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OCR
Oxford Cambridge and RSA

A LEVEL

Exemplar Candidate Work

MATHEMATICS A

H240
For first teaching in 2017

H240/02 Summer 2018 examination series

Version 1

Contents

Introduction

These exemplar answers have been chosen from the summer 2018 examination series.

OCR is open to a wide variety of approaches and all answers are considered on their merits. These exemplars, therefore, should not be seen as the only way to answer questions but do illustrate how the mark scheme has been applied.

Please always refer to the specification http://www.ocr.org.uk/Images/308723-specification-accredited-a-level-gce-mathematics-a-h240.pdf for full details of the assessment for this qualification. These exemplar answers should also be read in conjunction with the sample assessment materials and the June 2018 Examiners' report or Report to Centres available from Interchange https://interchange.ocr.org.uk/Home.mvc/Index

The question paper, mark scheme and any resource booklet(s) will be available on the OCR website from summer 2019. Until then, they are available on OCR Interchange (school exams officers will have a login for this and are able to set up teachers with specific logins – see the following link for further information http://www.ocr.org.uk/administration/support-and-tools/interchange/managing-user-accounts/).

It is important to note that approaches to question setting and marking will remain consistent. At the same time OCR reviews all its qualifications annually and may make small adjustments to improve the performance of its assessments. We will let you know of any substantive changes.

Question 1 (i)

1 (i) Express $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$. [4]

Exemplar 1 4 marks

1(i)

$2(x^2 - 6x) + 23$

$2((x-3)^2 - 9) + 23$

$2(x-3)^2 - 18 + 23$

$2(x-3)^2 + 5$

Exemplar 2 4 marks

1(i)

$2x^2 - 12x + 23$

$= 2[x^2 - 6x + 11.5]$

$= 2[(x-3)^2 - 9 + 11.5]$

$= 2[(x-3)^2 + 2.5]$

$= 2(x-3)^2 + 5$

Exemplar 3 2 marks

1(i)

$2(x-3)^2 + c$ B1 B1

$2x^2 - [illegible]x + \cancel{36}\ 72$

$2(x-3)^2 + 49$ M0 A0

Examiner commentary

Most candidates were able to answer this part well (exemplars 1 and 2). The best solutions gave sufficient detail so that the candidates were unlikely to make errors in moving from one line of working to the next. Those low scoring candidates who lost marks generally did so because they tried to perform two or more stages of working simultaneously (exemplar 3, where the initial error of giving b as -6 was corrected in the first line of working but this was not correctly followed through to the second line).

Question 1 (ii)

1 (ii) Use your result to show that the equation $2x^2 - 12x + 23 = 0$ has no real roots. [1]

Exemplar 1 1 mark

1(ii)	Turning point at $(-b, c)$. As $c = 5$ and the x^2 term is positive (so the curve is upturned), the curve never has a y co-ordinate below 5 so never crosses $y=0$ so has no real roots.

Exemplar 2 1 mark

1(ii)	$2(x-3)^2+5=0$ $2(x-3)^2=-5$ $(x-3)^2=-\frac{5}{2}$ $x-3=\sqrt{-\frac{5}{2}}$ cannot square root negative.

Exemplar 3 0 marks

1(ii)	Discriminant $= b^2-4ac$ $= (-12)^2 - 4(2)(23)$ $= -40$ BO Discriminant negative ∴ 0 real roots

Exemplar 4 0 marks

1(ii)	turning point = (3,5) 5>0 so the curve does not pass through the x-axis. ^

Examiner commentary

Two forms of answer were acceptable here, both required the use of part (i) as specified in the question. Some (exemplar 1) referred to the turning point and the fact that the curve is a positive quadratic whilst others (exemplar 2) rearranged the equation to show that there are no real roots. Many others (exemplar 3), whilst using correct mathematics, did not answer the question in the way stated (considering the discriminant of the quadratic rather than using the result from (i)) and thus do not gain credit. Many lower ability candidates attempted to use the turning point method but gave only a partial answer failing to state that the curve is a positive quadratic (exemplar 4).

Question 1 (iii)

1 (iii) Given that the equation $2x^2 - 12x + k = 0$ has repeated roots, find the value of the constant k. [2]

Exemplar 1 2 marks

1(iii)

$2x^2 - 12x + k = 0$ $\quad b^2 - 4ac = 0$

repeated roots

$a = 2 \quad b = -12 \quad c = k$

$144 - 4 \times 2 \times k = 0$

$144 - 8k = 0$

$144 = 8k \quad \underline{k = 18}$

Exemplar 2 1 mark

1(iii)

turning point = 3, 14

therefore to sit on the line

y = 0 (for a repeated root), y value

must equal 0 so take 5~~7~~ away

from 23.

M1

k = ~~[illegible]~~

A0

^

Examiner commentary

Generally, candidates were able to correctly answer this part, generally by considering the discriminant of the quadratic (see exemplar 1). Some looked to use a correct method utilising the turning point of the original quadratic but did not complete the working (exemplar 2).

Question 2 (i)

2 The points A and B have position vectors $\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix}$ respectively.

(i) Find the exact length of AB. **[2]**

Exemplar 1 — 2 marks

2(i)	$\lvert\overrightarrow{AB}\rvert = \sqrt{(-3-1)^2 + (-1+2)^2 + (2-5)^2}$
	$= \sqrt{16+1+9} = \sqrt{26}$

Exemplar 2 — 1 mark

2(i)	$\sqrt{(1--3)^2 + (-2-1)^2 + (5-2)^2} = \sqrt{34}$ M1

Examiner commentary

All candidates generally answered this well, often with clear details of how the answer was obtained (exemplar 1). Those who lost marks generally did so due to a minor error in dealing with negative values (exemplar 2).

Question 2 (ii)

2 (ii) Find the position vector of the midpoint of AB. **[1]**

Exemplar 1 — 1 mark

2(ii)

Midpoint at $\left(\frac{1-3}{2}, \frac{-2-1}{2}, \frac{5+2}{2}\right) = (-1, -1.5, 3.5)$

$\overrightarrow{OM} = \begin{pmatrix}-1\\-1.5\\3.5\end{pmatrix}$

Exemplar 2 — 0 marks

2(ii)

$\overrightarrow{AB} = \begin{pmatrix}-4\\1\\-3\end{pmatrix} \div 2 \quad \begin{pmatrix}-2\\\frac{1}{2}\\-\frac{3}{2}\end{pmatrix}$

Examiner commentary

Most, of all abilities, were able to answer this correctly, recognising the similarity to finding the midpoint in coordinate geometry, as in exemplar 1. Some confused position vectors with direction vectors and looked to find just half of the vector AB (exemplar 2).

Question 2 (iii)

2 The points P and Q have position vectors $\begin{pmatrix}1\\2\\0\end{pmatrix}$ and $\begin{pmatrix}5\\1\\3\end{pmatrix}$ respectively.

(iii) Show that $ABPQ$ is a parallelogram. [3]

Exemplar 1 — 3 marks

2(iii)

$\overrightarrow{AB} = \begin{pmatrix}-4\\1\\-3\end{pmatrix} = \underline{B} - \underline{A}$

$\overrightarrow{QP} = \begin{pmatrix}-4\\1\\-3\end{pmatrix} = \underline{P} - \underline{Q}$

$\overrightarrow{AB} = \overrightarrow{QP}$

$\overrightarrow{AQ} = \underline{Q} - \underline{A} = \begin{pmatrix}4\\3\\-2\end{pmatrix}$

$\overrightarrow{BP} = \underline{P} - \underline{B} = \begin{pmatrix}4\\3\\-2\end{pmatrix}$

$\overrightarrow{AQ} = \overrightarrow{BP}$

$\overrightarrow{AB} = \overrightarrow{QP}$ so opposite sides are equal in length and parallel. hence, ~~parallelogram~~ ABPQ is a parallelogram

Exemplar 2 — 3 marks

2(iii)

$\overrightarrow{AB} = \begin{pmatrix}-4\\1\\-3\end{pmatrix}$ $\overrightarrow{PQ} = \begin{pmatrix}4\\-1\\3\end{pmatrix}$

clearly $\overrightarrow{PQ} = -\begin{pmatrix}-4\\1\\-3\end{pmatrix} = -\overrightarrow{AB}$ $\therefore$ $\overrightarrow{AB}$ and $\overrightarrow{PQ}$ are parallel ... [1]

