AQA

A-LEVEL

MATHEMATICS

7357/1

Paper 1

Mark scheme

June 2018

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186A73571/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q1	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	$\frac{dy}{dx} = -\frac{2}{x^3}$
	Total		**1**	

Q2	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer	AO1.1b	B1	$y = 5 \times 5^x$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Circles correct answer	AO1.1b	B1	4
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Takes logs of an equation. Must be correct use of logs.	AO1.1a	M1	$y = e^{x-4}$ $\ln y = x - 4$ $4 + \ln y = x$ $f^{-1}(x) = 4 + \ln x, x > 0$
	Obtains correct inverse function in any correct form	AO1.1b	A1	
	Deduces correct domain	AO2.2a	B1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Differentiates 2^t or 2^{-t} to obtain $\pm A\ln 2\times 2^{\pm t}$	AO1.1a	M1	
	Obtains $\frac{dy}{dt}=(\pm A\ln 2)2^t$ and $\frac{dx}{dt}=(\pm B\ln 2)2^{-t}$	AO1.1b	A1	$\frac{dy}{dt}=(3\ln 2)2^t$ $\frac{dx}{dt}=(-4\ln 2)2^{-t}$
	Uses chain rule with correct $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and completes rigorous argument to obtain fully correct printed answer	AO2.1	R1	$\frac{dy}{dx}=\frac{(3\ln 2)2^t}{(-4\ln 2)2^{-t}}$ $=-\frac{3}{4}\times 2^{2t}$
(b)	Rearranges to write 2^{-t} in terms of x or 2^t in terms of y Or Writes given expression in terms of t	AO3.1a	M1	$2^t=\frac{y+5}{3}$ $2^{-t}=\frac{x-3}{4}$ $1=\left(\frac{y+5}{3}\right)\left(\frac{x-3}{4}\right)$ $12=xy+5x-3y-15$ $xy+5x-3y=27$
	Eliminates t Or Compares coefficients PI by a=5 or b=-3	AO1.1a	M1	
	Completes rigorous argument to obtain correct values of a, b and c and write the Cartesian equation in the required form ISW	AO2.1	R1	**ALT** $xy+ax+by=(4\times 2^{-t}+3)(3\times 2^t-5)+a(4\times 2^{-t}+3)+b(3\times 2^t-5)$ $=12-15+(4a-20)2^{-t}+(3b+9)2^t+3a-5b$ $a=5, b=-3$ $xy+5x-3y=-3+15+15$ $=27$
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Writes in a form to which the binomial expansion can be applied Accept $A(1+\frac{x}{4})^{-\frac{1}{2}}$	AO3.1a	M1	$\frac{1}{\sqrt{4+x}}=\frac{1}{2}(1+\frac{x}{4})^{-\frac{1}{2}}$ $\approx\frac{1}{2}\left[1+\left(-\frac{1}{2}\right)\frac{x}{4}+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{x}{4}\right)^2}{2!}\right]$ $\approx\frac{1}{2}\left[1-\frac{x}{8}+\frac{3x^2}{128}\right]$ $\approx\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2$
	Uses binomial expansion for their $(1+kx)^{\pm\frac{1}{2}}$ with at least two terms correct (can be unsimplified)	AO1.1a	M1	
	Obtains correct simplified answer No need to expand brackets CAO	AO1.1b	A1	
(b)	Substitutes $-x^3$ in their three term expansion from part (a)	AO1.1a	M1	$\frac{1}{\sqrt{4-x^3}}\approx\frac{1}{2}-\frac{1}{16}\left(-x^3\right)+\frac{3}{256}\left(-x^3\right)^2$ $\approx\frac{1}{2}+\frac{x^3}{16}+\frac{3x^6}{256}$
	Obtains correct expansion. FT their (a)	AO1.1b	A1F	
(c)	Uses their three term expansion as the integrand ignore limits PI by next mark	AO1.1a	M1	$\int_0^1\frac{1}{\sqrt{4-x^3}}\mathrm{d}x\approx\int_0^1\frac{1}{2}+\frac{x^3}{16}+\frac{3x^6}{256}\mathrm{d}x$ $\approx\left[\frac{x}{2}+\frac{x^4}{64}+\frac{3x^7}{1792}\right]_0^1$ $\approx\frac{1}{2}+\frac{1}{64}+\frac{3}{1792}$ ≈ 0.5172991
	Integrates (at least two terms correct)	AO1.1a	M1	
	Obtains correct value CAO	AO1.1b	A1	
(d)(i)	Explains that each term in the expansion is positive	AO2.4	E1	Each term in the expansion is positive. So increasing the terms will increase the estimated value hence the value must be an underestimate.
	Deduces that increasing the number of terms will increase the estimated value and that the value must be an underestimate. (Condone inference if evidence given ie value calculated numerically and compared)	AO2.2a	R1	
(d)(ii)	States the validity of their binomial expansion for part (b) Provided their $k\neq\pm1$	AO3.1a	B1F	The binomial expansion is valid for $\lvert x\rvert<\sqrt[3]{4}$ $2>\sqrt[3]{4}$
	Compares integral lower limit with validity of correct expansion CAO	AO2.3	E1	
