

LEVEL 2 CERTIFICATE Further Mathematics

8360/1 – Paper 1 Non-calculator Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Μ	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
Α	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
В	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent.
	eg, accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between <i>a</i> and <i>b</i> inclusive.
3.14	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

Q	Answer	Mark	Comments	
	$\frac{6x^5}{2}$ or $3x^5$ or $\frac{4x^3}{4}$ or x^3	M1	oe eg $\frac{12x^5}{4}$	
_	$3x^5 + x^3$	A1	or a correct factorised version eg $x^{3}(3x^{2} + 1)$	
1	Ad	ditional G	uidance	
	Do not ignore further work, eg correct a	answer foll	owed by $4x^8$ scores M1 A0	
	They must use the powers of <i>x</i> as give possible here	n in the qu	estion, so no misread	
	Alternative method 1			
	$\frac{6 + (-12)}{2} \text{ or } \frac{-2 + b}{2}$	M1	oe eg $18 \div 2 = 9$ and $6 - 9$ eg $6 \times 2 = 12$ and $-2 + 12$ These come from distances of 18 and 6, as seen in a diagram and used correctly	
	<i>a</i> = -3	A1		
	<i>b</i> = 10	A1		
	Alternative method 2			
	a - (-12) = 6 - a or 4 - b = -2 - 4	M1	oe eg 4+42	
2	<i>a</i> = -3	A1		
2	<i>b</i> = 10	A1		
	Alternative method 3			
	eg $6 - (-12) = 2(a12)$ eg $-2 - b = 2(-2 - 4)$	M1	for using an equation relating the "gap" between the points	
	<i>a</i> = -3	A1		
	<i>b</i> = 10	A1		
	Add	litional gu	idance	

Either answer correct, but no working, implies the M mark, eg a = -3, b = 6 scores M1 A1 A0 Correct answer seen with no working scores full marks

a = 10 and b = -3 (correct values but the wrong way round) with no working scores SC1

Q	Answer	Mark	Comments	
	$ \left(\begin{array}{cc} 4 & 16 \\ -8 & 17 \end{array}\right) $	B2	B1 for any two or three correct elements in the correct position in a 2×2 matrix	
	Additional Guidance			
	Correct answer followed by further wor	k, eg (20 9)	scores B1 only	
3	Matrices multiplied the wrong way rour	nd can sco	ore SC1 if correct	
	$BA = \begin{pmatrix} 14 & -14 \\ 7 & 7 \end{pmatrix}$			
	Condone no brackets around the numb	pers in the	ir 2 \times 2 matrix	
	atrix			
	Do not follow through on any misreads	of the nu	mbers in the given matrices	

Q	Answer	Mark	Comments	
	Alternative method 1			
	$1.25 \times 4x$ or $5x$	M1	ое	
	$0.6 \times 7x$ or $4.2x$	M1	ое	
	their $5x - $ their $4.2x = 28$		oe eg their $5x =$ their $4.2x + 2x$	8
	or $0.8x = 28$	M1dep	dep upon at least one of previo earned	us M marks
	<i>x</i> = 35	A1		
	Alternative method 2			
	two numbers in the ratio 4 : 7	M1		
4	correct increase by 25% and decrease by 40% calculations and comparison with 28	M1dep	If difference is not 28, then first must be clearly rejected	numbers
	second trial with correct calculations and comparison	M1dep	correct first trial means 2nd an marks scored automatically	d 3rd M
	<i>x</i> = 35	A1		
	Additional Guidance			
	Mark the better of their two versions if	they try bo	oth methods.	
	In alt 2 for the 2nd M1 (dep on 1st M1) the % calculations must be correct. If the difference is not 28 they must reject them. Attempting another two % calculations is sufficient evidence of this.			
	In alt 2 for the 3rd M1 (dep on the first two M1's) the difference must be closer than their first attempt. They can have more than one attempt at this so as to eventually score the 3rd M1. To score this mark they need to indicate clearly that this further attempt is better than their first attempt.			
	In alt 2 if it isn't clear in which order untidy working written all over the page better attempt, then they can score a n	e) and the	y do not indicate which is the	

