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A LEVEL

Exemplar Candidate Work

MATHEMATICS A

H240
For first teaching in 2017

H240/01 Summer 2018 examination series

Version 1

Contents

Introduction

These exemplar answers have been chosen from the summer 2018 examination series.

OCR is open to a wide variety of approaches and all answers are considered on their merits. These exemplars, therefore, should not be seen as the only way to answer questions but do illustrate how the mark scheme has been applied.

Please always refer to the specification http://www.ocr.org.uk/Images/308723-specification-accredited-a-level-gce-mathematics-a-h240.pdf for full details of the assessment for this qualification. These exemplar answers should also be read in conjunction with the sample assessment materials and the June 2018 Examiners' report or Report to Centres available from Interchange https://interchange.ocr.org.uk/Home.mvc/Index

The question paper, mark scheme and any resource booklet(s) will be available on the OCR website from summer 2019. Until then, they are available on OCR Interchange (school exams officers will have a login for this and are able to set up teachers with specific logins – see the following link for further information http://www.ocr.org.uk/administration/support-and-tools/interchange/managing-user-accounts/).

It is important to note that approaches to question setting and marking will remain consistent. At the same time OCR reviews all its qualifications annually and may make small adjustments to improve the performance of its assessments. We will let you know of any substantive changes.

Question 1

1 The points A and B have coordinates (1, 5) and (4, 17) respectively. Find the equation of the straight line which passes through the point (2, 8) and is perpendicular to AB. Give your answer in the form $ax + by = c$, where a, b and c are constants. **[4]**

Exemplar 1 — 4 marks

1 $A(1,5)$, $B(4,17)$.

The gradient of AB : $m_{AB} = \dfrac{17-5}{4-1} = 4$

$\therefore$ The gradient of the line which is perpendicular to AB is $-\dfrac{1}{4}$.

The equation of the straight line is :

$y - 8 = -\dfrac{1}{4}(x-2)$.

$\Leftrightarrow y - 8 = -\dfrac{1}{4}x + \dfrac{1}{2}$

$\Leftrightarrow y = -\dfrac{1}{4}x + \dfrac{17}{2}$

$\Leftrightarrow 4y = -x + 34$. $\Leftrightarrow x + 4y = 34$.

Examiner commentary

The candidate has shown clear working in their solution, and has also given the final answer in the requested form.

Exemplar 2 — 3 marks

1 $A(1,5)$ and $B(4,17)$

$\therefore$ gradient $= \dfrac{17-5}{4-1} = \dfrac{12}{3} = 4$ M1

normal gradient $= -\dfrac{1}{4}$ M1

$y - 8 = -\dfrac{1}{4}(x-2)$ M1

$4y - 32 = -x + 2$

$x + 4y - 34 = 0$ A0

Examiner commentary

This is an example of a candidate not gaining full marks for neglecting to give the final answer in the requested form.

Exemplar 3 — 2 marks

1 Gradient of AB $= \frac{4}{12} = \frac{1}{4}$ M0

Perpendicular gradient $= -4$ M1 FT

$y - y_1 = M(x - x_1)$

$y - 8 = -4(x - 2)$ M1

$y = -4x + 16$

~~4x~~ $4x - y + 16 = 0$ A0

Examiner commentary

The fraction is the wrong way around when attempting the gradient so the method mark is not credited. The candidate then uses their incorrect gradient to make at attempt at the perpendicular gradient so M1 is given. They also gain the following method mark for a clear attempt at the equation of the line. Had they simply written down an equation, with no method shown, then the method mark would not have been credited. This is a good example of where clear working gains partial credit once a mistake has been made.

Question 2 (i)

2 **(i)** Use the trapezium rule, with four strips each of width 0.5, to estimate the value of

$$\int_0^2 e^{x^2}\,dx$$

giving your answer correct to 3 significant figures. **[3]**

Exemplar 1 — 3 marks

2(i)

$x = 0 \qquad y = e^{x^2} \qquad \therefore y = e^0 = 1$

$x = \frac{1}{2} \qquad y = e^{(\frac{1}{2})^2} = e^{1/4}$

$x = 1 \qquad y = e$

$x = 3/2 \qquad y = e^{9/4}$

$x = 2 \qquad y = e^4$

$\frac{0.5}{2}\left(1 + e^4 + 2\left(e^{1/4} + e + e^{9/4}\right)\right)$

$= 20.644$

$= 20.6 \ (3sf)$

Examiner commentary

This question was generally very well answered. This candidate shows clear detail for their method, including explicit substitution into the correct trapezium rule. They have demonstrated good practice in using exact y values; whilst working with decimal equivalents can still gain full credit, candidates are more likely to lose marks through copying errors and not using y values to a sufficient degree of accuracy.

Exemplar 2 — 0 marks

2(i)

$\int_0^2 e^{x^2}dx \qquad \int_a^b y\,dx \approx h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots y_{n-1})\}$

$h = \frac{b-a}{n} \qquad h = \frac{2-0}{4} = \frac{1}{2}$

$\approx \frac{1}{2}\{(y_0 + y_4) + 2(y_1 + y_2 + y_3)\}$

B0

$\approx \frac{1}{2}\{(\frac{1}{2} \times 82.6$ M0

≈ 41.3

Examiner commentary

The trapezium rule is one of the formulae given at the start of the question paper; this candidate is either not aware of this or has copied it incorrectly. Either way, they do not gain the method mark, as the correct trapezium rule is not used. This solution also highlights the importance of showing clear detail of the method used. Whilst the 82.6 suggests that the candidate has done something sensible with the y values, they are not shown explicitly so the B1 cannot be credited.

Question 2 (ii)

(ii) Explain how the trapezium rule could be used to obtain a more accurate estimate. **[1]**

Exemplar 1 1 mark

2(ii) By using more strips within the same interval

Examiner commentary

Most candidates were able to give a good explanation of how a more accurate estimate could be obtained. This candidate gives a good explanation by suggesting the use of more strips and also explicitly saying that it has to be over the same interval (others referred to narrower strip widths).

Question 3

3 In this question you must show detailed reasoning.

Find the two real roots of the equation $x^4 - 5 = 4x^2$. Give the roots in an exact form. **[4]**

Exemplar 1 — 4 marks

3

$x^4 - 5 = 4x^2$

$\Leftrightarrow x^4 - 4x^2 - 5 = 0$

$\Leftrightarrow (x^2)^2 - 4x^2 - 5 = 0$

$\Leftrightarrow (x^2 - 5)(x^2 + 1) = 0$

$\Leftrightarrow x^2 = 5$ or $x^2 \neq -1$ $(x^2 \geq 0)$

$\Leftrightarrow x = \pm\sqrt{5}$

$\therefore$ The two real roots of the equation are

$\sqrt{5}, -\sqrt{5}$

Examiner commentary

This was a 'detailed reasoning' so candidates were expected to show clear evidence of how they solved the disguised quadratic, rather than just obtaining the roots from their calculator, and the majority of candidates did so. This candidate simply factorised the quartic into two brackets. A full solution required a reason for rejecting $x^2 = -1$, as seen from this candidate. On this script the reason given was that x^2 is always greater than or equal to 0; other reasons referred to being unable to square root – 1 or that the square root of – 1 would not give any real solutions. The reason given had to be fully accurate, so stating that square numbers are always positive was not condoned.

Exemplar 2 — 3 marks

3

$x^4 - 5 = 4x^2$

$\cancel{4x^2}$ $x^4 - 4x^2 - 5 = 0$ let $y = x^2$

$y^2 - 4y - 5 = 0$ $\times -5$ $+ -4$ $1\ -5$

$(y-5)(y+1)$

$y = 5$ $y = -1$ $\sqrt{-1}$ impossible

$x^2 = 5$ $\cancel{x^2 = -1}$

no roots from ~~this solution~~ BO

$x = \pm\sqrt{5}$

$x = \sqrt{5}$ or $-\sqrt{5}$

Examiner commentary

This candidate used an explicit substitution to solve the equation, which was a fairly common approach. They have discarded $x^2 = -1$, stating that there are no roots from this solution, but no reason is given for this statement so they do not get the mark.

Question 4

4 Prove algebraically that $n^3 + 3n - 1$ is odd for all positive integers n. [4]

Exemplar 1 4 marks

4 $n^3 + 3n - 1 = n(n^2+3) - 1$

If n is odd, (then n^2+3 is even

$\Rightarrow n(n^2+3)$ is even.)

then $n = 2k+1 \iff n^2+3 = (2k+1)^2+3$

$= 4k^2+4k+4 = 4(k^2+k+1)$.

$4(k^2+k+1)$ is even $\iff n^2+3$ is even. B1

$\therefore n(n^2+3) = (2k+1)(k^2+k+1)4$ which is even. B1

$\therefore n(n^2+3) - 1$ is odd.

If n is even, then $n = 2k \iff n^2+3 = (2k)^2+3$

$= 4k^2+3$ which is odd

$n(n^2+3) = 2k(4k^2+3)$ which is even. B1

$\therefore n(n^2+3) - 1$ is odd B1

So $n^3 + 3n - 1$ is odd for all positive integers n.

