**GCE AS Further Mathematics (8FM0) – Paper 21**

**Further Pure Mathematics 1**

**Summer 2019 student-friendly mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.**

**This document is intended for guidance only and may differ significantly from the final mark scheme published in July 2019.**

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| **Guidance on the use of codes within this document** |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.  A1 – accuracy mark. This mark is generally given for a correct answer following correct working.  B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.  Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

**Question 1 (Total 9 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | sin *x* = | B1 | This mark is given for the *t*-formulae for sin *x* correctly stated |
| (b)(i) | sin *x* = | M1 | This mark is given for substituting √2 into the *t*-formulae for sin *x* |
| sin *x* = √2 | A1 | This mark is given for a correctly simplified answer |
| (b)(ii) | tan *x* =  ⇒ cos *x* =  cos *x* = | M1 | This mark is given for using the correct trigonometric identity to find an expression for cos *x* in terms of *t* |
| cos *x* = | A1 | This mark is given for a fully correct proof |
| (c) | 7 + 9 + 3 = 0 | M1 | This mark is given for using the *t*‑formulae to write the equation in terms of *t* |
| Multiplying all terms by 1 + *t* 2 gives  14*t* + 9 – 9*t*2 + 3 + 3*t*2 = 0  Collecting terms gives  6*t*2 – 14*t* – 12 = 0  Dividing all terms by 2 gives  3*t*2 – 7*t* – 6 = 0 | M1 | This mark is given for multiplying through by 1 + *t* 2 and rearranging to find a three-term equation in *t* |
| (3*t* + 2)(*t* – 3) = 0  ⇒ *t* = – and *t* = 3  ⇒ *θ* = 2 arctan – and *θ* = 2 arctan 3 | M1 | This mark is given for factorising and solving for *t* and thus finding two values for *θ* |
| *θ* = 292.6° and *θ* = 143.1° | A1 | This mark is given for finding two correct values of *θ* |

**Question 2 (Total 6 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (b)(i) | Line 3 is missing brackets or  Sign error: – 24 should be + 24 | B1 | This mark is given for identifying the error made in line 3 |
| (b)(ii) | The inequality has been reversed | B1 | This mark is given for identifying the error made in line 7 |
| (b) | (*x* – 24)(*x* + 11)[*x*(*x* + 11) – (*x* – 24)] > 0  (*x* – 24)(*x* + 11)[*x* + 10*x* + 24)] > 0  (*x* – 24)(*x* + 11)(*x* + 6)(*x* + 4) > 0 | M1 | This mark is given for multiplying out the equation given to find an expression in four terms |
| Critical values: *x* = –11, –6, –4, 24 | A1 | This mark is given for finding all four critical values for *x* |
| {*x* ∈ ℝ: *x* < –11} ∪ {*x* ∈ ℝ: –6 < *x* < –4}  ∪ {*x* ∈ ℝ: *x* < –11} | M1 | This mark is given for an attempt to produce two “outside” intervals and one “middle” interval |
| A1 | This mark is given for three correct intervals |

**Question 3 (Total 7 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | *R*0 = 20 and *t*0 = 0  Population after 4 months required  ⇒ *h* = | B1 | This mark is given for identifying correct initial conditions and requirements for *h* |
| *R*0 = 20 and *t*0 = 0  = (2 × 20) + 4 sin 0 = 40 | M1 | This mark is given for using the model to evaluate  at *t*0 |
| *R*1 ≈ *R*0 + *h* ≈ 20 +  × 40 | M1 | This mark is given for applying the approximation formula |
| = | A1 | This mark is given for finding an approximation after the first iteration |
| ≈ 2 ×  + 4 sin  = 53.9969… | M1 | This mark is given for applying the approximation formula a second time |
| *R*2 ≈ *R*1 + *h* ≈  +  × 54  = 35.7 | A1 | This mark is given for finding an approximation after the second iteration |
| *R*2 = 36 rabbits, < 40  Julie will not be able to start selling her rabbits after 4 months | B1 | This mark is given for a correct comparison of *R*2 with 40 and conclusion |

**Question 4 (Total 8 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** | |
| (a) | **=** ± ,  **=** ± ,  **=** ± | | M1 | This mark is given for finding vectors **,** and |
| ×  or  ×  or  × | | M1 | This mark is given for attempting a cross product between any two appropriate vectors |
| **=** | A1 | This mark is given for finding a correct cross product | |
| Area of triangle = √(182 + 362) = 9√5 | A1 | This mark is given for finding the area of the triangle *ABC* | |
| (b) | Volume =  = | M1 | This mark is given for a method to find the volume of the tetrahedron (could be using vectors  or ) | |
| =  ⎢18 + 36*d* – 108 ⎢ | A1 | This mark is given for a correct expression for the volume of the tetrahedron | |
| ⎢36*d* – 90 ⎢ = 21 | M1 | This mark is given for putting an expression equal to 21 and solving for *d* | |
| *d* = 6 | A1 | This mark is given for a correct value for *d* | |

**Question 5 (Total 10 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | *y* = *c*2*x*–1  ⇒  = –*c*2*x*–2 =  = – | M1 | This mark is given for recognising that the equation of the tangent at the point *P* is required |
| *y* –  = –*m*(*x – ct*) | M1 | This mark is given for finding the equation of the tangent at the point *P* and its gradient *m* |
| *y* –  = –(*x – ct*) | A1 | This mark is given for the correct equation of the tangent |
| Substituting *x* = 0, *y* =  Substituting *y* = 0, *x* = 2*ct* | M1 | This mark is given for a method to find the coordinates of *A* and *B* |
| *A* = (2*ct*, 0) and *B* = | A1 | This mark is given for finding the coordinates of the points *A* and *B* |
| = 2 × 2*ct*  2*c* = 4*ct* 2 | M1 | This mark is given for using *OB* = 2 × *OA* to form and solve an equation for *t* |
| *t*2 =  ⇒ *t* = | A1 | This mark is given for the correct value of *t* |
| ×  × 2*ct* = 32  4*c*2 = 64, *c* = 4 | M1 | This mark is given for forming an equation using the area of the triangle *OAB* to solve and find the value of *c* |
| Substituting *c* = 4 and *t* =  into | M1 | This mark is given for a method to find the coordinates of *P* using values of *t* and *c* |
| *x* = 2√2 and *y* = 4√2 | A1 | This mark is given for finding the exact coordinates of *P* |