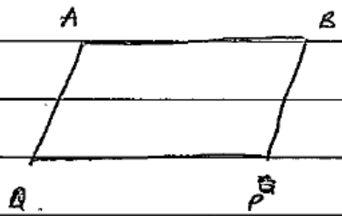
$\overrightarrow{BP} = \begin{pmatrix}4\\3\\-2\end{pmatrix}$ $\overrightarrow{AQ} = \begin{pmatrix}4\\3\\-2\end{pmatrix}$

clearly $AQ = 1(\overrightarrow{BP})$ $\therefore$ $\overrightarrow{BP}$ and $\overrightarrow{AQ}$ are parallel ... [2]

From [1] and [2], ABPQ must be a parallelogram

Exemplar 3 — 2 marks

2(iii)



$\overrightarrow{AB} = \overrightarrow{PQ}$ ✗ A0

$\overrightarrow{AB} = \begin{pmatrix}-4\\1\\-3\end{pmatrix}$ M1 $\quad \overrightarrow{PQ} = \begin{pmatrix}4\\-1\\3\end{pmatrix}$ M1

therefore parallel

Exemplar 4 — 0 marks

2(iii)

if ABPQ is a parallelogram

AB and PQ are parallel and BP and QA are parallel

m of AB $= (5-2) \div \frac{-2-1}{(-3)}$ ✗

$= -12$

M of PQ $= 3 \div \frac{1-2}{5-1}$ ✗

$= -12$

M of BP $= 2 \div \frac{-1-2}{-3-1}$ ✗

$= 8/3$

M of AQ $= 2 \div \frac{-2-1}{(-3)}$ ✗

$= 8/3$

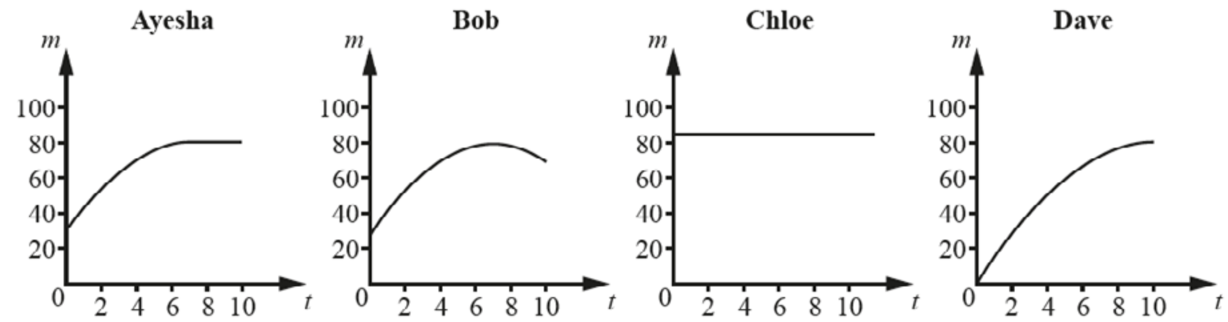
with opposite sides of lines are parallel ∴ its parallelogram.

Examiner commentary

Only a minority of candidates answered this question in the most efficient way, using the vector forms of one pair of sides. Many found the vector forms of both pairs of sides. Some then correctly stated that this then showed that opposite sides are equal and parallel (exemplar 1) others merely stated that this demonstrated that the opposite sides were parallel (exemplar 2), both of these are acceptable for full marks. Some confused vectors with coordinate geometry and tried to find a "gradient" for the vectors and then state that these were the same (exemplar 4). Partial marks were gained by some (exemplar 3) who correctly found appropriate vectors but then did not give sufficient detail in explaining why this showed that ABPQ is a parallelogram (in this case incorrectly stating that vector AB equals vector PQ, rather than QP).

Question 3 (i)

3 Ayesha, Bob, Chloe and Dave are discussing the relationship between the time, t hours, they might spend revising for an examination, and the mark, m, they would expect to gain. Each of them draws a graph to model this relationship for himself or herself.



(i) Assuming Ayesha's model is correct, how long would you recommend that she spends revising? **[1]**

Exemplar 1 — 1 mark

3(i)	7 hours

Examiner commentary

Almost all candidates answered this correctly (exemplar 1).

Question 3 (ii)

3 **(ii)** State one feature of Dave's model that is likely to be unrealistic. **[1]**

Exemplar 1 — 1 mark

3(ii)	If he didnt revise he would get 0 marks is unrealistic.

Exemplar 2 — 0 marks

3(ii)	That ~~there~~ eventually, he can gain over 100 marks.

Examiner commentary

Candidates of all abilities were generally able to gain this mark (exemplar 1). However, some of the middle ability candidates tried to read more into the question than was intended and they thus misinterpreted the given diagram (exemplar 2).

Question 3 (iii)

3 **(iii)** Suggest a reason for the shape of Bob's graph as compared with Ayesha's graph. **[1]**

Exemplar 1 — 1 mark

3(iii)	Bob could be predicting ~~[illegible]~~ he will become fatigued after too much revision and so he mustn't overdo it. Ayesha predicts that after a certain point she won't pick up any more marks, but her predicted mark wouldn't drop like bob's.

Exemplar 2 — 1 mark

3(iii)	Bob's result might be less if he revises for too many hours as stress may inhibit his performance on the day.

Examiner commentary

This was generally correctly answered by all candidates with the most common answers relating to tiredness (exemplar 1) or stress (exemplar 2).

Question 3 (iv)

3 (iv) What does Chloe's model suggest about her attitude to revision? **[1]**

Exemplar 1 — 1 mark

3(iv) Chloe believes that no matter how much revision she puts in, her grade will stay the same

Exemplar 2 — 0 marks

3(iv) It depends on the person but overall it is helpfull to a point.

Examiner commentary

Most candidates gave a good answer (exemplar 1) although some seemed not to give any thought to the given model (exemplar 2).

Question 4

4 Prove that $\sin^2(\theta+45)^\circ - \cos^2(\theta+45)^\circ \equiv \sin 2\theta^\circ$. [4]

Exemplar 1 4 marks

4 $\sin^2(\theta+45)^\circ - \cos^2(\theta+45)^\circ$

$= (\sin(\theta+45)^\circ)^2 - (\cos(\theta+45)^\circ)^2$

$= (\sin\theta\cos45^\circ + \cos\theta\sin45^\circ)^2 - (\cos\theta\cos45^\circ - \sin\theta\sin45^\circ)^2$

$= \left(\frac{\sqrt2}{2}(\sin\theta+\cos\theta)\right)^2 - \left(\frac{\sqrt2}{2}(\cos\theta-\sin\theta)\right)^2$

$= \frac12(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta)$

$- \frac12(\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta)$

$= \frac12(2\sin\theta\cos\theta + 2\sin\theta\cos\theta) = 2\sin\theta\cos\theta = \sin2\theta$

Exemplar 2 2 marks

4

$\sin^2(\theta+45) - \cos^2(\theta+45)$

$\sin2\theta = 2\sin\theta\cos\theta$

$\cos2\theta = \cos^2\theta - \sin^2\theta$

~~$-2\cos2\theta = \sin^2$~~

$-\cos(2(\theta+45)) = \sin^2(\theta+45) - \cos^2(\theta+45)$

$-\cos(2\theta+90)$ $\cos(\theta+90) = \sin\theta$

$= \sin(2\theta)$

Exemplar 3 — 2 marks

4

$\sin^2(\theta+45)^\circ - \cos^2(\theta+45)^\circ = \sin 2\theta^\circ$

$2\sin A\cos A$

$(\sin\theta\cos 45^\circ + \cos\theta\sin 45^\circ)^2$

$\left(\frac{\sqrt{2}}{2}\sin^2\theta + \frac{\sqrt{2}}{2}\cos\theta\right)^2 - (\cos\theta\cos 45 - \sin\theta\sin 45)^2$

M1 A1

$\left(\frac{\sqrt{2}}{2}\sin\theta + \frac{\sqrt{2}}{2}\cos\theta\right)^2 - \left(\frac{\sqrt{2}}{2}\cos\theta - \frac{\sqrt{2}}{2}\sin\theta\right)^2$

$\frac{1}{2}\sin^2\theta + \frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}\cos^2\theta - \frac{1}{2}\cos^2\theta + \frac{1}{2}\sin\theta\cos\theta - \frac{1}{2}\sin^2\theta$

$\sin\theta\cos\theta$ M0

^ A0

Exemplar 4 — 0 marks

4

$\sin^2(\theta+45^\circ) - \cos^2(\theta+45^\circ) \equiv \sin 2\theta$.

