**GCE A level Pure Mathematics (9MA0) – Paper 1**

**Pure Mathematics 1**

**Summer 2019 student-friendly mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.**

**This document is intended for guidance only and may differ significantly from the final mark scheme published in July 2019.**

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| **Guidance on the use of codes within this document** |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.  A1 – accuracy mark. This mark is generally given for a correct answer following correct working.  B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.  Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

**Question 1 (Total 3 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | |
|  | f(–3) = 3 × (–3)2 + 2*a* × (–3)2 – 4 × – 3 + 5*a* = 0 | | M1 | | This mark is given for a method to set f(–3) = 0 |
| f(–3) = 23*a* – 69 = 0  23*a* = 69 | M1 | | This mark is given for finding an equation to solve for *a* | |
| *a* = 3 | A1 | | This mark is given for finding the correct value of *a* | |

**Question 2 (Total 5 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) |  | M1 | This mark is given for plotting the line *y*= 2*x* +  on the diagram with a correct gradient and intercept |
| Only one intersection means that there is one root | A1 | This mark is given for a reason why there is only one real root |
| (b) | 1 –  – 2*x* –  = 0 | M1 | This mark is given for using the small angle approximation cos *x* = 1 –  in the given equation |
| *x*2 + 4*x* – 1 = 0 | M1 | This mark is igvne for rearranging to find a quadratic equation to solve |
| 0.236 or –2 + √5 | A1 | This mark is given for finding the correct (positive) solution for *x* |

**Question 3 (Total 5 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | |
| (a) | = | | M1 | | This mark si given for an attempt to differentiate the expression for *y* |
| A1 | | This mark is given for correctly differentiating the expression for *y* |
| = | M1 | | This mark sis given for cancelling the expression through by (*x* + 1) | |
| = | A1 | | This mark is given for a fully correct expression for | |
| (b) | If *A* > 0 and *n* = 1, 3 then *x* < –1 | B1 | | This mark is given for deducing that  < 0 ⇒ *x* < –1. | |

**Question 4 (Total 6 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | **=  =** | M1 | This mark is given for rearranging to attempt a binomial expansion |
| **=**  1 + **+** | M1 | This mark is given for an attempt at a binomial expansion |
| A1 | This mark is given for a fully correct binomial expansion |
| **=  +** *x* +*x*2 | A1 | This mark is given for a fully correct expansion with the first three terms |
| (b)(i) | *x* = –14, since the expansion is only valid for ⎪*x*⎪ < 4 | B1 | This mark is given for the correct value chosen with a correct reason |
| (b)(ii) | *x* = –, since the smaller value will give the more accurate approximation | B1 | This mark is given for the correct value chosen with a correct reason |

**Question 5 (Total 10 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | 2*x*2 + 4*x* + 9 = 2(*x* + *b*)2 + *c* | B1 | This mark is given for writing f(*x*) in the form *a*(*x* + *b*)2 + *c* with *a* = 2 |
| 2*x*2 + 4*x* + 9 = 2(*x* + 1)2 + *c* | M1 | This mark is given for writing f(*x*) in the form *a*(*x* + *b*)2 + *c* with *a* = 2 and *b* = 1 |
| 2*x*2 + 4*x* + 9 = 2(*x* + 1)2 + 7 | A1 | This mark is given for writing f(*x*) in the form *a*(*x* + *b*)2 + *c* with *a* = 2, *b* = 1 and *c*= 7 |
| (b) | (0, 9)  (–1, 7) | B1 | This mark is given for a U shaped curve in any position |
| B1 | This mark is given for a *y*-intercept shown at (0, 9) |
| B1 | This mark is given for a minimum shown at (–1, 7) |
| (c)(i) | g(*x*) = 2(*x* – 2)2 + 4(*x* – 2) + 5 | M1 | This mark is given for writing g(*x*) in the form *a*(*x* + *b*)2 + *c* and comparing to f(*x*) |
| Translation of | A1 | This mark is given for deducing the translation of *y* = f(*x*) to *y* = g(*x*) |
| (c)(ii) | h(*x*) =  Maximum value =  (when *x* = –1) | M1 | This mark is given for writing h(*x*) in the form  and finding its maximum value |
| 0 < h(*x*) ≤ 3 | A1 | This mark is given for finding the correct range of the function h(*x*) |

**Question 6 (Total 8 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | 5 sin 2*θ* = 9 tan *θ* ⇒  10 sin *θ* cos *θ* = 9 × | M1 | This mark is given for a method to substitute terms to form an equation in terms of cos *θ* |
| 10 cos2 *θ* = 9 | M1 | This mark is given for a correct equation in terms of cos *θ* |
| *θ* = arcos ± | M1 | This mark is given for finding a a value for *θ* in terms of arccos |
| *θ* = ±18.4°, ±161.6° | A1 | This mark is given for any one value of 18.4° or 161.6° found. |
| A1 | This mark is given for four values of *θ* found correctly |
| *θ* = ±0°, 180° | B1 | This mark is given for the deduction of the two other solutions for *θ* |
| (b) | 10 cos2 (*x* – 25) = 9  *x* has smallest positive value when  *x* – 25° = –18.4° | M1 | This mark si given for finding an equation to solve for *x* |
| *x* = 6.6° | A1 | This mark is given for correctly finding the smallest positive solution to the equation |