	Total		**12**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	Uses a technique which could lead to showing two lines are perpendicular. Obtains at least one correct distance (or distance^2) or gradient.	AO3.1a	M1	$AB^2 = (8-15)^2 + (17-10)^2$ $= 98$ $AC^2 = (8--2)^2 + (17--7)^2$ $= 676$ $CB^2 = (15--2)^2 + (10--7)^2$ $= 578$ $AB^2 + BC^2 = 98 + 578$ $= 676$ $= AC^2$ Angle ABC is a right angle.
	Obtains three correct distances (or distance^2) or two gradients. Lengths: $7\sqrt{2}, 17\sqrt{2}, 26$ Gradients: $AB = -\frac{7}{7}, BC = \frac{17}{17}$	AO1.1b	A1	
	Completes correct rigorous argument to show required result Uses Pythagoras OR Multiplies gradients to show product is -1 AND Writes a concluding statement.	AO2.1	R1	
(b)(i)	Explains why AC is a diameter Must reference angle subtended by diameter (condone "angle in a semi-circle") or give full explanation.	AO2.4	E1	The angle subtended by a diameter is 90° $\therefore AC$ must be a diameter of the circle
(b)(ii)	Deduces correct radius (or radius^2)	AO2.2a	B1	Radius $\frac{\sqrt{676}}{2} = 13$ Centre $\left(\frac{8-2}{2}, \frac{17-7}{2}\right) = (3,5)$ Distance from centre to D $(3--8)^2 + (5--2)^2 = 11^2 + 7^2$ $= 170 > 169$ So D lies outside the circle.
	Obtains mid-point of diameter	AO1.1b	B1	
	Uses $D(-8,-2)$ to find the distance or (distance^2) from *their* centre OE	AO1.1a	M1	
	Completes rigorous argument by comparing $\sqrt{170} > 13$ (or $170 > 169$) to show that D lies outside the circle	AO2.1	R1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Uses $A=\frac{1}{2}ab\sin C$ for triangle OAC or OAB PI by equation	AO1.2	B1	$\frac{1}{2}r\times\frac{r}{2}\sin\theta=\frac{1}{4}\left(\frac{1}{2}r^2\theta\right)$ $\Rightarrow\frac{r^2}{4}\sin\theta=\frac{1}{8}r^2\theta$ $\Rightarrow 2r^2\sin\theta=r^2\theta$ $\Rightarrow 2\sin\theta=\theta$ **AG**
	Forms an equation relating the area of OAC and ABC in the form $Ar^2\sin\theta=Br^2\theta$	AO3.1a	M1	
	Obtains fully correct equation ACF	AO1.1b	A1	
	Simplifies to obtain required equation, only award if all working correct with rigorous argument.	AO2.1	R1	
(b)	Rearranges to the form $\mathrm{f}(\theta)=0$ PI by correct θ_2 or θ_3	AO1.1a	M1	$\mathrm{f}(\theta)=\theta-2\sin\theta=0$ $\theta_{n+1}=\theta_n-\frac{\theta_n-2\sin\theta_n}{1-2\cos\theta_n}$ $\theta_2=2.094395...$ $\theta_3=1.913222...$ $\theta_3=1.91322\,(5\text{ d.p.})$
	Differentiates their $\mathrm{f}(\theta)$ or uses calculator PI correct θ_2 or θ_3	AO1.1b	A1	
	Obtains correct θ_3	AO1.1b	A1	
(c)	Obtains percentage error for θ_3 AWRT 0.94%	AO3.2b	B1	0.935%
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)	Uses S_n for arithmetic sequence with n = 6 or n = 36	AO1.1a	M1	$S_6 = 3(2a+5d)$ $= 6a+15d$
	Finds correct expressions for S_6 and S_{36}	AO1.1b	A1	$S_{36} = 18(2a+35d)$ $= 36a+630d$
	Forms equation in a and d using *their* S_{36} = (*their* S_6)^2	AO3.1a	M1	
	Expands quadratic and collects like terms to obtain printed answer Only award for completely correct solution with no errors	AO2.1	R1	$36a+630d = (6a+15d)^2$ $36a+630d = 36a^2+90ad+90ad+225d^2$ $4a+70d = 4a^2+20ad+25d^2$
(b)	Uses u_n for arithmetic sequence with n = 6	AO1.1b	B1	$a+5d = 25 \Rightarrow d = \dfrac{25-a}{5}$
	Eliminates a or d using *their* 'a+5d = 25' and the printed result in part (a) to obtain a quadratic in one variable	AO1.1a	M1	$4a+70\left(\frac{25-a}{5}\right) = 4a^2+20a\left(\frac{25-a}{5}\right)+25\left(\frac{25-a}{5}\right)^2$ $4a+350-14a = 4a^2+100a-4a^2+625-50a+a^2$
	Obtains correct quadratic equation Need not be simplified	AO1.1b	A1	350-10a=100a+625-50a+ a^2
	Solves their quadratic a =-5, a = -55 (*or* d=6, d=16)	AO1.1a	M1	a^2+60a+275=0
	Deduces min value a=-55 **NMS** a=-55 5/5	AO3.2a	A1	a =-5, a = -55 (*or* d=6, d=16) a = -55
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)	Uses model to form an equation to find k with t=5.7, $m = ½ m_0$	AO3.4	M1	$200 = 400e^{-kx\ 5.7}$ k=0.1216047… $m = 400\ e^{-0.1216…\ x\ 4}$ m = 250
	Obtains correct value of k	AO1.1b	A1	
	Uses model to find m with t=4, m_0 =400 and *their* k *(Condone* m_0=200)	AO3.4	M1	
	Obtains correct value of m **CAO** (245.9296…) AWRT 250	AO1.1b	A1	
(b)	Uses model to set up inequality or equation using *their* k and 280	AO3.1b	M1	$400e^{-0.1216t} \le 280$ $e^{-0.1216t} \le 0.7$ $-0.1216t \le \ln(0.7)$ $t \ge 2.933$