$(\sin^2\theta\cos^2 45 + \cos^2\theta\sin^2 45) - (\cos^2\theta\cos^2 45 - \sin^2\theta\sin^2 45)$ M0

$\sin^2\theta\cos^2 45 + \cos^2\theta\sin^2 45 - \cos^2\theta\cos^2 45 + \sin^2\theta\sin^2 45$

$\frac{1}{2}\sin^2\theta + \cancel{\frac{1}{2}\cos^2\theta} - \cancel{\frac{1}{2}\cos^2\theta} + \frac{1}{2}\sin^2\theta$

$\sin^2\theta \qquad \sin\theta \times \sin\theta$

$\cancel{\sin(\theta+\theta)}$

$\sin\theta \times \sin\theta = \sin 2\theta$ M0

Examiner commentary

A large variety of correct methods were seen (1). Some started with correct use of appropriate formula but having obtained a correct partial solution then jumped to the given answer with insufficient justification for a proof (exemplar 2). Some tried to move too quickly through the stages of the proof, either after initially correct work (exemplar 3) or initially starting with misquoted formulae, (exemplar 4).

Question 5 (i)

5 Charlie claims to have proved the following statement.

"The sum of a square number and a prime number cannot be a square number."

(i) Give an example to show that Charlie's statement is not true. **[1]**

Exemplar 1 — 1 mark

5(i)	$2^2 = 4$ 5 is a prime number
	$4 + 5 = 9 = 3^2$

Exemplar 2 — 1 mark

5(i)	$\cancel{a^2}$ ~~x~~
	$3^2 = 9$ ← square number
	7 is prime $9+7=\underline{\underline{16}}$ ← this is a square
	number.

Examiner commentary

All ability groups were able to give good answers to this question (exemplars 1 and 2).

Question 5 (ii)

5 Charlie's attempt at a proof is below.

Assume that the statement is not true.
$\Rightarrow$ There exist integers n and m and a prime p such that $n^2 + p = m^2$.
$\Rightarrow p = m^2 - n^2$
$\Rightarrow p = (m - n)(m + n)$
$\Rightarrow p$ is the product of two integers.
$\Rightarrow p$ is not prime, which is a contradiction.
$\Rightarrow$ Charlie's statement is true.

(ii) Explain the error that Charlie has made. **[1]**

Exemplar 1 — 1 mark

5(ii)	[illegible] A prime number is a product of two integers, 1 and itself. $(m-n)$ could be 1 and $(m+n)$ itself, in which case p is prime. Charlie has not considered this.

Exemplar 2 — 1 mark

5(ii)	Charlie has assumed that when P is the product of two integers, one of them integers is not one, which it may be and is therefore prime. BOD

Examiner commentary

This was often answered equally well by all ability levels (exemplars 1 and 2).

Question 5 (iii)

5 (iii) Given that 853 is a prime number, find the square number S such that $S + 853$ is also a square number. [4]

Exemplar 1 4 marks

5(iii)

$853 = (m-n)(m+n) = 1 \times 853$ (only factors if 853 is prime)

$m - n = 1$ $m = 1 + n$

$m + n = 853$

$1 + 2n = 853$

$2n = 852$

$n = 426$ $m = 427$

$426^2 + 853 = 427^2$

$S = 426^2 = 181,476$

Exemplar 2 4 marks

$n^2 + m^2 = 853$ / Trial and improvement on calculator until find difference of 853 between consecutive primes

$(427)^2 - (426)^2 = 853$

$\therefore$ S is $(426)^2$ is 181476

Exemplar 3 0 marks

5(iii)	$S^2+853=n^2$	$900-853=47$ ×
		$961-853=108$ ×
	$(S-n)(S+n)=853$	$1024-853=171$ ×
		$1089-853=286$ ×
		$1156-853=303$ ×
		$1225-853=372$ ×
		$1296-853=443$ ×
		$1369-853=516$ ×
		$1444-853$ 591 ×
		$1521-853$ 668 ×

Examiner commentary

Some candidates recognised that the starting point was $m-n=1$. Most of these proceeded to obtain the correct answer (exemplar 1). Many candidates, however, did not appreciate the link with part (ii) and attempted trial and improvement (exemplar 2, correct working despite initial sign error), although generally without success (exemplar 3).

Question 6 (i)

6 In this question you must show detailed reasoning.

A curve has equation $y = \frac{\ln x}{x}$.

(i) Find the x-coordinate of the point where the curve crosses the x axis. **[2]**

Exemplar 1 2 marks

6(i)

$y = 0$

$\frac{\ln x}{x} = 0$

$\ln x = 0 \quad \therefore \quad x = 1$

Exemplar 2 2 marks

6(i)

$y = \frac{\ln(x)}{x} \qquad 0 = \frac{\ln(x)}{x}$

when $x = 1 \leftarrow$ when $\ln(x) = 0$

answer: $(1, 0)$

Examiner commentary

This question was well answered by all candidates (exemplars 1 and 2).

Question 6 (ii)

6 (ii) The points A and B lie on the curve and have x coordinates 2 and 4. Show that the line AB is parallel to the x-axis. **[2]**

Exemplar 1 — 2 marks

6(ii)

when $x = 2$, $y = \frac{\ln 2}{2} = \frac{1}{2}\ln 2 = \ln 2^{\frac{1}{2}} = \ln\sqrt{2}$

when $x = 4$, $y = \frac{\ln 4}{4} = \frac{1}{4}\ln 4 = \ln 4^{\frac{1}{4}} = \ln\sqrt{2}$

Since the ~~y co-ordinates~~ y values are the same, a straight line between the points $(2, \ln\sqrt{2})$ and $(4, \ln\sqrt{2})$ must be parallel to the x axis.

Exemplar 2 — 0 marks

6(ii)

$y = \frac{\ln 2}{2}$ $y = \frac{\ln 4}{4}$

$\frac{\ln 2}{2} = \frac{\ln 4}{4}$

$A(2, \frac{\ln 2}{2})$ $B(4, \frac{\ln 4}{4})$

$\therefore$ 0 gradient, so they are parallel to the x-axis

Exemplar 3 — 0 marks

6(ii)

$x = 2$ $\frac{\ln 2}{2} = 0.347$

$\frac{\ln 4}{4} = 0.347$

$(2, 0.35)$ $(4, 0.35)$

this means they have a gradient of 0 ~~line~~. a gradient of 0.

this is the line $y = 0.35$ $y = 0$ is parallel to $y = 0.35$ as they both have

Examiner commentary

To gain any marks there had to be at least one line of valid working showing why $\frac{1}{4}\ln 4$ and $\frac{1}{2}\ln 2$ are equal (exemplar 1). Many candidates just stated or implied that $\frac{\ln 2}{2} - \frac{\ln 4}{4} = 0$, either without justification or by using their calculator and decimals (exemplars 2 and 3). These candidates scored no marks, because of the "**detailed reasoning**" instruction.

Question 6 (iii)

6 (iii) Find the coordinates of the turning point on the curve. **[4]**

Exemplar 1 — 4 marks

6(iii)

$$\frac{dy}{dx} = \frac{x\left(\frac{1}{x}\right) - \ln x\,(1)}{x^2}$$ quotient rule

$$= \frac{1 - \ln x}{x^2} = 0$$

$x^2 \neq 0$

$1 - \ln x = 0$

$1 = \ln x$

$x = e^1 = e$

$$\left(e, \frac{1}{e}\ln e\right) = \left(e, \frac{1}{e}\right)$$

Exemplar 2 — 3 marks

6(iii)

$$\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$$

let $u = \ln x$ $\quad v = x$

$\frac{du}{dx} = \frac{1}{x}$ $\quad \frac{dv}{dx} = 1$

$$\frac{1 - \ln x}{x^2} = 0$$

$$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$1 - \ln x = 0$

$1 = \ln x$

$e^1 = x$

$e = x$

Examiner commentary

This question was well answered (exemplar 1). A few candidates did not find the y-coordinate (exemplar 2).