Q	Answer	Mark	Comments	
	$(x^{3}) + 5x^{2} + kx - 3x^{2} - 15x (-3k)$	M1	allow one sign error in the x^2 or x terms	
	their (5 − 3) = their (<i>k</i> − 15)	M1dep	$5x^2 - 3x^2 = kx - 15x$ on its own, is not enough for M1dep	
	17	A1		
5	Additional Guidance			
	For the first M1, we do not need to see need to see the other 4 terms (3 terms			
	The terms of the expansion might appear in a grid, which can score the first M1			
	Mark positively terms in a grid might terms mark the better version.	t differ fror	n terms written as a string of	

	$(x+1)^2 + (y-2)^2 = 25$	B1	tick in 3rd box	
6	Additional Guidance			

Q	Answer	Mark	Comments	
	Alternative method 1			
	reflex angle $AOC = 2 \times 2x$ or $4x$	M1		
	their $4x + x + 75 = 360$	M1dep	oe If they start with this equation, the first M1, for reflex angle $AOC = 4x$, is implied	
	(<i>x</i> =) 57	A1		
	Alternative method 2		l	
	reflex angle $AOC = 360 - (x + 75)$ or $285 - x$	M1	oe	
	360 - (x + 75) = 2(2x) or their $285 - x = 2(2x)$	M1dep	oe	
	(<i>x</i> =) 57	A1		
7	Alternative method 3			
1	angle at circumference = $180 - 2x$	M1	creating a cyclic quadrilateral	
	x + 75 = 2(180 - 2x) or $x + 75 = 360 - 2(2x)$	M1dep	oe	
	(<i>x</i> =) 57	A1		
	Alternative method 4			
	angle at circumference = $\frac{x + 75}{2}$	M1	oe creating a cyclic quadrilateral	
	$\frac{x+75}{2} + 2x = 180$	M1dep	oe $\frac{x}{2} + \frac{\text{their 75}}{2} + 2x = 180$ scores this mark	
	(<i>x</i> =) 57	A1		
	٩	Additional G	Guidance	
	4x = x + 75 (ans $x = 25$) and $x + 75 + 2x = 180$ (ans $x = 35$) both score 0 marks			

Q	Answer	Mark	Comments
	$4 - \sqrt{5} + 8\sqrt{5} - 2\sqrt{5}\sqrt{5}$	M1	oe allow one incorrect term in a four term expansion
8	$-6 + 7\sqrt{5}$	A1	
	Additional Guidance		
	Any incorrect further work loses the A	mark, so tl	ney can only score M1 A0

	Alternative method 1		
	$14 - (2x)^2$ or $14 - 4x^2$	M1	or $14 - (2x)^2 = 5$ or $14 - 4x^2 = 5$
	$14-5=(2x)^2$ or $9=4x^2$		
	or $9 - 4x^2 = 0$ or $4x^2 - 9 = 0$	M1dep	
	or $(2x + 3)(2x - 3) = 0$		
	$(x =) \frac{3}{2}$ or 1.5 $(x =) -\frac{3}{2}$ or -1.5	A1	
	$(x =) -\frac{3}{2}$ or -1.5	A1	
	Alternative method 2		
9	$14 - x^2 = 5$ and $x = \pm 3$	M1	
	$2x = \pm 3$	M1dep	
	$(x =) \frac{3}{2}$ or 1.5	A1	
	$(x =) -\frac{3}{2}$ or -1.5	A1	
	Ad	ditional G	buidance
	A final answer of $\sqrt{\frac{9}{4}}$ scores M1 M1 A	0 A0	
	A final answer of $\pm \sqrt{\frac{9}{4}}$ scores M1 M1	A1 A0	

Q	Answer	Mark	Comments
	A correct first step using algebra	M1	Here are some of the possible alternatives $\frac{1}{x} = y\left(4 - \frac{3}{y}\right) \text{multiplying through by } y$ $1 = xy\left(4 - \frac{3}{y}\right) \text{multiplying through by } xy$ $1 = 4xy - \frac{3xy}{y} \text{multiplying through by } xy$ $y = 4xy^2 - 3xy \text{multiplying through by } xy^2$ $\frac{1}{xy} = \frac{4y - 3}{y} \text{multiplying through by } xy^2$ $\frac{1 + 3x}{xy} = 4 \text{rearranging and making the } LHS \text{ an algebraic fraction}$
10	Further correct algebra which leads to an equation that is one step from the final answer.	M1dep	Following two of the above alternatives $y = 4xy^2 - 3xy$ $y = x(4y^2 - 3y)$ M1dep gained $\frac{1 + 3x}{xy} = 4$ xy 1 + 3x = 4xy 1 = 4xy - 3x 1 = x(4y - 3) M1dep gained
	A correct final answer in any form	A1	$x = \frac{1}{4y - 3} \qquad x = \frac{-1}{3 - 4y}$ $x = \frac{y}{4y^2 - 3y} \qquad x = \frac{-y}{3y - 4y^2}$ $x = \frac{1}{y\left(4 - \frac{3}{y}\right)} \qquad x = \frac{-1}{y\left(\frac{3}{y} - 4\right)}$ $x = \frac{1}{\left(4 - \frac{3}{y}\right)} \div y$