Examiner commentary

This candidate gained full marks for this clearly reasoned solution. They take a slightly unusual approach in writing the cubic as $n(n^2 + 3) - 1$ and building up an argument, whereas most candidates considered the entire expression in one go. They started by explicitly stating that if n is odd then it can be represented by $2k + 1$; this was a step that many candidates omitted. They substitute it into the cubic and consider, in stages, whether the expression will be odd or even, explaining their reasoning throughout. The process is then repeated with $2k$ representing an even number.

Exemplar 2 2 marks

4 $n^3 + 3n - 1$

Let n be even $\therefore n = 2k$ where $k \in \mathbb{N}$

$n^3+3n-1 = (2k)^3 + 3(2k) - 1 = 8k^3+6k-1$ B1

$= 2(4k^3+3k) - 1$

$4k^3+3k \in \mathbb{N}$ $\therefore$ n^3+3n-1 is odd for even n ... [1] B0

Let n be odd $\therefore n = 2k+1$ where $k \in \mathbb{N}$

$n^3+3n-1 = (2k+1)^3 + 3(2k+1) - 1 = 8k^3+12k^2+6k+1+6k+3-1$ B1

$= 2(4k^3+6k^2+6k)+1$

$4k^3+6k^2+6k \in \mathbb{N}$ $\therefore$ n^3+3n-1 is odd for odd n ... [2] B0

From (1) and (2), n^3+3n-1 is odd for all $n \in \mathbb{N}$

Examiner commentary

On this script, the initial explanation of odd and even is clearly explained. The substitution and algebraic manipulation is accurate, but the final conclusions lack sufficient detail. To gain full credit the candidate should explain that $2(4k^3 + 3k)$ is even, hence $2(4k^3 + 3k) - 1$ is odd; and similar when considering odd n.

Exemplar 3 — 2 marks

4	n^3+3n-1	$(2k+1)(2k+1)(2k+1)$
		$8k^3+4k^2+4k^2+4k^2$
	n^3+3n-1 is odd.	$+2k+2k+2k+1$
	$n=2k$ **B0**	$n=2k+1$ **B0**
	$(2k)^3+3\times 2k-1$	$(2k+1)^3+3(2k+1)-1$
		$8k^3+12k^2+6k+1+6k+3-1$
	$8k^3+6k-1$	$8k^3+12k^2+12k+2+1$
	$2(4k^3+3k)-1$	$2(4k^3+6k^2+6k+1)+1$
	$2(4k^3+3k)$ is even **B1**	$2(4k^3+6k^2+6k+1)$ is even **B1**
	$\therefore 2(4k^3+3k)-1$ is odd	$\therefore 2(4k^3+6k^2+6k+1)+1$ is odd
	$\therefore n^3+3n-1$ is odd for all positive integers.	

Examiner commentary

This candidate also gained two marks, but in this example they introduce $2k$ and $2k + 1$ with no explanation of what they are representing so lose the first mark in each part. The algebraic manipulation is fine, and the conclusions are well reasoned, so they can still gain the second mark for each part.

Exemplar 4

1 mark

4 An odd number can be defined as

$2k+1$ where k is ~~an~~ positive integer

Sub ~~[illegible]~~ $2k+1$ for n

$(2k+1)^3 + 3(2k+1) - 1$ B1

$8k^3 + {}^3C_1 (2k)^2(1) + {}^3C_2 (2k)(1)^2 + (1)^3$

$+ 6k + 3 - 1$

$= 8k^3 + 12k^2 + 6k + 1$

$= 2(4k^3 + 6k^2 + 3k) + 1$

$\because$ ~~B~~ k is a positive integer, B0

$(4k^3 + 6k^2 + 3k)$ is a positive integer

$\therefore$ 2(integer) = an even number

and ~~2 × even numb~~

an even number + 1 = Odd

Examiner commentary

This candidate, with an overall very high score on this paper, starts this question well. The first mark gained for saying that they are using $2k + 1$ to represent an odd number, and substituting into the cubic. However, there are a couple of slips in the algebraic manipulation, meaning that the second mark cannot be given as the expression is not correct. This is unfortunate, as the accompanying explanation is suitably detailed and would have gained the mark had the expression been correct. The candidate then stops at this point rather than also considering when *n* is even.

Question 5 (i), (ii) and (iii)

5 The equation of a circle is $x^2+y^2+6x-2y-10=0$.

(i) Find the centre and radius of the circle. **[3]**

(ii) Find the coordinates of any points where the line $y=2x-3$ meets the circle $x^2+y^2+6x-2y-10=0$. **[4]**

(iii) State what can be deduced from the answer to part (ii) about the line $y=2x-3$ and the circle $x^2+y^2+6x-2y-10=0$. **[1]**

Exemplar 1 8 marks

5(i)

$x^2+y^2+6x-2y-10=0$

$x^2+y^2+6x-2y=10$

$(x+3)^2-9+(y-1)^2-1=10$

$(x+3)^2+(y-1)^2-10=10$

$(x+3)^2+(y-1)^2=20$

∴ The circle centre = $(-3,1)$

$x^2+y^2=r^2$

∴ $r^2=20$ ∴ radius = $\sqrt{20}=\cancel{4\sqrt{5}}\ 2\sqrt{5}$

5(ii)

$x^2+y^2+6x-2y=10$ $y=2x-3$ $\cancel{(2x-3)(2x-3)}$

$x^2+(2x-3)^2+6x-2(2x-3)=10$

$x^2+4x^2-12x+9+6x-4x+6=10$

$5x^2-10x+15=10$ $y=2x-3$

$5x^2-10x+5=0$ $y=2-3=-1$

$x^2-2x+1=0$ ∴ Coordinates = $(1,-1)$

$(x-1)^2$ ∴ $x=1$

5(iii)

The line $y=2x-3$ meets the circle at only one point, therefore $y=2x-3$ is tangent to the circle at $(1,-1)$.

Examiner commentary

This question was well answered, with many fully correct solutions seen. In part (i), this candidate shows clear detail of factorising the equation into usable form, and hence stating the correct centre and radius. In part (ii), they substitute the equation of the line into the equation of the circle. It was expected that candidates would use their calculators to solve the quadratic but this candidate, along with many others, still showed explicit detail of their solution method. In part (iii), they deduce that the line must be a tangent, and also explain why they know this. As there was only one mark available for the question, any mention of tangent was sufficient to gain the mark, but it is advised that candidates get into the habit of always giving a reason for any statements made.

Exemplar 2 — 4 marks

5(i)

$(x+3)^2-(3)^2+(y-1)^2-(-1)^2=10$ M1

$x^2+6x+y^2-2y=10$

Centre $(-3,1)$ B1

radius $=\sqrt{10}$ A0

5(ii)

$(y-1)^2=10-(x+3)^2$ $\qquad -((x+3)^2)$

$y-1=\sqrt{10-(x+3)^2}$ $\qquad =-[x^2+6x+9]$

$y=\sqrt{10-(x+3)^2}+1$

M1 $2x-3=\sqrt{10-(x+3)^2}+1$

$=\sqrt{-x^2-6x+1}$

$2x-4=\sqrt{x^2-6x+1}$

$(2x-4)^2=-x^2-6x+1$

$4x^2-16x+16+x^2+6x-1=0$

$5x^2-10x+15=0$

No real roots

5(iii)

The line $y=2x-3$ and the circle do ~~not~~ not intersect FT

Examiner commentary

In part (i), the candidate gains one mark for the correct centre of the circle, and one mark for attempting to write the equation of the circle in usable form. This candidate should have included a further line of working, simplifying the constant terms in the equation of the circle, and then the error with the radius may not have happened. In part (ii), they attempt to use their incorrect equation, rather than the expanded version given in the question; it is good practice to use the equation of the circle given in the question as opposed to the factorised form, just in case an error had occurred when factorising. This means that the marks are limited to the one method mark only. In part (iii), the candidate makes a correct deduction from their roots in part (ii), so gets the follow-through mark.

Question 6 (i)

6 The cubic polynomial $f(x)$ is defined by $f(x) = 2x^3 - 7x^2 + 2x + 3$.

(i) Given that $(x - 3)$ is a factor of $f(x)$, express $f(x)$ in a fully factorised form. **[3]**

Exemplar 1 3 marks

6(i)

$2x^2 - x - 1$

$x - 3 \;\big)\; 2x^3 - 7x^2 + 2x + 3$

$2x^3 - 6x^2$

$-x^2 + 2x + 3$

$-x^2 + 3x$

$-x + 3$

$-x - 3$

$(x-3)(2x^2 - x - 1)$

$(2x^2 - x - 1) = (2x+1)(x-1)$

$f(x) = (x-3)(2x+1)(x-1)$

Examiner commentary

This part of the question was very well answered, with many fully correct solutions seen. This candidate used algebraic long division which, along with the grid method, tended to be the most common methods. Other methods included coefficient matching or inspection, and all four methods were equally successful. Some candidates appeared to find the roots of the cubic using their calculator and then attempt to work backwards to find the factors. This was rarely successful, as they did not take into account the coefficient of x^3 resulting in a final answer of $(x - 3)(x - 1)(x + 0.5)$. The question asked for the cubic to be given in fully factorised form; on this script the final mark is only gained on the last line. Prior to that, the correct three factors have been seen but not yet with all three as a product.