Question 6 (iv)

6 (iv) Determine whether this turning point is a maximum or a minimum. **[5]**

Exemplar 1 — 5 marks

$$\frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - 2x(1-\ln x)}{x^4} = \frac{-x-2x+2x\ln x}{x^4}$$

at $e = x$ $\frac{d^2y}{dx^2} = -0.0498$ negative so we expect max point

checking either side:

$\frac{dy}{dx}$ at $e-0.1 = 5.47\times10^{-3}$ at $e+0.1 = -4.55\times10^{-3}$

positive $\rightarrow 0 \rightarrow$ negative So it's a maximum.

Exemplar 2 — 4 marks

6(iv)

$\cancel{\frac{1-\ln x}{x^2}}$ $u = 1-\ln x$ $u' = -\frac{1}{x}$

$v = x^2$ $v' = 2x$

$\frac{d^2y}{dx^2} = \frac{-x-2x+2x\ln x}{x^4}$ M1 $\frac{-1-2+2\ln x}{x^3}$ A1

$\frac{2\ln x - 3}{x^3}$

$\frac{2\ln e^1 - 3}{(e^1)^3}$ M1 $= -0.37$ A0 ← Maximum point. B1

answer space continues opposite

$$\frac{d^2y}{dx^2} = \frac{2\ln x - 3}{x^3} \qquad x = e^1$$

$$\frac{2\ln e^1 - 3}{e^{13}} = -0.37$$

max point..

Exemplar 3 — 3 marks

6(iv)

$\frac{d^2y}{dx^2} = \frac{x - 2x(1-\ln x)}{x^4}$ M1 $=$ A0 $\frac{2x\ln x - x}{x^4} = \frac{2\ln x - 1}{x^3}$

$x = e$: $\frac{d^2y}{dx^2} =$ M1 ~~max me~~ $\frac{2\ln e - 1}{e^3}$ A0 $= \frac{1}{e^3} > 0$

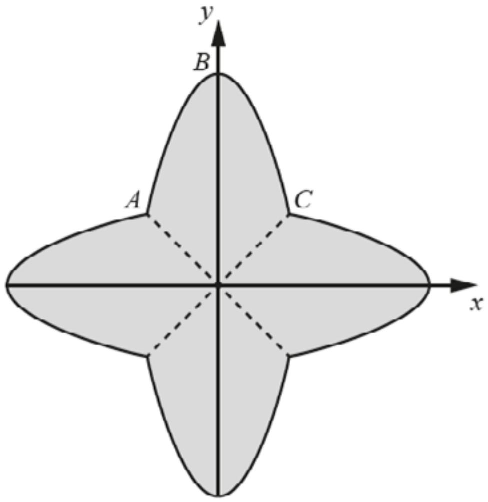
$\therefore$ turning point at $\left(e, \frac{1}{e}\right)$ is a minimum FT B1

Examiner commentary

Most candidates attempted a correct method (exemplar 1). Some made numerical errors when substituting $x = e$ into d 2 y (exemplar 2). Others made mistakes in the differentiation (exemplar 3).

Question 7

7 The diagram shows a part ABC of the curve $y = 3 - 2x^2$, together with its reflections in the lines $y = x$, $y = -x$ and $y = 0$.



Find the area of the shaded region. [7]

Exemplar 1 7 marks

7 reflection in $y = x$: $x = 3 - 2y^2$

$$y = 3 - 2(3 - 2y^2)^2$$

$$= 3 - 2(9 - 12y^2 + 4y^4)$$

$$= -15 + 24y^2 - 8y^4$$

$$8y^4 - 24y^2 + y + 15 = 0$$

$y = 1$: $8(1)^4 - 24(1)^2 + 1 + 15 = 0$

$$8y^4 - 24y^2 + y + 15 = (y-1)(8y^3 + ay^2 + by + c)$$

comparing coefficients: y^3: $a - 8 = 0 \therefore a = 8$

y^2: $b - 8 = -24 \therefore b = -16$

y: $c + 16 = 1 \therefore c = -15$

$\therefore (y-1)(8y^3 + 8y^2 - 16y - 15)$

sub. $y = 1$ into $x = 3 - 2y^2 \Rightarrow x = 1$

graph segments are all symmetrical reflections of each other so points of intersection should be equidistant from both axes

$\therefore$ area of central square $= 4$

area under $y = (3-1) - 2x^2 = 2\int_0^1 2 - 2x^2 \, dx$

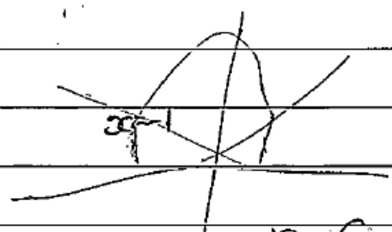
$= 2\left[2x - \frac{2}{3}x^3\right]_0^1 = \frac{8}{3}$

$\therefore$ total area $= 4 + 4\left(\frac{8}{3}\right) = \boxed{\frac{44}{3}}$

Exemplar 2

5 marks

7 $y = 3 - 2x^2 \quad y = -x \quad y = x$



$3 - 2x^2 = -x$

$2x^2 - x - 3 = 0$ $\quad \times -6 \quad + -1 \quad 2 \; -3$

$2x^2 + 2x - 3x - 3 = 0$ $\quad 3 - 2x^2 = x \quad \times -6 \quad +1 \; -2 \; 3$

$2x(x+1) - 3(x+1) = 0$ $\quad 2x^2 + x - 3 = 0$

$(2x-3)(x+1) = 0$ $\quad 2x^2 - 2x + 3x - 3 = 0$

$x = -1$ M1 A1 $\quad 2x(x-1) + 3(x-1) = 0$

$(2x+3)(x-1) = 0$

$\left(\int_{-1}^{1} 3 - 2x^2 \, dx - 2\int_0^1 x \, dx\right) \times 4 + (2 \times 2)$ M1 M1

$\left(\left[3x - \frac{2}{3}x^3\right]_{-1}^{1} - 2\left[\frac{1}{2}x^2\right]_0^1\right) 4$

$\left(+\frac{7}{3}\right) - \left(-\frac{7}{3}\right) \quad \left(\frac{1}{2}\right) - (0)$

A1 $\frac{14}{3} - 2\left(\frac{1}{2}\right) = \frac{11}{3} \times 4 = \frac{44}{3} + 4 = \frac{56}{3}$ M0 A0

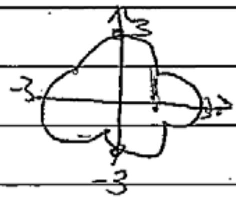
$= \frac{56}{3}$ square units

Exemplar 3 1 mark

7

$\int_{-3}^{3} 3 - 2x^2$ M1 $\left[3x - \frac{2}{3}x^3\right]_{-3}^{3}$

$\cancel{\int_{-3}^{3} 3x - \frac{2}{3}x^3 + c}$



$(9 - 18) - (-9 + 18)$

$9 - 18 + 9 - 18$

$9 + 9 - 18 - 18$

$\underline{\underline{-18}}$ area $= 18 \text{ cms}^2$

Examiner commentary

A large variety of correct methods were seen. Some were unnecessarily long (Exemplar 1).

A common misconception was to treat the full diagram as a single function and integrate $y = 3 - 2x^2$ using incorrect limits (exemplar 3). Many made mistakes in trying either to add or subtract all or part of the area of the middle square (exemplar 2).

Question 8 (i)(a)

8 **(i)** The variable X has the distribution N(20, 9).