Additional Guidance	
There are many ways of scoring the first M mark. They do not need to give any reasons but you need to check that what they do is valid.	
For the M1dep mark you must check that their algebra is correct and will lead to a result that is one step from the final answer. 'One step from' means that when they divide through, they have a correct version where <i>x</i> is the subject.	
Some of the final answers are more compact than others, but we didn't ask for any simplification so we have to accept a correct answer in any form.	
and, finally, one to look out for correct answer from wrong working 0 marks	
$\frac{1}{xy} = 4 - \frac{3}{y} \rightarrow xy = \frac{1}{4} - \frac{y}{3} \rightarrow x = \frac{1}{4y} - \frac{1}{3} \rightarrow x = \frac{1}{4y - 3} \text{(creative thinking !)}$	

Q	Answer	Mark	Comments
	4x + 3 or gradient = -5 seen	M1	
	4x + 3 = -5	M1dep	
	x = -2	A1	
11	y = -7	A1ft	ft their x only if M2 earned
	Ad	ditional G	ouidance

	<u>5</u> × 15 3	M1		
	or			
	25 seen as the length of <i>OB</i> or the coordinates of <i>B</i>			
	gradient $AB = 0 - \text{their } 25 \text{ or } -5 $ 15 - 0 3	M1	ое	
	gradient $BC = -1 \div (\text{their} - \frac{5}{3}) \text{ or } \frac{3}{5}$	M1	oe	
	$y = \frac{3}{5}x + 25$	A1	oe eg $y = \frac{15}{25}x + 25$ or $5y = 3x + 125$	
	Ad	ditional G	Guidance	
12	We must see y = for A1 (or any other correct equation)Look for this in their working if it isn't written on the answer line.			
	expression, can be recovered.			
	eg gradient $AB = \frac{0-25}{15-0} = \frac{25}{15} = \frac{5}{3}$			
	gradient $BC = \frac{3}{5}$ (positive gradient be	cause the	ey can see it from the diagram)	
	equation BC is $y = \frac{3}{5}x + 25$ this scores 4 marks			
	<u>- 25</u> – 0			
	and can still lead to 4 marks			

Q	Answer	Mark	Comments		
	Alternative method 1				
	$3x + 5 = \frac{2}{x}$ or $x(3x + 5) = 2$	M1	oe		
	$3x^2 + 5x - 2 (= 0)$ or $3x^2 + 5x = 2$	M1dep			
	(3x + a)(x + b) (= 0)	M1dep	ab = -2 or $a + 3b = 5$		
	(3x - 1)(x + 2) (= 0)	A1			
	$x = \frac{1}{3}$ $x = -2$ or $x = \frac{1}{3}$ $y = 6$ or $x = -2$ $y = -1$	A1			
	or $x = -2$ $y = -1$ $x = \frac{1}{3}$ $x = -2$ $x = \frac{1}{3}$ $y = 6$ or y = 6 $y = -1$ $x = -2$ $y = -1$	A1	either correct x's and correct y's or correct coordinate pairs		
	Alternative method 2				
13	$3x + 5 = \frac{2}{x}$ or $x(3x + 5) = 2$	M1	oe		
	$3x^2 + 5x - 2 (= 0)$ or $3x^2 + 5x = 2$	M1dep			
	$x = \frac{-5 \pm \sqrt{[(5)^2 - 4(3)(-2)]}}{2(3)}$	M1dep	allow one sign error but the 2×3 term must be beneath the full numerator		
	$x = \frac{-5 \pm 7}{6}$	A1			
	$x = \frac{1}{3}$ $x = -2$ or $x = \frac{1}{3}$ $y = 6$ or $x = -2$ $y = -1$	A1			
	$x = \frac{1}{3} x = -2 \qquad x = \frac{1}{3} y = 6$ or $y = 6 y = -1 \qquad x = -2 y = -1$	A1	either correct <i>x</i> 's and correct <i>y</i> 's or correct coordinate pairs		