Exemplar 2 3 marks

6(i)

$(x-3)(2x^2 - x - 1)$ factorisation by inspection

$(x-3)(x-1)(2x+1)$

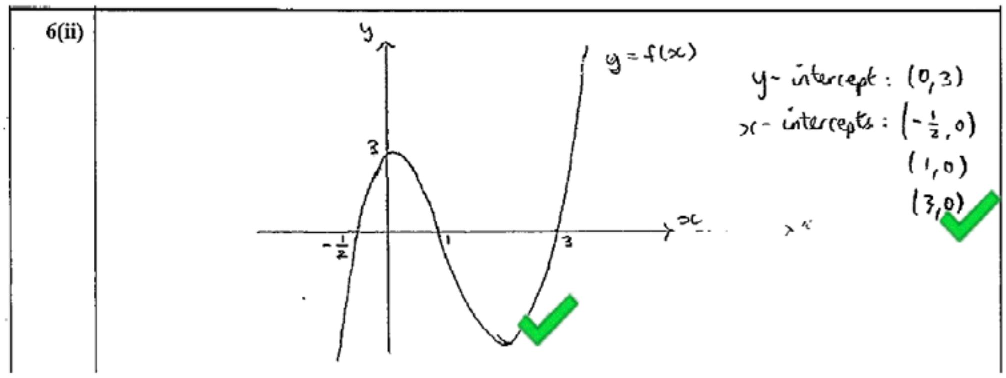
Examiner commentary

There was no specific command word used in this question to signal a clear mathematical justification needed and this able candidate has obtained full marks by solving by inspection. There is an obvious risk associated with not showing any working, where a wrong answer only response has no potential partial credit opportunities. However, there is no reason to suggest this candidate did not perform a check using their calculator.

Question 6 (ii)

(ii) Sketch the graph of $y = f(x)$, indicating the coordinates of any points of intersection with the axes. **[2]**

Exemplar 1 2 marks



Examiner commentary

This part was also well answered, with many candidates gaining full credit. This candidate has ruled axes, and has then drawn a smooth curve of the correct general shape. All points of intersection have been given, both as values on the axes and also as explicit coordinates; either would be sufficient to gain the marks.

Question 6 (iii)

(iii) Solve the inequality $f(x) < 0$, giving your answer in set notation. **[2]**

Exemplar 1 — 2 marks

6(iii) Crit values $3, -\frac{1}{2}, 1$ $\quad y < 0$

$\therefore\ x < -\frac{1}{2}$ or $1 < x < 3$

$\{x : x < -\frac{1}{2}\} \cup \{x : 1 < x < 3\}$.

Examiner commentary

This candidate was able to express their solution clearly using set notation.

Exemplar 2 — 2 marks

6(iii) $(x-3)(2x+1)(x-1) < 0$

$x < -\frac{1}{2} \quad 1 < x < 3$

$\{x : x < -\frac{1}{2}\} \cup \{x : x > 1\} \cap \{x : x < 3\}$

Examiner commentary

This candidate used a slightly longer, but equally valid, expression.

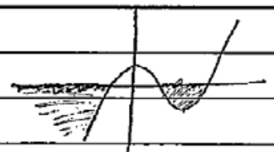
Exemplar 3 — 1 mark

6(iii) $2x^3 - 7x^2 + 2x + 3 < 0$

$(x-3)(2x+1)(x-1) < 0$

$x < -\frac{1}{2}$

$1 < x < 3$ $\quad x : (-\infty, -\frac{1}{2})$ ~~and~~ $x : (1, 3)$



M1

A0

Examiner commentary

The statement on the left, using inequalities, is sufficient for the first mark. This candidate then attempts to use interval notation, but it is spoilt by the use of 'and' rather than the union symbol, and also by the x: at the start of each interval.

Question 6 (iv)

(iv) The graph of $y = f(x)$ is transformed by a stretch parallel to the x-axis, scale factor $\frac{1}{2}$.
Find the equation of the transformed graph. **[2]**

Exemplar 1 — 1 mark

6(iv)	$2(2x)^3 - 7(2x)^2 + 2(2x) + 3$
	A0 $16x^3 - 28x^2 + 4x + 3$ ✓ M1
	^

Examiner commentary

This candidate gains one mark for attempting f(2*x*), but gives their final answer as an expression not an equation so does not get the final mark. Failing to give the final answer as an equation is a surprisingly common error, especially on coordinate geometry questions and trigonometric proofs.

Question 7 (i)

7 Chris runs half marathons, and is following a training programme to improve his times. His time for his first half marathon is 150 minutes. His time for his second half marathon is 147 minutes. Chris believes that his times can be modelled by a geometric progression.

(i) Chris sets himself a target of completing a half marathon in less than 120 minutes. Show that this model predicts that Chris will achieve his target on his thirteenth half marathon. **[4]**

Exemplar 1 — 4 marks

7(i)

$r = \frac{147}{150} = 0.98$

$U_n = 150(0.98)^{n-1}$

$120 > 150(0.98)^{n-1}$

$0.8 > 0.98^{n-1}$

$\log 0.8 > \log 0.98^{n-1}$

$\log 0.8 > (n-1)\log 0.98$

$n > \frac{\log 0.8}{\log 0.98} + 1$

$n > 12.04 \quad \therefore n = 13 = 13^{th}$ Half Marathon

Examiner commentary

This candidate finds the correct common ratio and then sets up and solves an inequality, dealing with inequality signs correctly throughout. A common error is not to reverse the inequality sign when dividing through by the logarithm of a number less than 1, which would have been penalised as the conclusion of 13 would then be inconsistent with a statement of $n < 12.05$. On this script, the value for n is slightly inaccurate (it should be 12.0452…) but benefit of doubt is given. The candidate appreciates that it now needs to be rounded up to an integer value, and also puts the answer in context to gain full marks. Had the conclusion simply been $n = 13$, with no mention of half marathons (or similar) then the final mark would have been withheld.

Exemplar 2 — 3 marks

7(i)

$U_1 = 150, \quad U_2 = 147.$

$U_2 = U_1 \times r \iff 147 = 150r \iff r = 0.98$ B1

$U_{13} = U_1 \times r^{12} = 150 \times 0.98^{12} = \cancel{117}\ 118$ (mins). M1 A1

$118 < 120.$

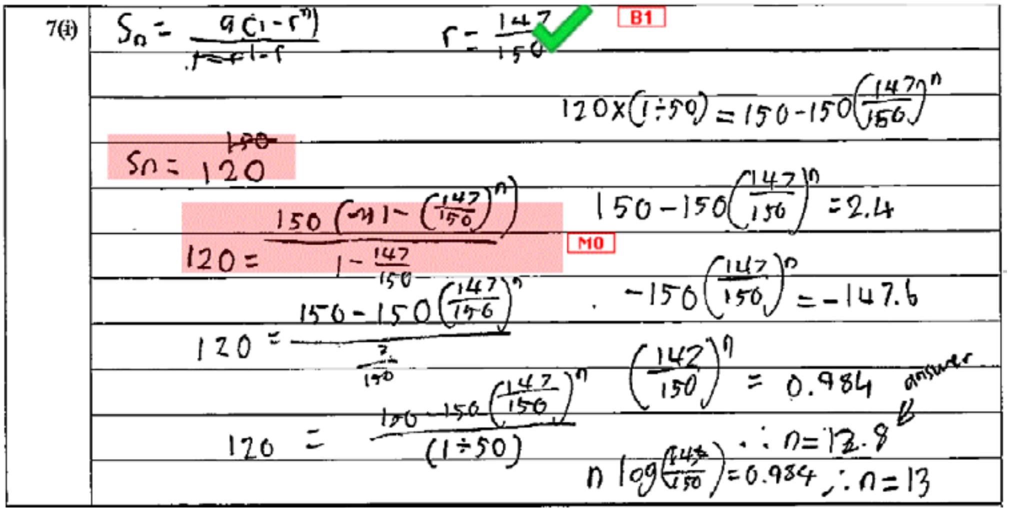
$\therefore$ The model predicts Chris will achieve his target on his thirteenth half marathon. ^ A0

Examiner commentary

Whilst most candidates attempted the approach in exemplar 21, namely setting up and solving an equation or inequality, some candidates did attempt to verify the given answer. On this script, the candidate has shown that the model does indeed predict a time of less than 120 minutes on the thirteenth half marathon, but they have not shown that this is the first time that it occurs. To gain full credit candidates also had to demonstrate that the model predicted that the time for the twelfth half marathon was greater than 120 minutes. The specification includes a section on the meaning of some instructions and words used in the specification; "Show that …" questions have the answer given, and candidates need to provide a full explanation, which has sufficient detail to cover every step of their working.