(a) Find $P(X > 25)$. **[1]**

Exemplar 1 — 1 mark

8(i)(a)	$P(X>25)$ $X \sim N(20, 9)$ $\sigma = 3$
	$P = 0.04779035$
	$= 0.0478$

Exemplar 2 — 0 marks

8(i)(a)	$X \sim N(20, 9)$
	$P(X > 25) =$ ~~[illegible]~~ 0.289

Examiner commentary

Most candidates answered this question correctly, generally just stating the answer directly from their calculator (exemplar 1). A few used a standard deviation of 9 (exemplar 2).

Question 8 (i)(b)

8 **(i)** **(b)** Given that $P(X > a) = 0.2$, find a.

Exemplar 1 — 1 mark

8(i)(b)	$P(X>a)=0.2$ a
	$a=22.5248637$
	$=\underline{\underline{22.525}}$

Exemplar 2 — 0 marks

8(i)(b)	$\Phi^{-1}(0.2) = 17.47\ldots$
	$\therefore a = 17.5$ (3s.f.)

Exemplar 3 — 0 marks

8(i)(b)	$a = 27.575$

Examiner commentary

Many gave correct solutions (exemplar 1). Some candidates found $\Phi^{-1}(0.2) = 17.5$ rather than $\Phi^{-1}(0.8) = 22.5$ (exemplar 2). A few used a standard deviation of 9 (exemplar 3).

Question 8 (i)(c)

8 (i) (c) Find b such that $P(20 - b < X < 20 + b) = 0.5$. [3]

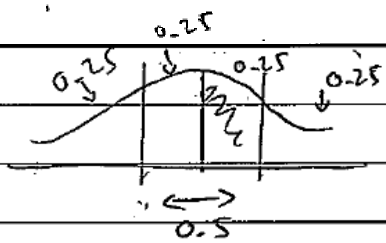
Exemplar 1 — 3 marks

8(i)(c)	$P(X > 20 + b) = 0.25$
	$20 + b = 22.02$ $\quad b = 2.02$

Exemplar 2 — 1 mark

8(i)(c)	$P(20 - 3b < X < 20 + b) = 0.5$
	when $b = 1$ $\quad P = 0.26$
	$b = 3$ $\quad P = 0.68$
	$b = 2$ $\quad P = 0.495$
	$P = 0.5$ to 3 sig fig
	$\therefore b = 2$
	M1
	^

Exemplar 3 — 1 mark

8(i)(c)	
	$P(20 < X < 20 + b) = 0.25$
	$0.5 - P(X > b) = 0.25$
	$P(X < b) = 0.75$
	$20 + b = 26.07041$
	$b = 6.07041$

Exemplar 4 0 marks

8(i)(c)

$P(20-b < X < 20+b) = 0.5$

~~P~~ ~~2b~~

when $B=5$, $P(X) = 0.422$
when $B=4$, $P(X) = 0.343$
when $B=6$, $P(X) = 0.495$
when $B=7$, $P(X) = 0.563$

therefore by ~~trail~~ trial and improvement, $B = 6$
so $P(14 < X < 26) = 0.5$

M0

Examiner commentary

Many candidates were able to give a correct solution (exemplar 1) but many could not make the first step, which is to move from the given probability of 0.5 to a probability of either 0.25 or 0.75 and thus used a trial and improvement method but often not continuing to sufficient accuracy to confirm their result (exemplar 2). A few used a standard deviation of 9 (exemplars 3 and 4).

Question 8 (ii)

8 **(ii)** The variable Y has the distribution $N(\mu, \frac{\mu^2}{9})$. Find $P(Y > 1.5\mu)$. **[3]**

Exemplar 1 — 3 marks

8(ii)

$$\Phi\left(\frac{1.5\mu - \mu}{\left(\frac{\mu}{3}\right)}\right) = 1 - P(Y > 1.5\mu)$$

$$\Phi\left(\frac{3 \times 0.5\cancel{\mu}}{\cancel{\mu}}\right) = \Phi(1.5) = 0.93319$$

$$1 - 0.93319 = 0.0668$$

$$P(Y > 1.5\mu) = 0.0668$$

Exemplar 2 — 2 marks

8(ii)

mean $= \mu$

$\sigma = \frac{\mu}{3}$

$Y \sim (\mu, \frac{\mu^2}{9})$

M1 A0

Let $\mu = 1$ $\therefore Y \sim (1, \frac{1}{9})$

$p(Y > 1.5) = 0.0668$ (3s.f.) A1

Exemplar 3 — 2 marks

8(ii)

$Y \sim N\left(\mu, \frac{\mu^2}{9}\right)$

$p(Y > 1.5\mu)$ let $\mu = 3$. **M1**

$Y \sim N(3, 1^2)$ **M0**

$p(Y > 4.5) = 0.0668$ (3sf) **A1**

Exemplar 4 — 1 mark

8(ii)

$N\left(\mu, \frac{\mu^2}{9}\right)$ $p(Y > 1.5\mu)$

$\frac{1.5\mu - \mu}{\frac{\mu}{3}} = 0.1$ $\frac{0.5\mu}{\frac{\mu}{3}} = 0.1$ $\frac{1.5\mu}{\frac{\mu}{3}} = 0.1$

M1

$1.5\mu = 0.1\mu$

A0

A0

Examiner commentary

The best candidates were able to correctly substitute $X = 1.5\mu$ into $\frac{X-\mu}{\frac{\mu}{3}}$ in order to find $P(X = 1.5\mu)$. (exemplar 1) but many others chose a value for μ (often 1 or 3), without justification. These could score two out of the three marks. (exemplars 2 and 3). Some could not handle the algebraic manipulation involved in substituting $X = 1.5\mu$ into $\frac{X-\mu}{\frac{\mu}{3}}$ (exemplar 4).

Question 9

9 Briony suspects that a particular 6-sided dice is biased in favour of 2. She plans to throw the dice 35 times and note the number of times that it shows a 2. She will then carry out a test at the 4% significance level. Find the rejection region for the test. **[7]**

Exemplar 1 — 7 marks

9 $H_0: p = \frac{1}{6}$ $\quad H_1: p > \frac{1}{6}$ where p is P(rolling 2)

$n = 35$ $\quad$ Significance level 4%.

Let X be the number of times a 2 is rolled.

If H_0 true, $X \sim B(35, \frac{1}{6})$

$P(X \geq 9) = 1 - P(X \leq 8) = 0.1162$

$P(X \geq 10) = 1 - P(X \leq 9) = 0.05501$

$P(X \geq 11) = 1 - P(X \leq 10) = 0.02318$

Critical value : $X = 11$

Critical region : $X \geq 11$

If $X \geq 11$ Briony will reject H_0, as there will be evidence at the 4% significance level that the dice may be biased.

Exemplar 2 — 5 marks

9 ~~H0~~ X is number of 2s

B0 B0

^

$X \sim B(35, \frac{1}{6})$

M1 M1

$P(X \geq 10) = 0.055 > 0.04$ so in acceptance region

A1

$P(X \geq 11) = 0.02318 < 0.04$

A1

so $X \geq 11$ is rejection region

A1

Exemplar 3

2 marks

9

$H_0 : p = \frac{1}{6}$ B1 test stat = ?

$H_1 : p > \frac{1}{6}$ ^ $n = 35$

expected = 6 $X \sim B(35, \frac{1}{6})$ M1

sig level = 4% $P(\cancel{X \geq}$

if $X \geq 10$ reject H_0

Examiner commentary

The best answers included a definition of p in the hypotheses and used an exact method, using the binomial distribution. This involved finding P($X \geq 10$) and P($X \geq 11$) or a similar method (exemplar 1). Some of the most able candidates did not include the hypotheses (exemplar 2). Some of the candidates did not define p in the hypotheses and sought to define a rejection region without giving relevant probabilities to justify their conclusions (exemplar 3). Clear definitions are required to support the statistical process and conclusions.

Question 10 (i)

10 A certain forest contains only trees of a particular species. Dipak wished to take a random sample of 5 trees from the forest. He numbered the trees from 1 to 784. Then, using his calculator, he generated the random digits 14 781 049. Using these digits, Dipak formed 5 three-digit numbers. He took the first, second and third digits, followed by the second, third and fourth digits and so on. In this way he obtained the following list of numbers for his sample.