Alternative method 3			
M1	ое		
M1dep			
M1dep			
A1			
A1			
A1	either correct <i>x</i> 's and correct <i>y</i> 's or correct coordinate pairs		
M1	oe		
M1dep			
M1dep	ab = -6 or $a + b = -5$		
A1			
A1			
A1	either correct <i>x</i> 's and correct <i>y</i> 's or correct coordinate pairs		
	M1dep M1dep A1 A1 A1 M1 M1dep M1dep A1 A1		

Alternative method 5		
$y = 3 \left(\frac{2}{y} \right) + 5 \text{ or } y (y - 5) = 2$ 3	M1	oe
$y^2 - 5y - 6 = 0$ or $y^2 - 5y = 6$	M1dep	
$y = \frac{5 \pm \sqrt{[(-5)^2 - 4(1)(-6)]}}{2(1)}$	M1dep	allow one sign error but the 2×1 term must be beneath the full numerator
$y = \frac{5 \pm 7}{2}$	A1	
$y = 6 \ y = -1$ or $y = 6 \ x = \frac{1}{3}$ or $y = -1 \ x = -2$	A1	
$x = \frac{1}{3} x = -2 \qquad x = \frac{1}{3} y = 6$ or $y = 6 y = -1 \qquad x = -2 y = -1$	A1	either correct <i>x</i> 's and correct <i>y</i> 's or correct coordinate pairs
Alternative method 6		
$y = 3\left(\frac{2}{y}\right) + 5 \text{ or } \frac{y(y-5)}{3} = 2$	M1	oe
$y^2 - 5y - 6 = 0$ or $y^2 - 5y = 6$	M1dep	
$(y - \frac{5}{2})^2$	M1dep	
$y - \frac{5}{2} = \pm \frac{7}{2}$	A1	
$y = 6$ $y = -1$ or $y = 6$ $x = \frac{1}{3}$ or $y = -1$ $x = -2$	A1	
$x = \frac{1}{3} x = -2 \qquad x = \frac{1}{3} y = 6$ or $y = 6 y = -1 \qquad x = -2 y = -1$	A1	either correct <i>x</i> 's and correct <i>y</i> 's or correct coordinate pairs

Additional Guidance	
Trial and improvement 0 marks No working shown 0 marks The instructions were clearly stated in the question.	

Q	Answer	Mark	Comments		
	Alternative method 1				
	$\frac{\frac{1}{3^{2}} \times \frac{1}{3^{2}} + \frac{1}{3^{2}} \times \frac{3}{3^{2}} + \frac{1}{3^{2}} \times \frac{3}{3^{2}} + \frac{3}{3^{2}} \times \frac{3}{3^{2}}}{\text{or}}$ or $\sqrt{3}\sqrt{3} + \sqrt{3}\sqrt{27} + \sqrt{3}\sqrt{27} + \sqrt{27}\sqrt{27}$	M1	oe allow an error in one term		
	3 or 9 or 27	M1dep			
	48	A1			
	Alternative method 2				
	$\sqrt{3}$ and $3\sqrt{3}$	M1	3√3 must come from correct working		
	$(4\sqrt{3})^2$	M1dep			
	48	A1			
14	Alternative method 3				
	$\left(3^{\frac{1}{2}}\right)^2 (1+3)^2$	M1	oe		
	3×4^2	M1dep	ое		
	48	A1			
	Additional Guidance				
	Alt 1 mark scheme likely to see a 3 (or 9 or 27) somewhere, so need to be careful that the M1 mark has been earned before awarding A1				
	In alt 1, for the first M1, we want to see an attempt at the full expansion of the correct terms.				
	Probably 4 terms, but there could be 3 eg $(\sqrt{3} + 27)(\sqrt{3} + 27)$ scores M0 beca	-			