Exemplar 3

1 mark


7(i) $S_n = \frac{a(1-r^n)}{1-r}$ $\quad r = \frac{147}{150}$ B1

$120 \times (1 \div 50) = 150 - 150\left(\frac{147}{150}\right)^n$

$S_n = \cancel{150}\ 120$

$120 = \frac{150\left(1-\left(\frac{147}{150}\right)^n\right)}{1-\frac{147}{150}}$ M0

$150 - 150\left(\frac{147}{150}\right)^n = 2.4$

$120 = \frac{150 - 150\left(\frac{147}{150}\right)^n}{\frac{3}{150}}$

$-150\left(\frac{147}{150}\right)^n = -147.6$

$\left(\frac{147}{150}\right)^n = 0.984$ answer

$120 = \frac{150 - 150\left(\frac{147}{150}\right)^n}{(1 \div 50)}$

$\therefore n = 13.8$

$n \log\left(\frac{147}{150}\right) = 0.984, \therefore n = 13$

Examiner commentary

On this script, the candidate gains one mark for the correct common ratio. No further credit is available as they use the sum formula not the term formula. On sequences and series questions, candidates should give careful thought as to whether it is an AP or a GP (in this question they were told) and also whether it is a sum or a term that is being considered. Even though $n = 13$ appears at the bottom of the solution, it does not follow a correct method so is ignored.

Question 7 (ii)

(ii) After twelve months Chris has spent a total of 2974 minutes, to the nearest minute, running half marathons. Use this model to find how many half marathons he has run. **[3]**

Exemplar 1 — 2 marks

7(ii)

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$2974 = \frac{150 - 150(0.98)^n}{0.02}$$

$$59.48 = 150 - 150(0.98)^n$$

$$90.52 = 150(0.98)^n$$

$$0.98^n = 0.603$$

$$\ln 0.98^n = \ln 0.603$$

$$n \ln 0.98 = \ln 0.603$$

$$n = \frac{\ln 0.603}{\ln 0.98}$$

$$n = 25$$ A0

Examiner commentary

This question was generally very well answered, with candidates able to set up and solve a relevant equation. Many candidates, such as this example, showed every step in their method rather than using their calculators efficiently as expected. As this is a modelling question, the final answer should be in context. Concluding with just 25, rather than 25 half marathons, resulted in the final mark being lost.

Question 7 (iii)

(iii) Give two reasons why this model may not be appropriate when predicting the time for a half marathon. **[2]**

Exemplar 1 — 2 marks

7(iii) The more half marathons chris runs the quicker his time will be however the model lowers the time taken each run, in reality chris would reach a point where he wouldn't be able to run any faster. Not every run would get a better time e.g on rainy days or when Chris is tired.

Examiner commentary

This candidate gives two detailed reasons, one of which refers to the limitations of the model (ie Chris will reach a point where he cannot go any faster) and one which refers to individual variations in Chris' times, with appropriate examples given.

Exemplar 2 — 1 mark

7(iii) As n gets large, his time will tend to 0 minutes because $0.98^{\infty} \to 0$ which is unrealistic. B1

It assumes his time will always decrease by a factor of 0.98, when in reality ~~his time~~ this is unlikely to happen. B0

~~Should Plateau out~~

Examiner commentary

The first reason identifies that the model predicts that his time will tend to 0 minutes, which is 'unrealistic', and hence gains the mark. Inaccurate statements, such as the time equalling 0 or eventually becoming negative, were not given any credit. The second reason alludes to individual variations but does not give any detail. To improve this answer the candidate should have given a specific example such as injury, terrain or weather conditions.

Exemplar 3 — 0 marks

7(iii) It may vary as he won't consistently get better B0

Something may stops him ~~w~~ running one as fast B0

Examiner commentary

The first reason refers to variation but does not get the mark as it is too vague; it would be improved by using a specific example. The second reason is equally unclear, but it seems to be another reference to variations in individual race times. When two reasons are requested, candidates should try to find two different themes for their answers and to also give specific examples.

Question 8 (i)

8 **(i)** Find the first three terms in the expansion of $(4-x)^{-\frac{1}{2}}$ in ascending powers of x. **[4]**

Exemplar 1 — 4 marks

8(i)

$$(4 - x)^{-\frac{1}{2}}$$
$$= 4^{-\frac{1}{2}}\left(1 - \frac{x}{4}\right)^{-\frac{1}{2}}$$
$$= \frac{1}{2}\left(1 - \frac{x}{4}\right)^{-\frac{1}{2}}$$
$$= \frac{1}{2}\left(1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{x}{4}\right)^2}{2!}\right)$$
$$= \frac{1}{2}\left(1 + \frac{x}{8} + \frac{3x^2}{128}\right) + \ldots$$
$$= \frac{1}{2} + \frac{x}{16} + \frac{3x^2}{256} + \ldots$$

Examiner commentary

This is a well-structured solution, in which the candidate has made effective use of brackets. One of the most common mistakes on binomial expansion questions is a failure to use brackets, which then often results in sign errors and/or not applying the relevant power to the coefficient of *x*.

Exemplar 2 — 2 marks

8(i)

$(4-x)^{\frac{1}{2}}$ MR $\qquad (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2$

$$\left(1 - \frac{x}{4}\right)^{\frac{1}{2}} = 1 + \frac{-x}{4} \times \frac{1}{2} + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)}{2}\left(\frac{x}{4}\right)^2$$ M1

$$= 1 + \frac{-x}{8} + \frac{-\frac{1}{4}}{2} \times \frac{x^2}{16}$$ B0 MR

$$= \cancel{1 +} 1 - \frac{x}{8} - \frac{x^2}{128}$$ A1

Examiner commentary

This candidate appears to have misread the question, as evidenced by their very first statement. Had they simply written down an expansion it would have been unclear whether it was a misread or an error when applying the binomial expansion, so credit could not have been credited. This candidate also makes effective use of brackets and the expansion is correct for their misread so they get 2 of the first 3 marks, after the MR penalty is applied. This script is also an example of one of the common misconceptions for the binomial expansion – the candidate appreciates the need to write it in the required form before expanding, but do so by cancelling each term by 4 rather than factorising. The other common misconception is omitting to apply the power to the common factor that has been taken out.

Question 8 (ii)

(ii) The expansion of $\frac{a+bx}{\sqrt{4-x}}$ is $16-x\ \ldots$. Find the values of the constants a and b. **[3]**

Exemplar 1

3 marks

8(ii)

$$(a+bx)(4-x)^{-\frac{1}{2}}$$

$$= (a+bx)\left(\frac{1}{2}+\frac{x}{16}+\frac{3x^2}{256}+\ldots\right)$$

$$= \frac{a}{2}+\frac{ax}{16}+\frac{bx}{2}+\ldots$$

$$\frac{a}{2}=16 \quad \therefore \quad a = 32$$

$$\frac{a}{16}+\frac{b}{2}=-1$$

$$\frac{32}{16}+\frac{b}{2}=-1 \qquad \frac{b}{2}=-3$$

$$b=-6$$

$$a=32,\ b=-6$$

Examiner commentary

The candidate has appreciated the link between this part of the question and the expansion that they attempted in part (i). They have set up and solved two relevant equations, and their solution is well structured and easy to follow.

Exemplar 2

0 marks

8(ii)

$$\frac{a+bx}{\sqrt{4-x}} = 16-x\ldots \qquad (3x\quad)(x\quad)$$

$$\frac{1}{2}+\frac{x}{16}+\frac{3x^2}{256}+\ldots$$

$$\frac{a+bx}{\frac{1}{2}+\frac{x}{16}+\frac{3x^2}{256}} \qquad = \frac{128}{256}+\frac{16x}{256}+\frac{3x^2}{256}+\ldots$$

$$= \frac{3x^2+16x+128}{256}$$

M0

$$\frac{a+bx}{1}\times\frac{256}{3x^2+16x+128}$$

$$\frac{256a+256bx}{3x^2+16x+128}$$

Examiner commentary

This candidate has replaced the denominator with the expansion from the previous part of the question, and has hence made no progress. This solution would have been improved had they given more thought to the implications of the index in part (i) being –0.5 rather than rushing into a solution attempt in this part.

Question 9 (i)

9 The function f is defined for all real values of x as $f(x) = c + 8x - x^2$, where c is a constant.

(i) Given that the range of f is $f(x) \leqslant 19$, find the value of c. **[3]**

Exemplar 1 — 3 marks

9(i)

$f(x) = c + 8x - x^2 . = -(x^2 - 8x - c).$

$= -(x^2 - 8x + 16 - 16 - c).$

$= -[(x-4)^2 - 16 - c]$

$= -(x-4)^2 + 16 + c$

$= c + 16 - (x-4)^2$

$f(x) \leqslant 19 \iff c + 16 = 19$

$\iff c = 3.$

Examiner commentary

A fully correct solution, identifying the maximum point by completing the square on f(x).