147 478 781 104 49

(i) Explain why Dipak omitted the number 810 from his list. **[1]**

Exemplar 1 — 1 mark

10(i)	the sample size is ~~o~~ only up to 784 and 810 is greater than 784 so can not be included as there is no data for it.

Examiner commentary

This question was very well answered by most candidates (exemplar 1).

Question 10 (ii)

10 (ii) Explain why Dipak's sample is not random. **[1]**

Exemplar 1 — 1 mark

10(ii) the 3-digit numbers are not independent of one another – they each share 2 digits with the numbers before and after them.
the probability of getting a specific number depends on the numbers that come before it.

Exemplar 2 — 0 marks

10(ii) ~~Because~~ Because when he formed the 3 digit ~~Aa~~ ~~num~~ numbers, it wasn't random.

Exemplar 3 — 0 marks

10(ii) ~~the~~ only the trees which 3 digits have a higher chance of Being picked since Dipak can only pick one two digit tree per draw.

Examiner commentary

Only a limited number of candidates were correctly able to identify why this sample was not random (exemplar 1). Many merely restated the information given with no explanation (exemplar 2). Others gave answers which suggested a misunderstanding of the given information (exemplar 3).

Question 10 (iii)

10 (iii) Carry out the test at the 2% significance level. **[7]**

Exemplar 1 — 7 marks

10(iii)

μ = mean height of trees in his forest

$H_0: \mu = 4.2$ B1 $\quad \bar{X}$ = sample mean

$H_1: \mu < 4.2$ B1

$\bar{X} \sim N\left(4.2, \frac{0.8^2}{50}\right)$ ← central limit theorem

$P(\bar{X} \leq 4.0) = 0.0385 > 0.02$ ← (significance level)

M1 A1 A1

there is not sufficient ~~of~~ evidence to suggest the mean height of his forest's trees is less than 4.2

accept H_0, reject H_1 M1 A1

Exemplar 2 — 6 marks

10(iii)

Assumed that the trees is normally distributed.
H is the height of the tree.

$H \sim N(4.2, 0.8^2)$ μ is the mean height of the tree.

BOD

~~The~~ $\mu \sim N(4.2, \frac{0.8^2}{50})$

$H_0: \mu = 4.2$
$H_1: \mu < 4.2$ at 2%

$p(M < 4) = 0.0385$ (3sf)

$0.0385 > 0.02$

$\therefore$ accept H_0, there is enough evidence to suggest that the mean height of the tree is 4.2 m.

A0

Exemplar 3 — 6 marks

10(iii)

$\mu = 4.2m$ $\mu < 4.2m$ $n = 50$ ~~$\bar{x}$~~ $= 4.0m$
$\sigma = 0.8$ 2% sig level
$H_0: \mu = 4.2m$ μ = mean height of trees of this species.
$H_1: \mu < 4.2m$

$X \sim N(4.2, \frac{0.8^2}{50})$ $\sigma = \frac{2\sqrt{2}}{25}$ or 0.11314

$P(X \leq 4)$ $P = 0.0385499 > 0.02$

not enough evidence to suggest H_1, reject H_1 and accept H_0.

^

Exemplar 4 1 mark

10(iii)	Hypothesis test using a normal distribution
	B1
	H_0 : $\mu = 4.2$ $\sigma = 0.8$
	B0 ^
	H_1 : $\mu < 4.2$ sig level = 2%
	test stat = 4.0
	M1 A0
	A1 BOD
	$P(X < 4.0)$ ^ > 0.02
	M1
	∴ we do not reject H_0 as there is sufficient A0 evidence at the 2% significance level to suggest that the mean tree hight is lower than 4.2m.

Examiner commentary

This question was well answered by many candidates (exemplar 1). Some draw the incorrect conclusion that there is sufficient evidence that the mean height of the trees is 4.2m rather than there is insufficient evidence that the mean height is less than 4.2m (exemplar 2). Others fail to give any conclusion in context (exemplar 3). A number did not give sufficient detail to be able to gain marks, not defining μ nor the variance or the relevant probability (exemplar 4).

Question 11 (i)(a)

11 Christa used Pearson's product-moment correlation coefficient, r, to compare the use of public transport with the use of private vehicles for travel to work in the UK.

(i) Using the pre-release data set for all 348 UK Local Authorities, she considered the following four variables.

Number of employees using public transport	x
Number of employees using private vehicles	y
Proportion of employees using public transport	a
Proportion of employees using private vehicles	b

(a) Explain, in context, why you would expect strong, positive correlation between x and y. **[1]**

Exemplar 1 — 1 mark

11(i)(a) Because if the no. of employees using public transport increase, it ~~wer~~ would suggest a larger no. of employees, so therefore there is a higher ~~change~~ chance that more people will use a private vehicle.

Examiner commentary

Most candidates answered correctly (exemplar 1).

Question 11 (i)(b)

11 (b) Explain, in context, what kind of correlation you would expect between a and b. [2]

Exemplar 1 2 marks

11(i)(b) A strong negative correlation. If a greater proportion use public transport, there must be a lower proportion using private vehicles, as the sum of the proportions must always be 1. However $r \neq -1$ because there might be employees who fit in neither category (such as those who walk or work from home) so ~~they~~ ~~are~~ $a + b \neq 1$.

Exemplar 2 1 mark

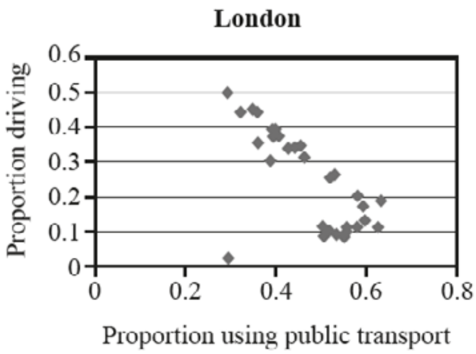
11(i)(b) Inversely proportional, B0 because if the proportion of employees using a public transport increases, there are ~~a~~ less proportion of the ~~week~~ employees using a private vehical. B1

Examiner commentary

Many candidates understood the point (exemplar 1), although some seemed to be unfamiliar with term correlation, referring instead to proportionality (exemplar 2).

Question 11 (ii)(a)

11 **(ii)** Christa also considered the data for the 33 London boroughs alone and she generated the following scatter diagram.



One London Borough is represented by an outlier in the diagram.

(a) Suggest what effect this outlier is likely to have on the value of r for the 32 London Boroughs. **[1]**

Exemplar 1 — 1 mark

11(ii)(a)	It will raise the value of r (so ~~the~~ r is closer to zero).

Exemplar 2 — 0 marks

11(ii)(a)	reduces r quite significantly.

Examiner commentary

Some candidates ensured that there was no ambiguity by saying that r would "move closer to 0" in addition to stating the value of r increases (exemplar 1). Some candidates stated that r would decrease, which is incorrect. It is possible that what they meant was that the size of r would decrease, which is correct, but these candidates could not be credited the mark (exemplar 2).

Question 11 (ii)(b)

11 (b) Suggest what effect this outlier is likely to have on the value of r for the whole country. [1]

Exemplar 1 1 mark

11(ii)(b)	near to no effect since there are so many cities.

Examiner commentary

Many good answers were seen (exemplar 1).

Question 11 (ii)(c)

11 (c) What can you deduce about the area of the London Borough represented by the outlier? Explain your answer. [1]

Exemplar 1 — 0 marks

11(ii)(c) there is good links to public transport within that area and more people use this transport than those who drive BO

Exemplar 2 — 1 mark

11(ii)(c) maybe people cannot afford private vehicles there.

Exemplar 3 — 1 mark

11(ii)(c) It may be in the centre of the city with lots of offices, so many people are able to walk to work and do not need to use public transport or private vehicles, because they live close to where they work.

Examiner commentary

Most candidates recognised the key factor - that a tiny proportion drive to work. But some candidates mistakenly suggested that this is because there is a great deal of public transport available (exemplar 1). To gain the mark answers had to fall into one of two types:

1. A sensible suggestion for a possible reason why in this particular area few people drive. (exemplar 2)
2. A statement that it is likely that a large proportion walk or cycle to work, or that jobs are generally close to home (exemplar 3).