Q	Answer	Mark	Comments

	2 - x or x - 2	M1	do not award M1 if you see evidence of incorrect method for finding a linear expression	
	y = 2 - x accurately drawn	M1		
	3.4	A1	accept 3.3 to 3.5	
	0.6	A1	accept 0.5 to 0.7	
	Ad	ditional 0	Suidance	
	For the first M1, start by looking for evidence of a correct method. eg $x^2 - 4x + 2 + 3x - x^2 = -x + 2$			
15	or			
	$x^{2} - 4x + 2 = 0 \rightarrow x^{2} - 3x - x + 2 = 0 \rightarrow -x + 2 = 3x - x^{2}$			
	Attempts to solve $x^2 - 4x + 2 = 0$ by using the quadratic formula or by completing the square or by drawing a new quadratic graph (for $y = x^2 - 4x + 2$) score 0 marks			
	You might see work which uses the quadratic formula or completing the square which leads to answers of $2 \pm \sqrt{2}$ and if this follows working using a correct method to find the linear graph, it can be ignored (they could be using it as a check on their answers obtained graphically), but if it looks like it is their main method, then award 0 marks, as stated above			
	Ignore any y coordinates that might accompany the final x values.			

Q	Answer	Mark	Comments	
	factorising to get (x + 3)(x - 1) (= 0) or completing the square and getting as far as $x + 1 = \pm 2$ or using the quadratic formula and getting as far as $x = -2 \pm 4$ 2	M1	(-3, 13) y	
	x = -3 and $x = 1$	A1		
16	$(-3, 13)$ as a maximum point and $(1, 2^{1}/_{3})$ as a minimum point, plotted	M1	/	
	Smooth correct curve which must have the stationary points plotted in the correct quadrants and must cross the negative <i>x</i> -axis	A1		
	Additional Guidance			
	SC1 for a fully correct sketch with the st but lacking any detail in terms of the <i>x</i> c with incorrect values of the stationary po- method to obtain $x = -3$ and $x = 1$	oordinates	s of the stationary points, or	

Q	Answer	Mark	Comments
	$f(2) = (2)^3 + 8(2)^2 + 5(2) - 50$ $= 8 + 32 + 10 - 50 = 0$	B1	substitutes $x = 2$ and verifies that $f(2) = 0$ the terms must be evaluated
17 (a)	Additional Guidance		
	Using the factor theorem is essential.	Using long	division here scores M0

	Alternative method 1				
	$x^{3} + 8x^{2} + 5x - 50$ = (x - 2)(x ² + kx + 25)	M1	Sight of a 3 term quadratic with x^2 and +25 as the first and last terms		
	$x^2 + 10x + 25$	A1			
	$(x-2)(x+5)^2$	A1	oe		
	Alternative method 2				
	Substitutes another value into the	M1	their value correctly worked out		
	expression and tests for '= 0'		eg $f(1) = -36$ $f(3) = 64$		
	(<i>x</i> + 5)	A1	coming from $f(-5) = -125 + 200 - 25 - 50 = 0$		
	$(x-2)(x+5)^2$	A1	oe		
	Alternative method 3				
17 (b)	Long division of polynomials getting as far as $x^2 + 10x$	M1			
	$x^2 + 10x + 25$	A1			
	$(x-2)(x+5)^2$	A1	oe		
	Alternative method 4				
	Using synthetic division to arrive at $x^2 + 10x$	M1	2 1 8 5 -50 0 2 20 50 1 10 25 0		
	$x^2 + 10x + 25$	A1			
	$(x-2)(x+5)^2$	A1	oe		
	Alternative method 5		-		
	$x^3 + 8x^2 + 5x - 50$	M1			
	$\equiv (x-2)(ax^2+bx+c)$				
	$\equiv ax^{3} - 2ax^{2} + bx^{2} - 2bx + cx - 2c$				

and any two of $a = 1, b = 10, c = 25$				
$x^2 + 10x + 25$	A1			
$(x-2)(x+5)^2$	A1	ое		
Additional Guidance				
This work might appear in 17a you can mark it having seen it in 17a unless there is a contradiction with any work in 17b. Also, mark from what you might see in 17a if there is no work in 17b				
Ignore further work which gives answers of 2, -5 and -5 (from solving $f(x) = 0$)				

