B, C$ and $D$ are points on a circle.
$D, E$ and $F$ are points on different circle, centre $C$.
$D C E, A D F$ and $B C F$ are straight lines.
angle $D E F=x$
Not drawn accurately


20 (a) Prove that angle $B A D=2 x$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

20 (b) In the case when $A B$ is parallel to $D E$, work out the size of angle $x$.
Do not write
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Answer $\qquad$ degrees

## Turn over for the next question

21 ABCDEFGH is a cuboid.

$$
B C=15 \mathrm{~cm} \quad C D=12 \mathrm{~cm} \quad D H=8 \mathrm{~cm}
$$

Work out the size of the angle between the line CE and the plane CDHG.
$\qquad$
$\qquad$
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Answer $\qquad$ degrees


22 (a) Show that $\frac{2 \sin ^{2} x-1+\cos ^{2} x}{\sin x \cos x}$ is equivalent to $\tan x$
Do not write

22 (b) Hence solve $\quad \frac{2 \sin ^{2} x-1+\cos ^{2} x}{\sin x \cos x}=-1 \quad$ for $0^{\circ} \leqslant x \leqslant 360^{\circ}$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Answer $\qquad$

23 A circle has centre $C$ and equation $(x-1)^{2}+(y+3)^{2}=25$
$P(4,-7)$ and $Q$ are points on the circle.
The tangent at $Q$ is parallel to the $x$-axis.
The tangents at $P$ and $Q$ intersect at point $R$.


Not drawn accurately

23 (a) Write down the coordinates of $C$.

Answer $\qquad$


24 Show that the curve $y=\frac{3}{5} x^{5}+x^{4} \quad$ has exactly two stationary points.
$\qquad$
$\qquad$
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$\qquad$
$25 \quad \mathrm{f}(x)=x^{3}-10 x-c \quad$ where $c$ is a positive integer. $(x+c)$ is a factor of $\mathrm{f}(x)$.

Use the factor theorem to work out the value of $c$.
$\qquad$
$\qquad$
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Answer $\qquad$
$26 \quad \mathrm{f}(x)$ is a function with domain all values of $x$.

$$
\mathrm{f}(x)=\sqrt{x^{2}+6 x-a} \quad \text { where } a \text { is a constant. }
$$

Work out the possible values of $a$.
Give your answer as an inequality.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
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$\qquad$
$\qquad$

Answer $\qquad$

## Turn over for the next question

27 The curve $y=\mathrm{f}(x)$ has $\frac{\mathrm{d} y}{\mathrm{~d} x}=(x+2)^{6}+(x+2)^{4}$
The curve has exactly one stationary point at $P$ where $x=-2$
Use the expression for $\frac{\mathrm{d} y}{\mathrm{~d} x}$ to show that $P$ is a point of inflection.
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## END OF QUESTIONS








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