Question 12 (i)

12 The discrete random variable X takes values 1, 2, 3, 4 and 5, and its probability distribution is defined as follows.

$$P(X=x)=\begin{cases} a & x=1, \\ \frac{1}{2}P(X=x-1) & x=2,3,4,5, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that $a=\frac{16}{31}$. **[2]**

Exemplar 1 — 2 marks

12(i)

X	1	2	3	4	5
$p(X=x$	a	$\frac{1}{2}a$	$\frac{1}{4}a$	$\frac{1}{8}a$	$\frac{1}{16}a$

$a+\frac{1}{2}a+\frac{1}{4}a+\frac{1}{8}a+\frac{1}{16}a=1$

$\frac{31}{16}a=1$

$a=\frac{16}{31}$

Exemplar 2 — 0 marks

12(i)

$P(x=5)=\frac{1}{31}$ $1+2+4+8$

$P(x=4)=\frac{2}{31}$ $=3+12$

$P(x=3)=\frac{4}{31}$ $=15$

$P(x=4)=\frac{8}{31}$

$\frac{31}{31}-\frac{15}{31}=\frac{16}{31}$

$a=\frac{16}{31}$

Examiner commentary

This question was well answered by many (exemplar 1), although a few candidates used the probabilities in the table, just finding $1-(\frac{8}{31}+\frac{4}{31}+\frac{2}{31}+\frac{1}{31})$ (exemplar 2).

Question 12 (ii)

12 The discrete probability distribution for X is given in the table.

x	1	2	3	4	5
$\mathrm{P}(X=x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

(ii) Find the probability that X is odd. **[1]**

Exemplar 1 1 mark

12(ii)

$p(X \text{ is odd}) = p(x=1) + p(x=3) + p(x=5)$

$= \dfrac{16+4+1}{31}$

$= \dfrac{21}{31}$

Examiner commentary

This question was well answered (exemplar 1).

Question 12 (iii)

12 Two independent values of X are chosen, and their sum S is found.

(iii) Find the probability that S is odd. **[2]**

Exemplar 1 — 2 marks

12(iii)

$P(S \text{ is odd}) = P(X_1 \text{ odd} \cap X_2 \text{ even}) + P(X_1 \text{ even} \cap X_2 \text{ odd})$

$= \left(\frac{21}{31} \times \frac{10}{31}\right) + \left(\frac{10}{31} \times \frac{21}{31}\right)$

$= \frac{420}{961} = 0.4370$ (to 4s.f.)

Exemplar 2 — 1 mark

12(iii)

$1+2=3 \approx 128$

$1+4=5 \approx 32$

$2+3=5 \approx 32$

$2+5=7 \approx 8$

$3+4=7 \approx 8$

$4+5=9 \approx 2$

$128 + 64 = 192$

$192 + 8 + 8 + 2 = 210$

$\frac{210}{961}$ M1 ^

Examiner commentary

Some candidates recognised that their answer to part (ii) could be used (exemplar 1). Others started from scratch but often omitted to consider all possibilities (exemplar 2).

Question 12 (iv)

12 **(iv)** Find the probability that S is greater than 8, given that S is odd. **[3]**

Exemplar 1 — 3 marks

12(iv)

$P(S>8 \mid S \text{ is odd}) = \dfrac{P(S>8 \cap S \text{ is odd})}{P(S \text{ is odd})}$

$P(S>8 \cap S \text{ is odd}) = P(S=9) = 2\left(\frac{2}{31} \times \frac{1}{31}\right)$

$= \frac{4}{961}$

$P(S>8 \mid S \text{ is odd}) = \dfrac{4/961}{420/961} = \frac{1}{105}$

Exemplar 2 — 1 mark

12(iv)

greater than 8 = 4+5

M0

$\dfrac{\left(\frac{2}{31} \times \frac{1}{31}\right)}{\frac{226}{961}} = \frac{1}{113}$ A0

M1

Examiner commentary

Most candidates recognised the need to find P(S = 9) with the most able producing excellent solutions (exemplar 1). Some omitted to include both 4, 5 and 5, 4 but then correctly divided by their answer to part (iii) (exemplar 2 where their answer to (iii) had been 226/961).

Question 12 (v)

12 Sheila sometimes needs several attempts to start her car in the morning. She models the number of attempts she needs by the discrete random variable Y defined as follows.

$$P(Y = y+1) = \frac{1}{2}P(Y = y) \quad \text{for all positive integers } y.$$

(v) Find $P(Y = 1)$. **[2]**

Exemplar 1 — 2 marks

12(v)

1	a
2	$a/2$
3	$a/4$
4	$a/8$
...	...

geometric series

$$\frac{a}{1-\frac{1}{2}} = 1$$

$$2a = 1$$

$$\boxed{a = \frac{1}{2}}$$

$$P(Y=1) = \frac{1}{2}$$

Exemplar 2 — 0 mark

12(v)

$1 = y + 1$

$0 = y$

$y = 0$

$p(Y=0) = \frac{1}{2}p(Y=0)$

$\cancel{p(Y=0) = 0}$

$p(Y=1) = 0$

Examiner commentary

Some candidates recognised the need for an infinite series and were able to sum to infinity (exemplar 1), but many could not cope with the fact that the first term is unknown and gave no response to this part. Some candidates demonstrated poor understanding of probability notation and hence resulted in concluding that $P(Y = 1) = 0$ (exemplar 2).

Question 12 (vi)

12 **(vi)** Give a reason why one of the variables, X or Y, might be more appropriate as a model for the number of attempts that Sheila needs to start her car. **[1]**

Exemplar 1 — 1 mark

12(vi)	Y is more appropriate as it is unlikely that if the car doesn't start 5 times in a row it never will, as X predicts.

Exemplar 2 — 1 mark

12(vi)	X may be more appropriate because it limits the number of attempts to start the car at 5. It is very unlikely that more than 5 attempts will be needed, ~~and~~ if Y assumes that if the car didn't start she would keep trying forever when in reality ~~B~~ she would probably give up and check if it was broken.

Examiner commentary

A choice of either *X* or *Y* with a reasonable justification was acceptable (exemplars 1 and 2). Many offered no response to this part.

Question 13 (i)

13 In this question you must show detailed reasoning.

The probability that Paul's train to work is late on any day is 0.15, independently of other days.

(i) The number of days on which Paul's train to work is late during a 450-day period is denoted by the random variable Y. Find a value of a such that $\mathrm{P}(Y > a) \approx \frac{1}{6}$. **[3]**

Exemplar 1 — 3 marks

13(i)

$a = 74$ M1 $Y \sim B(450, \frac{1}{6})$

$P(Y>74) = 0.177 = \frac{1}{6} + 0.0103$

$P(Y>73) = 0.212$

$P(Y>75) = 0.14575 = \frac{1}{6} - 0.021$ M1

$a = 74$ is best match for $P(Y>a) \approx \frac{1}{6}$ A1

Exemplar 2 — 1 mark

13(i)

$P(Y < a) \approx \frac{5}{6}$

from binomial CD list on calculator,

$P(Y \leq 74) = 0.8229$ M1

$\therefore P(Y < 75) = 0.8229$ ^ M0

$a = 75$

Examiner commentary

Because this question required "detailed reasoning", correct answers with insufficient detail did not necessarily score full marks (exemplar 2). Trial and improvement methods only scored full marks if they were very clearly explained, with the distribution fully described and with at least two values close to 75 being tried, with the relevant probabilities actually seen (exemplar 1).

Question 13 (ii)

13 In the expansion of $(0.15 + 0.85)^{50}$, the terms involving 0.15^r and 0.15^{r+1} are denoted by T_r and T_{r+1} respectively.