	Solves their inequality or equation to find t (Follow through their k only) (2.933067)	AO1.1b	A1F	
	Interprets *their* solution (Only follow through if time is earlier than 1:42 pm)	AO3.2a	A1F	10:56 am
(c)	States any sensible reason such as: Different people eliminate caffeine at different rates The model is based on an average person The length of time taken to drink two cups of coffee may have been significant The amount of caffeine in a “strong cup of coffee” may vary	AO3.5b	B1	Different people eliminate caffeine at different rates
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
11(a)(i)	Uses model to form equation with V=0	AO3.4	M1	$\therefore 10+100\left(\frac{T}{30}\right)^3-50\left(\frac{T}{30}\right)^4=0$
	Rearranges to isolate T^4 term	AO1.1a	M1	$\Rightarrow 50\left(\frac{T}{30}\right)^4=10+100\left(\frac{T}{30}\right)^3$
	Completes rigorous and convincing argument to clearly show the required result. Need to see evidence of division by T to isolate T^3 term Must be an equation throughout **AG**	AO2.1	R1	$\Rightarrow \frac{T^4}{16200}=10+\frac{T^3}{270}$ $\Rightarrow \frac{T^3}{16200}=\frac{10}{T}+\frac{T^2}{270}$ $\Rightarrow T=\sqrt[3]{\frac{162000}{T}+60T^2}$
11(a)(ii)	Calculates T_1 (44.96345…..)	AO1.1a	M1	T_1=44.963
	Calculates T_2 and T_3 (49.98742….) Condone greater than 3dp (53.50407….)	AO1.1b	A1	T_2 = 49.987 T_3 = 53.504
11(a)(iii)	Explains 38 in context	AO3.2a	B1	38 represents current year 2018
11(b)	Translates 2029 into t=49	AO3.3	B1	$10+100\left(\frac{t}{30}\right)^3-50\left(\frac{t}{30}\right)^4=4.5\times1.063^t$ $\Rightarrow t=49.009$
	Uses models to set up equation or evaluate both models at one value of t	AO3.4	M1	1980 + 49 = 2029
	Obtains correct values for **both** models for two appropriate values of t. $t\in[49,50]$ eg t=49 and t=50 t =49 gives: 89.89 and 89.81 t =50 gives: 87.16 and 95.47 Or Solves equation using any method to obtain AWFW 49.009 to 49.01	AO1.1b	A1	Therefore use of oil and production of oil will be equal in the year 2029
	Explains that the use of oil and the production of oil are equal when t = 49.009 Or Uses a change of sign argument OE to explain that the value of each model for two appropriate values of t shows that the production of oil and the use of oil are the same for $t\in(49,50)$	AO2.4	E1	
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
12(a)	Begins a proof using a valid method Eg. Factor theorem, algebraic division, multiplication of correct factors	AO1.1a	M1	$p\left(-\frac{1}{2}\right)=30\times\left(-\frac{1}{2}\right)^3-7\left(-\frac{1}{2}\right)^2-7\left(-\frac{1}{2}\right)+2$ $=0$ $\therefore 2x+1$ is a factor of $p(x)$
	Constructs rigorous mathematical proof. To achieve this mark: Factor theorem the student must clearly substitute and state that p(-1/2)=0 **and** clearly state that this implies that $2x$ + 1 is a factor Algebraic division OR Multiplication of correct factors The method must be completely correct with a concluding statement	AO2.1	R1	
(b)	Obtains quadratic factor PI	AO1.1a	M1	$p(x)=(2x+1)(15x^2-11x+2)$ $=(2x+1)(5x-2)(3x-1)$
	Obtains second linear factor	AO1.1b	A1	
	Writes $p(x)$ as the product of the correct three linear factors. NMS correct answer 3/3	AO1.1b	A1	
(c)	Rearranges to achieve a cubic equation in $\sec x$ (or $\cos x$)	AO3.1a	M1	$\frac{30\sec^2 x+2\cos x}{7}=\sec x+1$ $\Rightarrow 30\sec^2 x+2\cos x=7\sec x+7$ $\Rightarrow 30\sec^3 x+2=7\sec^2 x+7\sec x$ $30\sec^3 x-7\sec^2 x-7\sec x+2=0$ $\Rightarrow (2\sec x+1)(5\sec x-2)(3\sec x-1)=0$ $\Rightarrow \sec x=-\frac{1}{2},\frac{1}{3},\frac{2}{5}$ These values do not fall within the range of $\sec x$ as they are between -1 and 1 $\therefore \frac{30\sec^2 x+2\cos x}{7}=\sec x+1$ has no real solutions.
	Equates to zero and uses result from (b) or factorises	AO1.1a	M1	
	Deduces that if solutions exist they must be of the form $\sec x$ = -½, $\sec x$ =1/3 or $\sec x$ = 2/5 OE	AO2.2a	A1	
	Explains that the range of $\sec x$ is $(-\infty,-1]\cup[1,\infty)$ OE OE for $\cos x$	AO2.4	E1	
	Completes argument explaining that there cannot be any real solutions as values are outside of the function's range.	AO2.1	R1	
	Total		**10**	