Exemplar 2 — 3 marks

9(i)

$y(x) = -x^2 + 8x + c$

$f'(x) = -2x + 8$

At sp, $f'(x) = 0$

$2x = 8 \quad \therefore \quad x = 4$ = Maximum point $\therefore$ corresponding y value is max $= 19$

$19 = -(4)^2 + 8(4) + c$

$19 = -16 + 32 + c$

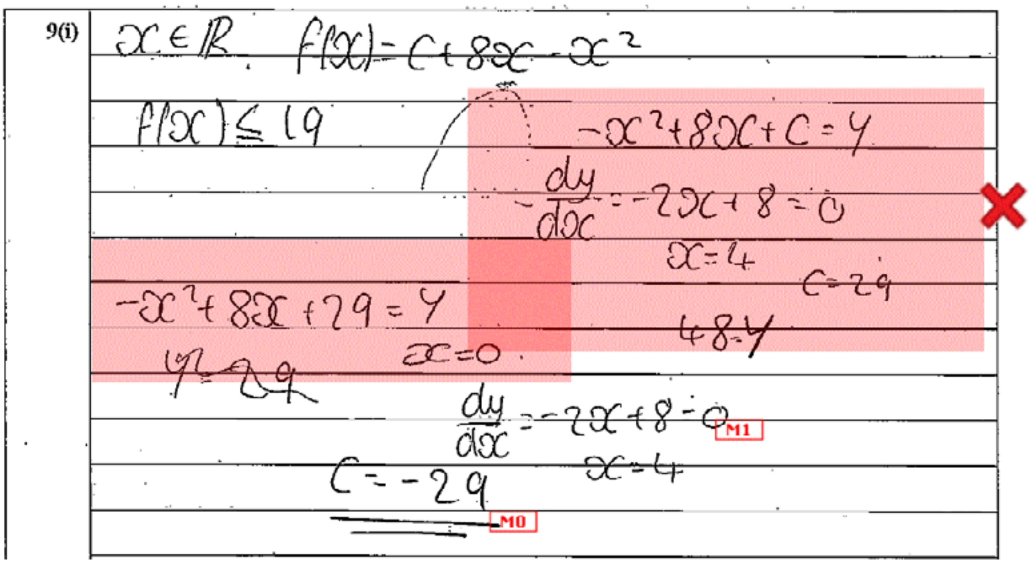
$\therefore \quad c = 3$

Examiner commentary

An alternative, equally successful, method instead using differentiation to identify the maximum point and then linking it to 19.

Exemplar 3

1 mark



9(i) $x \in \mathbb{R}$. $f(x) = c + 8x - x^2$

$f(x) \leq 19$

$-x^2 + 8x + c = 4$

$\frac{dy}{dx} = -2x + 8 = 0$

$x = 4$

$c = 29$

$48 = 4$

$-x^2 + 8x + 29 = 4$

$y = 29$

$x = 0$

$\frac{dy}{dx} = -2x + 8 = 0$ M1

$x = 4$

$c = -29$ M0

Examiner commentary

The candidate has made several attempts at the question, so it is the last attempt that has been marked. They gain the first M1 for differentiating f(*x*), equating it to 0 and solving for *x*. The value of *c* is incorrect and no method is shown so no further credit can be credited. Had the candidate shown how the value of *c* was attempted it is possible that they could have gained the second method mark as well.

Question 9 (ii)

(ii) Given instead that ff(2) = 8, find the possible values of c. **[4]**

Exemplar 1 — 4 marks

9(ii)

$$f(2) = -(2)^2 + 8(2) + c$$
$$= -4 + 16 + c$$
$$= 12 + c$$
$$f(12+c) = 8$$
$$f(12+c) = -(12+c)^2 + 8(12+c) + c = 8$$
$$= -(144 + 24c + c^2) + 96 + 8c + c = 8$$
$$-144 - 24c - c^2 + 96 + 8c + c = 8$$
$$-56 \cancel{+} -15c - c^2 = 0$$
$$c^2 + 15c + 56 = 0$$
$$(c+8)(c+7) = 0$$
$$\therefore\ c = -8 \text{ or } c = -7$$

Examiner commentary

This is an example of the more successful method when answering this question, namely finding an expression for f(2) and then substituting this into f(x) before equating to 8 and solving. It was expected that candidates would make efficient use of their calculators when solving the quadratic but this candidate, in common with many others, showed full detail of their method.

Exemplar 2 — 2 marks

9(ii)

$$f(f(2)) = 8 \qquad \cancel{-105 - 21c - c^2}$$
$$c + 8x - x^2$$
$$= c + 16 - 4 \quad \text{B0}$$
$$= \cancel{c+15}\ c + 15$$
$$f(c+15) = 8$$
$$c + 8(c+15) - (c+15)^2 = 8 \quad \text{M1}$$
$$c + 8c + 120 - (c^2 + 30c + 225) = 8$$
$$-c^2 - 21c - 105 = 8$$
$$c^2 + 21c + 113 = 0 \quad \text{M1}$$

~~x is imaginary~~

c has no real roots A0

Examiner commentary

This script emphasises the need to read the question carefully. The candidate has not appreciated the relevance of 'given instead' and has attempted to use an expression still associated with part (i).By showing clear detail of the subsequent method they were then able to gain both of the method marks. Obtaining complex roots to their quadratic should have indicated that a mistake had been made; time permitting, had the candidate checked back through their solution they may have been able to identify and correct the error.

Exemplar 3 1 mark

9(ii)	~~f(x)3x~~
	$f(x) = C + 8x - x^2$
	$C + 8(C + 8x - x^2) - (C + 8x - x^2)^2$ M1
	$C + 8C + 64x - 8x^2 -$

Examiner commentary

This candidate has attempted an algebraic expression for ff(x), but struggles with the subsequent expansion and manipulation and is unable to make further progress. Substituting $x = 2$ would have gained a further method mark and may also have simplified the expression sufficiently for the candidate to make further progress.

Question 10 (i)

10 A curve has parametric equations $x = t + \frac{2}{t}$ and $y = t - \frac{2}{t}$, for $t \neq 0$.

(i) Find $\frac{dy}{dx}$ in terms of t, giving your answer in its simplest form. **[4]**

Exemplar 1 — 4 marks

10(i)

$y = t - \frac{2}{t} \qquad x = t + \frac{2}{t}$

$y = t - 2t^{-1} \qquad x = t + 2t^{-1}$

$\frac{dy}{dt} = 1 + 2t^{-2} \qquad \frac{dx}{dt} = 1 - 2t^{-2}$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

$= 1 + \frac{2}{t^2} \div 1 - \frac{2}{t^2}$

$= \frac{t^2 + 2}{t^2} \div \frac{t^2 - 2}{t^2}$

$= \frac{(t^2 + 2)(t^2)}{t^2(t^2 - 2)} = \cancel{\frac{t^4 + 2t^2}{t^4 - 2t^2}}$

$\frac{dy}{dx} = \frac{t^2 + 2}{t^2 - 2}$

Examiner commentary

The candidate correctly finds the two derivatives and then combines them, gaining the first 3 marks with clear concise layout of working. Manipulation of algebraic fractions can be tricky for candidates, and this script demonstrates good practice by working with fractions rather than negative indices and also combining the derivatives as two separate fractions rather than fractions within fractions. Benefit of doubt was given regarding the missing brackets around the $(1+2/t^2)$ and $(1-2/t^2)$ due to clear intent in the subsequent working, and no instruction within the question to show a full mathematical argument.

Exemplar 2 — 3 marks

10(i)

$x = t + \frac{2}{t}$ $\quad 2t^{-1}$ $\qquad y = t - \frac{2}{t} \qquad t \neq 0$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

$\frac{dx}{dt} = 1 - 2t^{-2} \qquad \frac{dy}{dt} = 1 + 2t^{-2}$

$\frac{dy}{dx} = \frac{1+2t^{-2}}{1-2t^{-2}}$ $\quad 3 \qquad \frac{1+\frac{2}{t^2}}{1-\frac{2}{t^2}}$

$\frac{dy}{dx} = \frac{\left(1+\frac{2}{t^2}\right)}{\left(1-\frac{2}{t^2}\right)}$ A0

Examiner commentary

This is another script where the first three marks are gained with ease. This candidate does not appreciate that their final answer is not yet in its simplest form and makes no attempt to go any further.

Exemplar 3 — 1 mark

10(i)

$x = t + \frac{2}{t} \qquad y = t - \frac{2}{t}$

$x = t + 2t^{-1} \qquad y = t - 2t^{-1}$

$\frac{dx}{dt} = -2t^{-2} \qquad \frac{dy}{dt} = 2t^{-2}$

$= -\frac{2}{t^2} \qquad = \frac{2}{t^2}$

$\therefore \frac{dt}{dx} = \frac{t^2}{-2}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ M1

$= \frac{2}{t^2} \times \frac{t^2}{-2}$

$= -1$

$x = t + \frac{2}{t} \qquad y = t - \frac{2}{t}$

$x = t + 2t^{-1} \qquad y = t - 2t^{-1}$

$\frac{dx}{dt} = -2t^{-2} \qquad \frac{dy}{dt} = 2t^{-2}$

$\frac{dt}{dx} = \frac{1}{-2t^{-2}} \qquad \frac{dy}{dx} = \frac{2t^{-2}}{-2t^{-2}} = \frac{2}{-2} = -1$

Examiner commentary

The candidate correctly converts the equations to index form but then loses the first term both times when differentiating. They do still gain the method mark for clearly showing how they have combined their two derivatives. This is an example of where the candidate has realised that their final answer is unlikely to be correct and used this as a cue to check their method, but they repeat the same error in their second attempt.

Question 10 (ii)

(ii) Explain why the curve has no stationary points. **[2]**

Exemplar 1 — 2 marks

10(ii)

stationary point implies $\frac{dy}{dx} = 0$ ✓

$\therefore \frac{t^2+2}{t^2-2} = 0$

$t^2+2=0$ ✓

$t^2 = -2$ but $t^2 \neq -2$ for $t \in \mathbb{R}$ ✓

$\therefore \frac{t^2+2}{t^2-2} \neq 0$, $\frac{dy}{dx} \neq 0$ ✓

so no stationary points ✓

Examiner commentary

This is an excellent solution that contains all of the reasoning that the examiners expected to see. The candidate explains why they are equating their derivative to 0, why there are no solutions to $t^2 + 2 = 0$ and why it can be deduced that there are no stationary points.