(ii) Show that $\frac{T_r}{T_{r+1}} = \frac{17(r+1)}{3(50-r)}$. **[3]**

Exemplar 1 — 3 marks

13(ii)

~~0.85^{50} 0.85^{49} $\times 0.15$ $\times {}^{50}C_{r}$~~

① $0.85^{50-r}\ 0.15^{r}\ {}^{50}C_r = T_r$

$0.85^{49-r}\ 0.15^{r+1}\ {}^{50}C_{r+1} = T_{r+1}$

M1

② $\frac{T_r}{T_{r+1}} = \frac{0.85^{50}\ \cancel{0.85^{r}}\ \cancel{0.15^{r}}\ {}^{50}C_r}{0.85^{49}\ \cancel{0.85^{r}}\ 0.15^{1}\ \cancel{0.15^{r}}\ {}^{50}C_{r+1}}$

$= 0.85 \frac{{}^{50}C_r}{{}^{50}C_{r+1}}$

③ ${}^{n}C_r = \frac{n!}{r!(n-r)!}$ ${}^{50}C_r = \frac{50!}{r!(50-r)!}$ ${}^{50}C_{r+1} = \frac{50!}{(r+1)!(49-r)!}$

④ $0.85 \frac{\cancel{50!}}{r!(50-r)!} \times \frac{(r+1)!(49-r)!}{\cancel{50!}}$

$= 0.85 \frac{(r+1)!\ (49-r)!}{r!\ (50-r)!}$

$= \frac{0.85(r+1)}{50-r}$ M1

$= \boxed{\frac{17(r+1)}{3(50-r)}}$

A1

Exemplar 2 — 1 mark

13(ii)

~~[illegible]~~

$T_r = 50cr \cdot 0.15^r \cdot 0.85^{50-r}$

$T_{r+1} = 50c(r+1) \cdot 0.15^{r+1} \cdot 0.85^{50-r-1}$

$\frac{50Cr \times 0.15^r \times 0.85^{50-r}}{50C_{r+1} \times 0.15^{r+1} \times 0.85^{49-r}}$ M1

$= \frac{50cr \times 0.15^r \times 0.85^{50} \div 0.85^r}{50r+1 \times 0.15^r \times 0.15 \times 0.85^{49} \div 0.85^r}$ ^

~~$8/3 \times 4$~~ (4 reflected parabolas)

$8/3 \times 4 = 32/3$

$32/3 + 4 = 44/3$

parabolas square

$\therefore$ shaded area is $44/3$

13(ii) $\frac{50!}{r!\,(50-r)!} \times 0.15^{r} \times 0.85^{50-r}$

$\frac{50!}{(r+1)!\,(50-(r+1))!} \times 0.15^{r+1} \times 0.85^{50-r}$

A0

Exemplar 3

0 marks

13(ii) $(T + 0.85)^{50} = 0.85^{50}({}^{50}C_0)(T)^0 + {}^{50}C_1(T)^1(0.85)^{49} + {}^{50}C_2(T)^2(0.85)^{48} + \ldots$

$= 0.85^{50} + 50(0.85)^{49}T + 1225(0.85)^{48}T^2 + \ldots$

$(0.15 + 0.85)^{50} = 1$

$1 = 0.85^{50} + 50(0.85)^{49}T + 1225(0.85)^{48}T^2 + \ldots$

$\frac{T_r}{T_{r+1}} = \frac{\frac{50!}{(50-r)!\,r!} \times (0.85)^{50-r}}{\frac{50!}{(50-r_{+1})!\,r_{+1}!} \times (0.85)^{50-r_{+1}}} = \frac{\frac{50!}{(50-r)!\,r!} \times 0.85}{\frac{50}{(50-r_{+1})!\,r_{+1}!}}$

$= \frac{50! \times 0.85}{(50-r!)\,r!} \times \frac{(50-r_{+1})!\,r_{+1}!}{50} = \frac{0.85(50!)\,r_{+1}!\,(50-r_{+1})!}{50\,r!\,(50-r)!}$

$= \frac{0.85(50!)\,r}{50\;(50-r)}$

Exemplar 4 0 marks

13(ii)

npq

$$\frac{0.15^{50}}{50\times0.15^{49}\times0.85} = \frac{0.15^{50}}{\frac{85\times0.15^{49}}{2}} = \frac{2\times0.15^{50}}{85\times0.15^{49}}$$

$$= \frac{2\times0.15}{85}$$

$$= \frac{0.3}{85}$$

$$= \frac{3}{850}$$

Examiner commentary

The best candidates were able to state correct expressions for both T_r and T_{r+1} and were then able to show clearly how the given fraction cancels to the stated result (exemplar 1). Some candidates appeared not to understand the definition of T_r and T_{r+1} (exemplar 3). Others correctly stated T_r and T_{r+1} but were unable to cancel the fraction correctly (exemplar 2). Many, however, did not make any reference to T_r and T_{r+1} (exemplar 4) and a good number gave no response.

Question 13 (iii)(a)

13 (iii) The number of days on which Paul's train to work is late during a 50-day period is modelled by the random variable X.

(a) Find the values of r for which $P(X = r) \leqslant P(X = r+1)$. **[4]**

Exemplar 1 — 4 marks

13(iii)(a)

$P(X=r) = \frac{50!}{(50-r)!\,r!}(0.15)^r(0.85)^{50-r}$

$P(X=r+1) = \frac{50!}{[50-(r+1)]!\,(r+1)!}(0.15)^{r+1}(0.85)^{50-(r+1)}$

~~$P(X=r) \leq P(X=r+1)$~~

$\frac{50!}{(50-r)!\,r!}(0.15)^r(0.85)^{50-r} \leq \frac{50!}{[50-(r+1)]!\,(r+1)!}(0.15)^{r+1}(0.85)^{50-(r+1)}$

$17(r+1) \leq 3(50-r)$ **M1**

$20r \leq 133$ **A1**

$r \leq 6.65$ **A1**

so $r \leq 6$ **A1** because $r \in \mathbb{N}$

Exemplar 2 — 3 marks

13(iii)(a)

$17(r+1) \leq 3(50-r)$ **M1**

$17r + 17 \leq 150 - 3r$ **A1**

$20r \leq 133$

$r \leq \frac{133}{20}$ **A1**

$r \leq 6.65$

^ **A0**

$T_r \leq T_{r+1}$

$\frac{T_r}{T_{r+1}} \leq 1$

T_{r+1} is positive so don't flip sign

~~$T_r \leq T_{r+1}$~~

$\frac{17(r+1)}{3(50-r)} \leq 1$

$r < 50$ so $3(50-r) > 0$

~~so don't flip sign~~

$17(r+1) \leq 3(50-r)$

as $(0.15 + 0.85)^{50}$ gives terms which are probability of a train arriving late the same number of days as 0.15's power.

Exemplar 3 2 marks

13(iii)(a)	$X \sim B(50, 0.15)$
	$E(X) = 50 \times 0.15 = 7.5$
	$P(X=7) = 0.1575$ $P(X=8) = 0.1493$
	$P(X=6) = 0.1419$ SC B1
	$r \leq 6$

Exemplar 4 0 marks

13(iii)(a)	
	For $\boxed{r=6}$ trial and improvement
	^ SC B0

Examiner commentary

Some candidates carried out a correct method but stopped after obtaining $r \leq 6.65$ (exemplar 2). Others did not recognise how part (ii) could be used but were able to give a completely correct solution (exemplar 1). A trial and improvement method could score a maximum of 2 marks in this question (exemplar 3). Many gave no response or gave an answer with no detail, despite the question stating that detailed reasoning must be shown (exemplar 4). These score no marks.

Question 13 (iii)(b)

13 **(iii)** **(b)** Hence find the most likely number of days on which the train will be late during a 50-day period. **[2]**

Exemplar 1 2 marks

13(iii)(b)

$P(X=7) = 0.1575 \quad P(X=6) = 0.1419 \quad P(X=8) = 0.1493$

Most likely value of $X = 7$

Exemplar 2 0 marks

13(iii)(b)

probability increasing up to 6.65 days then decreasing

7 is likeliest number of days on which ~~[illegible]~~ train is late.

BO

BO

~~This is also the expected value nearest integer to of X, 6.65~~

$\boxed{7}$

Exemplar 3 0 marks

13(iii)(b)

8 days

Examiner commentary

Almost no candidates gave a correct solution based on their answer to part (iii)(a). Some used trial and improvement, considering sufficient values of *X*. (At least *X* = 6 and 7 were required) (exemplar 1). Some rounded their figure of 6.65 from part (iii)(a) to 7. This did not score any marks (exemplar 2). Most gave no response to this part or gave a value without any working shown (exemplar 3).

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