Q	Answer	Mark	Comments		
18	Angle $DST = x$	M1	' base angles of isosceles triangle <i>DST</i> but we do not require a reason for this mark		
	Angle $DFS = x$ angle in alternate segmentor $Angle RSF = x$ Corresponding	M1	either of these angles with a correct reason scores this mark no reason or an incorrect reason is M0		
	Further evaluation of angles, with correct reasons , to arrive at a stage where either it is possible to use the converse of a theorem or which leads to the fact that <i>DTSF</i> is a parallelogram	M1dep	Here is a complete example angle $DST = x$ angle $DSR = 180 - x$ angles on a straight line angle $RSF = x$ corresponding angle $FDS = x$ $FDS = RSF$, angle in		
	A statement of the angles, or the values of the angles, that will complete the proof the angles must be clearly identified	M1dep	alternate segment angle DSR + angle FDS = $180 - x + x$ = 180		
	A statement of the correct reason to accompany these angles, thus completing the proof	A1	<i>FD</i> is parallel to <i>RST</i> because these angles add to 180 using the (converse) of the co-interior angles theorem		
	Additional Guidance				
	Some methods are much shorter than others. Follow their reasoning to see if it is free from error.				
	The 3rd mark is dependent on both of the previous two M marks.				
	So, if they have not scored M1 M1 they cannot score any more marks				
	For the 3rd M mark when following their work, stop as soon as there is an error in the value of any angle or in the reason given for any particular angle. The proof breaks down at that point and they can only score a maximum of 2 marks.				
	The 4th mark (M1dep) and the 5th mark (A1) are closely related.				
	They can be thought of as a pair because the 4th mark is for identifying the two angles that make the completion of the proof possible and the 5th mark is for stating the correct reason. Condone the fact that you may not see the word 'converse'.				
	Any sign of them using the result (eg $DST = FDS$, alternate angles) means they have compromised the proof immediately and they can only score a maximum of 2 marks.				

	Alternative method 1				
	sight of $2(x^2 - 8x \dots)$	M1			
	sight of $2(x - 4)^2$	M1dep			
	$2[(x-4)^2-16]+13$				
	or $2(x-4)^2 - 32 + 13$ or $2[(x-4)^2 - 16 + 6.5]$	M1dep			
	$2(x-4)^2 - 19$	A1	or $a = 2, b = -4, c = -19$		
40	Alternative method 2				
19	<i>a</i> = 2	B1			
	-16 = 2ab or $-16 = 4bor 13 = ab^2 + c or 13 = 2b^2 + c$	M1			
	$-16 = 2ab$ and $13 = ab^2 + c$ or $-16 = 4b$ and $13 = 2b^2 + c$	M1dep	oe		
	$2(x-4)^2 - 19$	A1	or $a = 2, b = -4, c = -19$		
	Additional Guidance				

Q	Answer	Mark	Comments		
	Alternative method 1				
20	$\sin 45 = \frac{1}{2} \text{ or } \frac{\sqrt{2}}{2} \text{ or } \sin 60 = \frac{\sqrt{3}}{2}$	B1	look for these in their working one of them correct will gain the mark at any stage of their working		
	$\frac{x}{\sin 45} = \frac{6\sqrt{2}}{\sin 60}$	M1	oe		
	(x =) $\frac{6\sqrt{2} \times \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}}$ or $\frac{6\sqrt{2} \times \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}}$	M1dep	oe a correct expression or equation coming from both surds correct eg $\frac{\sqrt{3}}{2}x = 6\sqrt{2} \times \frac{1}{\sqrt{2}}$ or $\frac{6}{x} = \frac{\sqrt{3}}{2}$		
	$(x =) \frac{12}{\sqrt{3}}$ or $\sqrt{3}x = 12$	A1			
	$(x =) 4\sqrt{3}$	A1			
	Alternative method 2				
	$\sin 45 = 1$ or $\sqrt{2}$ or $\sin 60 = \sqrt{3}$ $\sqrt{2}$ 2 2 2	B1	look for these in their working one of them correct will gain the mark at any stage of their working		
	<u>AD</u> = sin 45 or (AD =) $6\sqrt{2} \times sin 45$ $6\sqrt{2}$ or (AD =) 6	M1	where D is the foot of the perpendicular from A to BC		
	(x =) $\frac{6\sqrt{2} \times \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}}$ or $\frac{6\sqrt{2} \times \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}}$	M1dep	oe a correct expression or equation coming from both surds correct eg $\frac{\sqrt{3}}{2}x = 6\sqrt{2} \times \frac{1}{\sqrt{2}}$ or $\frac{6}{x} = \frac{\sqrt{3}}{2}$		
	$(x =) \frac{12}{\sqrt{3}}$ or $\sqrt{3}x = 12$	A1			
	$(x =) 4\sqrt{3}$	A1			
	Additional Guidance				
	If one surd value is incorrect they can only score a maximum of 2 marks				