Q	Marking instructions	AO	Mark	Typical solution
13	Identifies and clearly defines consistent variables for length and width. Can be shown on diagram.	AO3.1b	B1	Width of rectangle = $2x$ Length of rectangle = $2y$
	Models the area of rectangle with an expression of the correct dimensions	AO3.3	M1	$A = 4xy$
	Eliminates either variable to form a model for the area in one variable.	AO1.1a	M1	$x^2 + y^2 = 16$
	Obtains a correct equation to model the area in one variable	AO1.1b	A1	$A = 4x\sqrt{16 - x^2}$
	Differentiates their expression for area. Condone one error	AO3.4	M1	$\frac{dA}{dx} = 4\sqrt{16 - x^2} - \frac{4x^2}{\sqrt{16-x^2}}$ $\frac{dA}{dx} = \frac{64-8x^2}{\sqrt{16-x^2}}$ For maximum point $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$
	Explains that their derivative equals zero for a maximum or stationary point.	AO2.4	E1	$\frac{64-8x^2}{\sqrt{16-x^2}} = 0$
	Equates area derivative to zero and obtains correct value for either variable. CAO	AO1.1b	A1	$x = 2\sqrt{2}$ When x = 2.8, $\frac{\mathrm{d}A}{\mathrm{d}x} = 0.448$
	Completes a gradient test or uses second derivative of their area function to determine nature of stationary point	AO1.1a	M1	When $x = 2.9$, $\frac{\mathrm{d}A}{\mathrm{d}x} = -1.191$ Therefore maximum
	Deduces that the area is a maximum at $x = 2\sqrt{2}$ or $\theta = \frac{\pi}{4}$ Values need not be exact	AO2.2a	R1	The maximum area is 32 sq in
	Obtains maximum area with correct units AWRT 32	AO3.2a	B1	
	Total		**10**	