Exemplar 2 — 1 mark

10(ii)

$1 + \frac{2}{t^2} = 0$ B0

$t^2+2=0$

$t^2=-2$ can't square root a negative

~~$t=\sqrt{-2}$ can't $\sqrt{\ }$ a negative~~

therefore no stationary points. B1

Examiner commentary

There is no explanation as to why the derivative is being equated to 0 so the first mark is not credited. The subsequent reasoning and conclusion is fine so the second mark is credited.

Exemplar 3

0 marks

10(ii)	Because $\frac{dy}{dx}$ will never be equal to 0

Examiner commentary

This candidate identifies that the derivative can never be equal to 0, but provides no evidence to justify this. This solution would be improved by equating the numerator of their derivative to 0 and explaining why it has no solutions and what can hence be deduced.

Question 10 (iii)

(iii) By considering $x + y$, or otherwise, find a cartesian equation of the curve, giving your answer in a form not involving fractions or brackets. **[4]**

Exemplar 1 4 marks

10(iii)

$x + y = t + \frac{2}{t} + t - \frac{2}{t} = 2t$

$x - y = \frac{4}{t} = \frac{8}{2t} = \frac{8}{x+y}$

$\therefore x^2\ y^2 = 8$

Examiner commentary

Having found an expression for *t* in terms of *x* and *y*, most candidates substituted back into one of the two parametric equations and attempted to simplify. This candidate, along with a few others, offered an alternative solution that was both elegant and efficient.

Exemplar 2 2 marks

10(iii)

~~$x = t + \frac{2}{t}$~~ ~~$xt = t^2 + 2$~~

~~$y = t - \frac{2}{t}$~~ ~~$2 = xt - t^2$~~

~~$2 = t(x - t)$~~

~~$x + y = 2t$~~ ~~$t = \frac{2}{x-t}$~~

~~$y + \frac{2}{t} = t$~~ ~~$t = y + \frac{2}{t}$~~

~~$y = 2t - x$~~ ~~$t = x - \frac{2}{t}$~~

~~$y = 2t - t$~~ ~~$t^2 - xt + 2 = 0$~~

~~$(t-2)(t-1)$~~

~~When $x = -3$~~

$x = t + \frac{2}{t}$ $\quad y = t - \frac{2}{t}$

$x + y = 2t$ B1 $\quad t^2 - ty - 2 = 0$

$x + y = 2\left(\sqrt{2 + \frac{y^2}{4}} + \frac{y}{2}\right)$ M1 $\quad$ ~~$(t - 2)(t + 1) = 0$~~

$\left(t - \frac{y}{2}\right)^2 - \frac{y^2}{4} - 2 = 0$

$x + y = 2\sqrt{2 + \frac{y^2}{4}} + y$ $\quad \left(t - \frac{y}{2}\right)^2 = 2 + \frac{y^2}{4}$

$x = 2\sqrt{2 + \frac{y^2}{4}}$ M0 $\quad t - \frac{y}{2} = \sqrt{2 + \frac{y^2}{4}}$

$t = \sqrt{2 + \frac{y^2}{4}} + \frac{y}{2}$ BOD

Examiner commentary

This candidate used the hint provided to obtain a correct expression for 2*t* but was unsure how to proceed with this solution. They then decide to make *t* the subject of the parametric equation for *y* and eliminate *t* in this manner instead. There is some attempt at simplifying, but the candidate does not obtain an equation not involving fractions or brackets as requested. This solution could have been improved by the candidate ensuring that their final answer matched the form requested in the question.

Exemplar 3

0 marks

10(iii)

$x = t + \frac{2}{t}$ $\qquad y = t - \frac{2}{t}$

$x - \frac{2}{t} = t$

$\left(x - \frac{2}{t}\right)\left(x - \frac{2}{t}\right)$

$y = \left(x - \frac{2}{t}\right) - \frac{2}{\left(x - \frac{2}{t}\right)}$ $\qquad x^2 + \frac{4}{t^2} - \frac{2x}{t} - \frac{2x}{t}$

$y = \frac{\left(x - \frac{2}{t}\right)^2}{\left(x - \frac{2}{t}\right)} - \frac{2}{x - \frac{2}{t}} = \frac{\left(x - \frac{2}{t}\right)^2 - 2}{\left(x - \frac{2}{t}\right)}$

B0

M0

$y = \frac{x^2 + \frac{4}{t^2} - \frac{2x}{t} - 2}{x - \frac{2}{t}}$

$y = \frac{x^2 + 4t^{-2} - 2xt^{-1} - 2}{x - 2t^{-1}}$

Examiner commentary

This solution attempt is a good example of why candidates should pay attention to any hints given in the question. Rarely will the 'or otherwise' option prove to be a more straightforward method. This candidate, along with others, ignored the suggestion to consider $x + y$ and was unable to make any progress with the question.

Question 11 (i)

11 In a science experiment a substance is decaying exponentially. Its mass, M grams, at time t minutes is given by $M = 300e^{-0.05t}$.

(i) Find the time taken for the mass to decrease to half of its original value. **[3]**

Exemplar 1 — 3 marks

11(i)	$300e^{-0.05t} = \frac{300}{2} = 150$
	$e^{-0.05t} = \frac{1}{2}$
	$-0.05t = \ln\frac{1}{2}$
	$t = \frac{\ln\frac{1}{2}}{-0.05} = 13.9$ minutes (3s.f.)
	$= 13$ minutes 52 seconds to nearest second

Examiner commentary

This question was very well answered, with many fully correct solutions seen. This candidate demonstrates good practice by working in exact values throughout, thus ensuring that their final answer is suitably accurate. The decimal answer of 13.9 minutes was sufficient for full marks but this candidate went one step further by giving their answer in minutes and seconds.

Question 11 (ii)

A second substance is also decaying exponentially. Initially its mass was 400 grams and, after 10 minutes, its mass was 320 grams.

(ii) Find the time at which both substances are decaying at the same rate. **[8]**

Exemplar 1 8 marks

11(ii)

$M_2 = Ae^{kt}$

When $t = 0 \Leftrightarrow$, $M_2 = 400$:

$400 = Ae^{0} \Rightarrow A = 400$.

$M_2 = 400e^{kt}$.

When $t = 10$, $M_2 = 320$:

$320 = 400e^{10k} \Leftrightarrow e^{10k} = \frac{4}{5}$.

$\Leftrightarrow 10k = \ln\left(\frac{4}{5}\right) \Leftrightarrow k = -0.0223$

$M_2 = 400e^{-0.0223t}$

$\frac{dM_2}{dt} = -0.0223 \times 400e^{-0.0223t} = -8.92e^{-0.0223t}$

$M = 300e^{-0.05t}$ $\frac{dM}{dt} = -0.05 \times 300e^{-0.05t}$

$= -15e^{-0.05t}$

$\frac{dM}{dt} = \frac{dM_2}{dt} \Leftrightarrow -15e^{-0.05t} = -8.92e^{-0.0223t}$

$\Leftrightarrow 1.68e^{-0.05t} = e^{-0.0223t}$.

$\Leftrightarrow \ln(1.68) - 0.05t = -0.0223t$

$\Leftrightarrow 0.0277t = \ln(1.68)$.

$\Leftrightarrow t = 18.7$ (mins).

Examiner commentary

This is a good example of a fully correct solution. The initial model is set up and values found for the relevant parameters. It is usually good practice to work with exact values but, in this question, it tended to result in slips and sign errors so working with decimal values was usually more successful. This candidate has worked to an appropriate degree of accuracy throughout resulting in an acceptable final answer. When the two derivatives are equated, this candidate combines the constant terms but not the exponential terms. It was anticipated that candidates would obtain an equation of the form $e^{f(t)} = c$ before introducing logarithms. On this script, logarithms are introduced at an earlier stage, but the candidate is able to manipulate the product correctly to gain full credit. Often, when the log of a product is taken the result is the product of two logs, which is why candidates are well advised to rearrange their equation to a usable form first.

Exemplar 2

5 marks

11(ii) B1

$M_2 = 400e^{-0.05(t)x}$

$400e^{x(10)} = 320$

$e^{10x} = 0.8$

$\ln e^{10x} = \ln 0.8$

$10x = \ln 0.8$ M1

$x = \frac{\ln 0.8}{10} = -0.022$ A1

$M_2 = 400e^{-0.022(t)}$

$M_1 = 300e^{-0.05(t)}$

$\frac{dM_1}{dt} = -15e^{-0.05(t)}$ M1 $\frac{dM_2}{dt} = -8.8e^{-0.022(t)}$ A1 FT

$-15e^{-0.05(t)} = -8.8e^{-0.022(t)}$

$-15e^{-0.05(t)} + 8.8e^{-0.022(t)} = 0$ M0

$t = 19.05$

Examiner commentary

This candidate is able to set up an appropriate expression for the mass of the second substance, find the relevant parameters and thus gain the first three marks. The first derivative is fully correct, and the second derivative is correct following their rounded value of – 0.022. Whilst interim values may be rounded when written down in a solution, the exact values should be stored on a calculator and used in any calculations. On this script, the candidate was unable to solve the equation resulting from the derivatives being equated. Even if they had managed to solve their equation, working with 2 significant figures would have resulted in a final answer that would not have been sufficiently accurate and this would have been penalised.