**Question 7 (Total 7 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | *V* = *A*e–*kt* | M1 | This mark is given for suggesting a suitable exponential model for *V* in terms of *t* |
| When *t* = 0 and *V* = 20 000, *A* = 20 000 | M1 | This mark is given for using the model to show the initial value for *A* is £20 000 |
| When *t* = 1 and *V* = 16 000,  16 000 = 20 000e–1*k*  *k* = ln 0.8 = –0.223 | M1 | This mark is given for using the value of the car after one year to find a value for *k* |
| *V* = 20 000e–0.223*t* | A1 | This mark is given for finding a fully correct exponential model |
| (b) | When *t* = 10, *V* = £2150 | M1 | This mark is given for finding a value for *V* when *t* = 10 |
| This model is reliable since the value £2150 is close to £2000 | A1 | This mark is given for a valid statement comparing the two possible values of the car after 10 years |
| (c) | For example:  The value of *k* should be increased (e.g. *V* = 20 000e–0.1*t* )  A constant should be added  (e.g. *V* = 20 000e–0.223*t*  + 2000) | B1 | This mark is given for a statement suggesting a valid adaptation |

**Question 8 (Total 10 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | *y* = *x*(*x* + 2)(*x* – 4) = *x*3 – 2*x*2 – 8*x* | B1 | This mark is given for expanding brackets as a first step to a solution |
|  | M1 | This mark is given for a method to find the exact are of *R*1 |
| = | M1 | This mark is given for a method to evaluate the integral |
| = 0 – (4 –  – 16) = | A1 | This mark is given for a full method to show the exact value of *R*1 |
| (b) | *b*4 – *b*3 – 4*b*2 = – | M1 | This mark is given for deducing the area of *R*2 = – |
| 3*b*4 – 8*b*3 – 48*b*2 + 80 = 0 | A1 | This mark is given for rearranging the equation to a quartic |
| (*b* + 2)2 (3*b*2 – 20*b* + 20)  = (*b*2 + 4*b* + 4)(3*b*2 – 20*b* + 20) | M1 | This mark is given for expanding the equation given |
| = 3*b*4 – 8*b*3 – 48*b*2 + 80 = 0  The two equations are the same, so verified | A1 | This mark is for showing, and stating, that the equations are the same |
| (c) | 5.442 | B1 | This mark is given for a sketch of the curve with *b* = 5.442 shown |
|  | Between *x* = –2 and *b* = 5.442, the area above the *­x­*-axis is the same as the area below the *x*-axis | B1 | This mark is given for a valid explanation of the significance of the root 5.442 |

**Question 9 (Total 5 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| (a) | log *a* – log *b* = log | B1 | This mark is given for restating the log equation using log |
| *a* – *b* =  *ab* – *b*2 = *a*  *ab* – *a* = *b*2 | M1 | This mark is given for rearranging so that terms in a are on one side of the equation |
| *a*(*b* – 1) = *b*2  *a* = | A1 | This mark is for rearranging to show the result required |
| (b) | *b* ≠ 1 | B1 | This mark is given for deducing that *b* ≠ 1 |
| Since *a* > 0,  > 0  *b* > 1 since *b*2 is positive | B1 | This mark is given for stating that *b* > 1 and explaining the reason for the restriction |

**Question 10 (Total 6 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| (i) | For even numbers *n* = 2*m*,  *n*2 + 2 = 4*m*2 + 2 | M1 | This mark is given for showing the case for all even numbers |
| This is a multiple of 4 with 2 added, so cannot be divisible by 4 | A1 | This mark is given for a correct conclusion with a reason why *n*2 + 2 is not divisible by 4 for all even numbers |
| For odd numbers *n* = 2*m* + 1,  *n*2 + 2 = (2*m* + 1)2 + 2 = 4*m*2 + 4*m* + 3  = 4(*m*2 + *m*) + 3 | M1 | This mark is given for showing the case for all odd numbers |
| This is a multiple of 4 with 3 added, so cannot be divisible by 4  Hence, for all *n* ∈ ℕ, *n* + 2 is not divisible by 4 | A1 | This mark is given for a correct conclusion with a reason why *n*2 + 2 is not divisible by 4 for all odd numbers and a full concluding statement that for all *n* ∈ ℕ, *n* + 2 is not divisible by 4 |
| (ii) | For example, for *x* = 9.4  ⎪3*x* – 28⎪ = 0.2 and (*x* – 9) = 0.4 | M1 | This mark is given for showing that the statement is not true for 9.25 < *x* < 9.5 |
| The statement is sometimes true;  For example, for *x* = 12  ⎪3*x* – 28⎪ = 8 and (*x* – 9) = 3 | A1 | This mark is given for a correct statement and an example where the statement is true |