Q	Marking instructions	AO	Mark	Typical solution
14(a)	Explains why $\angle EFQ = A$ Must be a fully correct explanation with reasons which may include: Vertically opposite angles and right angle implies similar triangles.	AO2.4	E1	$\angle OQR = \angle FQE$ vertically opposite angles $\angle ORQ = \angle FEQ = 90°$ So $\angle EFQ = A$
	Deduces $\frac{PF}{EF} = \cos(A)$ **AND** $\frac{EF}{OF} = \sin(B)$ Must have at least stated or implied that $\angle EFQ = A$ through similarity	AO2.2a	R1	Since $\angle EFQ = A$ $\frac{PF}{EF} = \cos(A)$ And $\frac{EF}{OF} = \sin(B)$ in triangle OEF
14(b)	Completes proof	AO2.2a	B1	$\frac{DE}{\mathbf{OE}} \times \frac{\mathbf{OE}}{OF} + \frac{PF}{EF} \times \frac{EF}{OF}$ $= \mathbf{sinAcosB} + \cos A \sin B$
14(c)	Explains that the proof is based on right angled triangles which limits A and B to acute angles	AO2.3	E1	Since the proof is based on the diagram which uses right-angled triangles it is assumed that A and B are acute. Therefore, the proof only holds for acute angles.
14(d)	Substitutes $-B$ into identity for $\sin(A+B)$ to give $\sin(A-B)$	AO2.1	R1	$\sin(A-B) = \sin A \cos(-B) + \cos A \sin(-B)$
	Recalls at least one of the identities $\sin(-B) = -\sin(B)$ $\cos(-B) = \cos(B)$ Must be explicitly stated	AO1.2	B1	$\sin(-B) = -\sin(B)$ $\cos(-B) = \cos(B)$
	Deduces correct identity with no errors. This must be clearly deduced from a correct argument and not simply stated.	AO2.2a	R1	Hence $\sin(A-B) = \sin A \cos B - \cos A \sin B$
	Total		**7**	

Q	Marking instructions	AO	Mark	Typical solution
15(a)	Forms expression of the correct form for the gradient of the line AB condone sign error	AO1.1a	M1	Gradient of AB $=\dfrac{(-4+h)^3-48(-4+h)-\left((-4)^3-48(-4)\right)}{h}$ $=\dfrac{h^3-12h^2+48h-64-48h+192-128}{h}$ $=\dfrac{h^3-12h^2}{h}$ $=h^2-12h$
	Obtains correct expansion of $(-4+h)^3$	AO1.1b	B1	
	Obtains correct expansion of numerator	AO1.1b	A1	
	Simplifies numerator and shows given result	AO2.1	R1	
15(b)	Explains that as $h \to 0$ the gradient of the line AB $\to$ the gradient of the curve or tangent to the curve Or gradient of curve is given by $\lim\limits_{h\to 0} h^2-12h$ Must not use $h=0$	AO2.4	E1	The gradient of the curve is given by $\lim\limits_{h\to 0} h^2-12h$
	Explains that $\lim\limits_{h\to 0} h^2-12h=0$ therefore A must be a stationary point	AO2.4	E1	As $h\to 0$, $h^2-12h\to 0$ therefore A must be a stationary point
	Total		**6**	

	TOTAL		**100**	