Exemplar 3

3 marks

11(ii)

$M_2 = 400e^{-kt}$ B1

$320 = 400e^{-10k}$

$0.8 = e^{-10k}$

M1

$\ln 0.8 = -10k$

$k = 0.0223$

$M_2 = 400e^{-0.0223t}$ A1

$vdu + udv$

$\frac{dM_1}{dt} = 300e^{-0.05t}$

$\frac{dm_1}{dt} = \cancel{t0.05te^{-0.05t}} \cancel{+300e^{-0.05t}}$

$\cancel{+0.05te^{-0.05t}}$

$-(300 \times -0.05t)e^{-0.05t}$ M0

$M_2 = 400e^{-0.0223t}$

$\frac{dm_2}{dt} = \cancel{t0.0223te^{-0.0223t}} \cancel{+400e^{-0.0223t}}$

$\cancel{-0.0223te^{-0.0223t}}$ $-(400 \times 0.0223t)e^{-0.0223t}$

$\cancel{-0.05te^{-0.05t}}$ =

M0

$-8.92te^{-0.0223t} = -15te^{-0.05t}$

$\frac{e^{-0.0223t}}{e^{-0.05t}} = \frac{-15t}{-8.92t}$

$e^{0.0277t} = 1.682$

$\ln 1.682 = 0.0277t$

$t = 18.76$

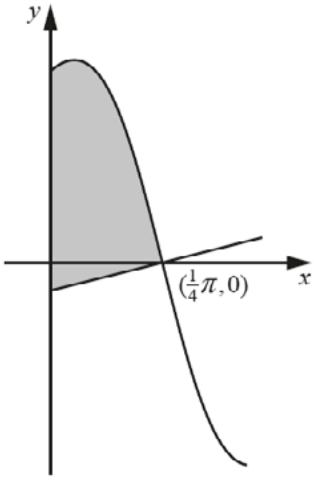
at 19 min

Examiner commentary

The candidate gains the first 3 marks for finding a correct equation for the mass of the second substance. They appreciate that 'rate' indicates the need to equate derivatives, but the attempt demonstrates the common misconception of multiplying by the entire index and not just the coefficient of *t*. Their derivatives are not of the correct form, so the method mark for attempting to solve cannot be credited despite this candidate demonstrating an understanding of how to combine like terms. The final answer of 18.76 is not given credit, coming from incorrect working due to the *t*s cancelling.

Question 12

12 In this question you must show detailed reasoning.



The diagram shows the curve $y = \dfrac{4\cos 2x}{3 - \sin 2x}$, for $x \geqslant 0$, and the normal to the curve at the point $(\frac{1}{4}\pi, 0)$. Show that the exact area of the shaded region enclosed by the curve, the normal to the curve and the y-axis is $\ln\frac{9}{4} + \frac{1}{128}\pi^2$. **[10]**

Exemplar 1

9 marks

12

$y = \dfrac{4\cos 2x}{3 - \sin 2x}$

$\dfrac{dy}{dx} = \dfrac{(3-\sin 2x)(-8\sin 2x) - (4\cos 2x)(-2\cos 2x)}{(3-\sin 2x)^2}$

$= \dfrac{-24\sin 2x + 8\sin^2 2x + 8\cos^2 2x}{9 - 6\sin 2x + \sin^2 2x}$

$= \dfrac{-24\sin 2x + 8}{(3 - \sin 2x)^2}$

$\left(\frac{1}{4}\pi, 0\right)$: $\dfrac{dy}{dx} = \dfrac{-24\sin\frac{\pi}{2} + 8}{\left(3 - \sin\frac{\pi}{2}\right)^2} = \dfrac{-24+8}{(3-1)^2} = -4$

normal gradient $= -\dfrac{1}{-4} = \dfrac{1}{4}$

$\therefore$ y-intercept of normal $= \frac{1}{4}\left(\frac{1}{4}\pi\right) = \frac{1}{16}\pi$

Area of triangle $= \frac{1}{2}\left(\frac{1}{4}\pi\right)\left(\frac{1}{16}\pi\right) = \frac{1}{128}\pi^2$

Let $u = \sin 2x$

$du = 2\cos 2x\, dx \quad \therefore dx = \dfrac{du}{2\cos 2x}$

Area between curve, x-axis and y-axis

$\displaystyle\int_0^{\frac{1}{4}\pi} \frac{4\cos 2x}{3 - \sin 2x}\, dx = \int_0^1 \frac{4\cos 2x}{3-u}\left(\frac{du}{2\cos 2x}\right) = \int_0^1 \frac{2}{3-u}\, du$

$\displaystyle = -2\int_0^1 \frac{-1}{3-u}\, du = -2\Big[\ln|3-u|\Big]_0^1$

$= -2(\ln 2 - \ln 3) = -2\ln\frac{2}{3} = \ln\frac{9}{4}$

$\therefore$ Total area $= \frac{1}{128}\pi^2 + \ln\frac{9}{4}$

A0

Examiner commentary

This was an unstructured, multi-step problem and candidates had to devise their own strategy, ensuring that they showed sufficient detail of the methods used to satisfy the 'detailed reasoning' instruction. On this script, the candidate finds the correct derivative and then shows explicitly how this was used to find the gradient of the tangent and hence the normal. This detail was lacking on a number of scripts. They attempt the area of the triangle, but the statement about the y-intercept is incorrect as it should be negative. This is penalised in the loss of the final mark only as the area of the triangle has not been fully justified. The integration is correct and the candidate has shown explicit use of log laws as expected.

Exemplar 2

4 marks

12

$$y = \frac{4\cos 2x}{3 - \sin 2x}$$

M1 A1

$$\frac{dy}{dx} = \frac{(3 - \sin 2x) \times (-8\sin 2x) - 4\cos 2x(-2\cos 2x)}{(3 - \sin 2x)^2}$$

$$= \frac{-24\sin 2x + 8\sin^2 2x + 8\cos^2 2x}{(3 - \sin 2x)^2}$$

$$= \frac{-24\sin 2x + 8(\sin^2 2x + \cos^2 2x)}{(3 - \sin 2x)^2}$$

$$= \frac{-24\sin 2x + 8}{(3 - \sin 2x)^2} = -\frac{(24\sin 2x - 8)}{(3 - \sin 2x)^2}$$

M0

$\therefore$ The gradient at $x = \frac{1}{4}\pi$ is -4.

B1

$\Rightarrow$ The gradient of the normal at $x = \frac{1}{4}\pi$ is $\frac{1}{4}$.

The equation of the normal is:

$$y - 0 = \frac{1}{4}\left(x - \frac{1}{4}\pi\right) \Leftrightarrow y = \frac{1}{4}x - \frac{1}{16}\pi$$

When $x = 0 \Leftrightarrow y = -\frac{1}{16}\pi$

M1

$$A_{\text{SHADED}} = \int_0^{\frac{1}{4}\pi} \frac{4\cos 2x}{3 - \sin 2x}\,dx + \frac{1}{2} \times \frac{1}{16}\pi \times \frac{1}{4}\pi$$

$$u = \frac{1}{3 - \sin 2x}$$

$$\frac{du}{dx} = \frac{2\cos 2x}{(3 - \sin 2x)^2}$$

$$\frac{dv}{dx} = 4\cos 2x$$

M0

$$v = 2\sin 2x$$

Examiner commentary

This candidate has found the derivative correctly and gains the first 2 marks. The gradient of the tangent is correct, but not justified, so the method mark is not given. They do gain credit for finding the area of the triangle, and this is clearly the second term in the given answer. The candidate is then unsure of the integration technique required and would have benefited from paying more attention to the given answer. It is clear that the first term relates to the area enclosed by the curve and the axes, which indicates that the integration will involve a natural logarithm. Appreciating the cue given in the answer would have allowed the candidate to make further progress.

Exemplar 3

2 marks

12

12 $y = \frac{4\cos 2x}{3-\sin 2x}$ $\quad x \geqslant 0$

$\frac{dy}{dx} = \frac{(3-\sin 2x)(-8\sin 2x) + 4\cos^2 2x}{(3-\sin 2x)^2}$

$u = 4\cos 2x \qquad v = 3-\sin 2x$

$\frac{du}{dx} = -8\sin 2x \qquad \frac{dv}{dx} = -2\cos 2x$

$= \frac{2\cos^2 2x + 4\sin 2x + 2\sin^2 2x}{9 + \sin^2 2x - 6\sin 2x}$ M0

when $x = \frac{\pi}{4}$

M1 BOD

$\frac{dy}{dx} = \frac{6 + 2 + 0}{9 + 1 - 6} = \frac{8}{4} = 2$

B1 FT

grad of line $= -\frac{1}{2}$ $\qquad y = -\frac{1}{2}x + c$

$0 = -\frac{1}{2}\left(\frac{\pi}{4}\right) + c \qquad c = \frac{\pi}{8}$

$\boxed{y = -\frac{1}{2}x + \frac{\pi}{8}}$

$\int_0^{\pi/4} \left(\frac{4\cos 2x}{3-\sin 2x}\right) - \frac{1}{2}x + \frac{\pi}{8}\, dx$

$u = (3-\sin 2x)^{-1} \qquad \frac{dv}{dx} = 4\cos 2x$ M0

$\frac{du}{dx} = (3\sin 2x)^{-2} x - \cos 2x \qquad v =$

Examiner commentary

The candidate makes one attempt at the quotient rule, and then replaces it with a second attempt. It appears to be a new attempt rather than an attempt to simplify the first one, so it is the latter that is marked. Whilst the denominator is correct, there is no evidence that the numerator is of the correct structure so the method mark is not given. There is an explicit attempt to find the gradient of the tangent, and the majority of the terms correctly follow their derivative so benefit of doubt is given for the method mark. The gradient of the normal correctly follows this value so credit is given. The candidate attempts to find the equation of the normal, but there is no attempt to use it to find the area of the triangle. This is another example of a candidate who would benefit from looking at the given answer when deciding on an appropriate integration technique.