**Question 11 (Total 7 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | 24 + (6 × 1.05) + (6 × 1.052) minutes | M1 | This mark is for a method to find the time taken for the competitor to run 6 km |
| = 96.915 minutes  = 36 minutes 55 seconds | A1 | This mark is given for finding the total time as required |
| (b) | For example,  5th km = 6 × 1.051  6th km = 6 × 1.052  7th km = 6 × 1.053 …  *r*th km = 6 × 1.05*r* – 4 | B1 | This mark is given for showing the time taken to run the *r*th km, as required |
| (c) | 24 + | M1 | This mark is given for showing the total time to run the race is the time taken for the first 4 km added to the time taken from 5th to 20th km |
| = 24 + 6.3 × | M1 | This mark is given for using *s*= *a* where *a* = 6 × 1.05 = 6.3, *r* = 1.05 and *n* = 20 – 4 = 16 |
| = 24 + 149.04 | A1 | This mark is given for a correct total time (represented decimally) |
| = 173 minutes and 3 seconds | A1 | This mark is given for finding a correct total time given in minutes in seconds |

**Question 12 (Total 10 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | f ′(*x*) = –2.5e–0.25*x* sin *x* + 10e–0.25*x* cos *x* | M1 | This mark is given for a method to differentiate to find an expression for f ′(*x*) |
| A1 | This mark is given for correctly differentiating to find an expression for f ′(*x*) |
| f ′(*x*) = 0  ⇒ e–0.25*x* (–2.5 sin *x* + 10 cos *x*) = 0  ⇒ (–2.5 sin *x* + 10 cos *x*) = 0 | M1 | This mark is given for setting f ′(*x*) = 0 and finding as method to solve for tan *x* |
| =  tan *x* = 4 | A1 | This mark is given for showing that tan *x*= 4 as required. |
| (b) |  | M1 | This mark is given for a graph with a correct shape |
| A1 | This mark is given for a graph with heights > 0 |
| (c) | tan *x* = 4, *x* = 1.326  *t* = *π* + 1.326 = 4.47 | M1 | This method is given for finding a value for *t* between the first and second bounce |
| H(4.47) = ⎪10e–0.25 × 4.47 sin 4.47⎪ | M1 | This mark is given for substituting the value of *t* = *π* + arctan 4 into H(*t*) |
| = ⎪3.27 × –0.97⎪  = 3.17 metres | A1 | This mark is given for finding the maximum height of the ball |
| (d) | The time between each bounce should not stay the same when the heights of each bounce are getting smaller | B1 | This mark is given for a valid explanation of why the model should not be used the predict the time of each bounce |

**Question 13 (Total 11 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | The asymptote is found where 2*x* – *q* = 0  Hence *q* = 4 | B1 | This mark is given for explaining that the asymptote at *x* = 2 is a solution of 2*x* – *q* = 0 |
| *y* =  = | M1 | This mark is given for substituting *x* = 3, *y* =  (and *q* = 4) |
| *p* – 9 = 6  *p* = 15 | A1 | This mark is given for solving for *p* and showing that *p* = 15, as required |
| (b) | =  + | M1 | This mark is given for a method to use partial fractions |
| =  – | M1 | This mark is given for finding values for *A* and *B* |
| =  – | A1 | This mark is given for a fully simplified expression |
| *I* =  = *m* ln (2*x* – 4) + *n* ln (*x* + 3) | M1 | This mark is given for a method to integrate to find the area of *R* |
| = 0.9 ln (2*x* – 4) + 2.4 ln (*x* + 3) | A1 | This mark is given for a correct expression for the area of *R* |
| Area *R* = | M1 | This mark is given for deducing an expression for the area of *R*  (*y* = 0 when *x* = 5) |
| = [0.9 ln 6 – 2.4 ln 8] – [0.9 ln 2 – 2.4 ln 6]  = [0.9 ln 6 + 2.4 ln 6] – [7.2 ln 2 + 0.9 ln 2]  = 3.3 ln 6 – 8.1 ln 2  = 3.3 ln 3 + 3.3 ln 2 ­– 8.1 ln 2 | M1 | This mark is given for a method to find the exact area of *R* |
| = 3.3 ln 3 – 4.8 ln 2 | A1 | This mark is given for a correct value of the area of *R* with *a* = 3.3 and *b* = 4.8 |

**Question 14 (Total 7 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| (a) | = 8 cos 2*y* ⇒  = | M1 | This mark is given for differentiating and inverting |
| At (0, 0),  = | A1 | This mark is given for finding  when *y* = 0 |
| (b)(i) | sin 2*y* ≈ 2*y* ⇒ *x* ≈ 8*y* | B1 | This mark is given for finding an approximation for *x* |
| (b)(ii) | When *x* and *y* are small, *x* = 4 sin 2*y* approximates to the line *x* = 8*y* | B1 | This mark is given for a valid explanation of the relationship between *x* and *y* when both are small |
| (c) | sin2 2*y* + cos2 2*y* = 1  ⇒ cos2 2*y* = 1 – sin2 2*y*  *x* = 4 sin 2*y* ⇒ sin2 2*y* = | M1 | This mark is given for a method to use find an expression for sin2 2*y* in terms of *x* |
| =  = | A1 | This mark is given for an unsimplified expression for |
| = | A1 | This mark is given for a fully correct answer with *a* = 2 and *b* = 16 |