Question 13 (i) (a) and (b)

13 A scientist is attempting to model the number of insects, N, present in a colony at time t weeks. When $t = 0$ there are 400 insects and when $t = 1$ there are 440 insects.

(i) A scientist assumes that the rate of increase of the number of insects is inversely proportional to the number of insects present at time t.

(a) Write down a differential equation to model this situation. **[1]**

(b) Solve this differential equation to find N in terms of t. **[4]**

Exemplar 1 5 marks

13(i)(a)

$$\frac{dN}{dt} = \frac{k}{N}$$

13(i)(b)

$$\frac{dN}{dt} = \frac{k}{N} \iff \int N \, dN = \int k \, dt$$

$$\iff \frac{1}{2}N^2 = kt + c$$

When $t = 0$, $N = 400$

$$\frac{1}{2} \times 400^2 = k0 + c \Rightarrow c = 80000$$

$$\therefore \frac{1}{2}N^2 = kt + 80000$$

When $t = 1$, $N = 440$:

$$\frac{1}{2} \times 440^2 = k + 80000 \iff k = 16800$$

$$\therefore \frac{1}{2}N^2 = 1600\ 16800t + 80000$$

$$\iff N^2 = 33600t + 160000$$

$$\iff N = \sqrt{33600t + 160000}$$

Examiner commentary

Most candidates were able to write down the correct differential equation and then attempt integration. This candidate has sensibly created a kt term which means that c can be found directly when using the boundary condition of $t = 0$. Those who left k on the other side then had to solve simultaneous equations to find c and k. Once the values of c and k have been found, the candidate has rearranged the equation to the required form. Some candidates ignored this request, and others spoilt an otherwise correct solution by square rooting term by term.

Exemplar 2

2 marks

13(i)(a)

$t=0$ 600 $t=1$ 640 $\quad 400 + \frac{t}{10}(400)$

$\frac{dN}{dt} = \frac{k}{n}$ $\quad \frac{dN}{dt} = \frac{k}{N}$ $\quad 400+40t$ $\quad \cancel{\frac{dN}{dt} = \frac{k}{600+40t}}$

13(i)(b)

$\cancel{\frac{dN}{dt} = \frac{k}{600+40t}}$

$\cancel{dN = \frac{k}{600+40t}\,dt}$

N= number of ineects

$\frac{dN}{dt} = \frac{k}{N}$

$dN = \frac{k}{N}\,dt$

$\int N\,dN = \int k\,dt$

M1

$\frac{1}{2}N^2 = kt + c$

$N^2 = 2kt + c$

$N = \sqrt{2kt+c}$

Examiner commentary

The candidate gained the mark in part (a) for stating a correct differential equation. In part (b), the integration attempt is correct, which gains a method mark, but there is no attempt to find *k* and *c*. This candidate should look at the question tariff of four marks and query whether their final answer would seem worthy of four marks. They should also question why the boundary conditions were given in the main stem of the question if they were not needed to answer the first part.

Question 13 (ii)

(ii) In a revised model it is assumed that $\frac{dN}{dt} = \frac{N^2}{3988e^{0.2t}}$. Solve this differential equation to find N in terms of t. **[6]**

Exemplar 1 — 6 marks

13(ii)

$$\frac{dN}{dt} = \frac{N^2}{3988e^{0.2t}}$$

$$\int \frac{1}{N^2}\,dN = \int \frac{1}{3988} e^{-0.2t}\,dt$$

$$-\frac{1}{N} = -\frac{5}{3988} e^{-0.2t} + c$$

$$t = 0,\ N = 400 \therefore -\frac{1}{400} = -\frac{5}{3988} + c \quad \therefore c = -\frac{497}{398800}$$

$$\therefore -\frac{1}{N} = -\frac{5}{3988} e^{-0.2t} - \frac{497}{398800}$$

$$-\frac{1}{N} = -\frac{500e^{-0.2t} + 497}{398800}$$

$$\therefore N = \frac{398800}{500e^{-0.2t} + 497}$$

Examiner commentary

This candidate separates the variables and carries out the integration correctly. Along with the majority of candidates the 3988 is left in the denominator; moving it to the other side and combining with N^{-2} would have made the subsequent manipulation easier. Candidates could choose either of the boundary conditions and the sensible choice, as per this script, was to use $t = 0$.

Exemplar 2 4 marks

13(ii)	$\dfrac{dN}{dt} = \dfrac{N^2}{3988e^{0.2t}}$
	$\Leftrightarrow \int \dfrac{1}{N^2}\,dN = \int \dfrac{1}{3988e^{0.2t}}\,dt$ M1
	$\Leftrightarrow -\dfrac{1}{N} = \dfrac{1}{3988}\int e^{-0.2t}\,dt$ M1
	$\Leftrightarrow -\dfrac{1}{N} = \dfrac{1}{3988} \times (-5e^{-0.2t}) + c$ A1
	$\Leftrightarrow -\dfrac{1}{N} = -\dfrac{5e^{-0.2t}}{3988} + c$
	$\Leftrightarrow N = \dfrac{3988}{5e^{-0.2t}} + c$ M0
	When $t = 0$, $N = 400$:
	$400 = \dfrac{3988}{5} + c \Rightarrow c = -\dfrac{1988}{5}$ M1
	$\therefore N = \dfrac{3988}{5e^{-0.2t}} - \dfrac{1988}{5}$

Examiner commentary

This candidate gains the first three marks for the integration. The subsequent error of inverting each fraction in their equation was a common misconception; candidates who already had a numerical value for *c* were less likely to make this error but it did still occur. Despite the equation now being incorrect, the method mark was still credited for the attempt to find *c*.

Exemplar 3 2 marks

13(ii)

$$\frac{dN}{dt} = \frac{N^2}{3988e^{0.2t}}$$

$$3988e^{0.2t}\,\frac{dN}{dt} = N^2$$

$$(3988e^{0.2t})(dN) = (N^2/dt)$$

$$\frac{dt}{3988e^{0.2t}} = \frac{dN}{N^2}$$

$$\int \frac{1}{N^2}\,dN = \int \frac{1}{3988e^{0.2t}}\,dt \checkmark$$

M1

$$-N^{-1} = \frac{1}{3988}\int \frac{1}{e^{0.2t}}\,dt = \frac{1}{3988} \times \frac{1}{5e^{0.2t}} + c$$

M1 A0

$$-N^{-1} = \cancel{\frac{1}{3988}} \frac{1}{19940e^{0.2t}}$$

$$-N = 19{,}940e^{0.2t} + c$$

M0 M0

$$N = -19{,}940e^{0.2t} + c$$

Examiner commentary

The candidate separates the variables and makes an attempt at integration. There is a suspicion that they have simply integrated the denominator on the right-hand side rather than first writing it in usable form. However, they do obtain an integral of the correct form so benefit of doubt is allowed. There is no attempt at *c*, and the attempt to make *N* the subject does not involve a + *c*, so no further credit is credited.

Question 13 (iii)

(iii) Compare the long-term behaviour of the two models. **[2]**

Exemplar 1 — 2 marks

13(iii) For the 1^{st} Model as t increases the population will tend to $\sqrt{\infty} = \infty$ ∴ tends to infinity.

For the revised model, $-5e^{-0.2(t)} \to 0$ as t gets large ∴ N longterm $= \frac{-3988}{-497/100}$
$= 802.4\ldots$ ∴ is tends to ≈ 802 insects.

Examiner commentary

Two correct statements made to gain full marks.

Exemplar 2 — 1 mark

13(iii) The revised model converges to a fixed population whereas the first model continues to increase. The revised model is more realistic as it takes into account death and amount of food.

B0 B1

Examiner commentary

Stating that the first model continues to increase was sufficient for the mark. The statement about the revised model tending to a limit is also correct but the candidate has not stated what the limit is so the mark is not credited. This question did not specifically ask how realistic the models were (unlike question 7 (iii)), but it was pleasing to see candidates considering this aspect of the models (although no credit available).

Exemplar 3

0 marks

13(iii)	In model 1 as t gets very large N will tend to T^{20} BO In model 2 as t gets large, N will tend to $3\sqrt{e^t}$ BO

Examiner commentary

Examiners expected to see real-life comments about the long-term behaviour rather than just trying to consider what their equations would be if the constant terms could be ignored as *t* tended to infinity.

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