Qualification Accredited

Oxford Cambridge and RSA

A LEVEL

Exemplar Candidate Work

MATHEMATICS A

H240
For first teaching in 2017

H240/03 Summer 2018 examination series

Version 1

Contents

Introduction	3
Question 1(i)	4
Question 1(ii)	5
Question 2	6
Question 3	8
Question 4(i)(a)	14
Question 4(i)(b)	15
Question 4(ii)	16
Question 5(i)	19
Question 5(ii)	21
Question 5(iii)	26
Question 6(i)	27
Question 6(ii)	30
Question 7	32
Question 8(i)	40
Question 8(ii)	42
Question 9(i)	44
Question 9(ii)	45
Question 9(iii)	47
Question 9(iv)(a)	49
Question 9(iv)(b)	50
Question 10(i)	51
Question 10(ii)	54
Question 10(iii)(a)	55
Question 10(iii)(b)	57
Question 11(i)	58
Question 11(ii)	59
Question 12(i)(a)	64
Question 12(i)(b)	68
Question 12(ii)	70

Introduction

These exemplar answers have been chosen from the summer 2018 examination series.

OCR is open to a wide variety of approaches and all answers are considered on their merits. These exemplars, therefore, should not be seen as the only way to answer questions but do illustrate how the mark scheme has been applied.

Please always refer to the specification http://www.ocr.org.uk/Images/308723-specification-accredited-a-level-gce-mathematics-a-h240.pdf for full details of the assessment for this qualification. These exemplar answers should also be read in conjunction with the sample assessment materials and the June 2018 Examiners' report or Report to Centres available from Interchange https://interchange.ocr.org.uk/Home.mvc/Index

The question paper, mark scheme and any resource booklet(s) will be available on the OCR website from summer 2019. Until then, they are available on OCR Interchange (school exams officers will have a login for this and are able to set up teachers with specific logins – see the following link for further information http://www.ocr.org.uk/administration/support-and-tools/interchange/managing-user-accounts/).

It is important to note that approaches to question setting and marking will remain consistent. At the same time OCR reviews all its qualifications annually and may make small adjustments to improve the performance of its assessments. We will let you know of any substantive changes.

Question 1(i)

1 A circle with centre C has equation $x^2 + y^2 + 8x - 2y - 7 = 0$.

Find

(i) the coordinates of C, **[2]**

Exemplar 1 — 2 marks

1(i)

$(x+4)^2 - 16 + (y-1)^2 - 1 - 7 = 0$

$(x+4)^2 + (y-1)^2 = 24$

centre $(-4, 1)$

~~radius~~

Exemplar 2 — 1 mark

1(i)

$x^2 + 8x + y^2 - 2y = 7$

$(x \; [illegible])^2 - 16 + (y-1)^2 - 1 = 7$ M1

$(4, 1)$ A0

Examiner commentary

This proved to be a good start for nearly all candidates (Exemplar 1) with the vast majority correctly completing the square (twice) to find the coordinates of the centre of the circle. When errors occurred these were nearly always down to sign errors inside the two brackets (Exemplar 2).

Question 1(ii)

(ii) the radius of the circle. [1]

Exemplar 1 — 1 mark

1(ii) radius $= \sqrt{24} = 2\sqrt{6}$

Exemplar 2 — 1 mark

1(ii) $r = \sqrt{24} \approx 4.899$ (3dp)

Exemplar 3 — 0 marks

1(ii) $(x+4)^2+(y-1)^2=7 \quad \therefore$ radius $= \sqrt{7}$

Examiner commentary

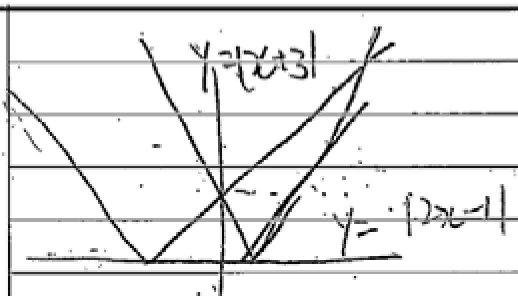
Nearly all candidates stated the radius of the circle correctly, either in surd form (Exemplar 1) or as a decimal approximation (Exemplar 2). Note that whilst the expectation was that candidates would leave their answer in surd form (and hence avoiding any risk of mis-rounding), questions would explicitly state if a specific format was required. A minority of candidates did not fully complete the squares in part (i) and hence had problems with finding the radius in part (ii), as seen in Exemplar 3.

Question 2

2 Solve the equation $|2x-1| = |x+3|$. [3]

Exemplar 1 — 3 marks

2



$4x^2 - 4x + 1 = x^2 + 6x + 9$

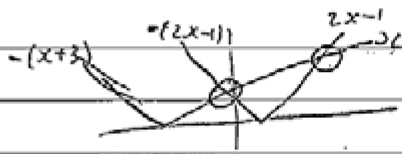
$3x^2 - 10x - 8 = 0$

$(3x+2)(x-4) = 0$

$x = 4, \quad x = -\frac{2}{3}$

Exemplar 2 — 3 marks

2 $|2x-1| = |x+3|$



$-(2x-1) = x+3$ $\qquad$ $x+3 = 2x-1$

$-2x+1 = x+3$ $\qquad$ $4 = x$

$-2 = 3x$

$-\frac{2}{3} = x$

Exemplar 3

2 marks

2

$|2x-1| = |x+3|$

$\cancel{x = po}$ $x \geq 0$ $\cancel{x < 0}$

$2x - 1 = x + 3$

$x - 1 = 3$

$x = 4$ A1

$x < 0$

M1 $-2x - 1 = x + 3$

$-3x - 1 = 3$

$-3x = 4$

$x = -\frac{4}{3}$

$\therefore x = 4$ or $x = -\frac{4}{3}$ A0

Examiner commentary

This question was answered extremely well with nearly all candidates correctly solving this equation involving the modulus function. Of the two main methods for solving this type of equation the first, which involved re-writing as $(2x - 1)^2 = (x + 3)^2$ was far more successful (Exemplar 1). The alternate method was to re-write as two linear equation as many made sign errors even though most started from the correct two equations $(2x - 1) = \pm (x + 3)$. Exemplar 2 has perhaps carefully negotiated this risk through an initial sketch. Exemplar 3 highlights the risk of this second method with a sign error on multiplying out the $(2x - 1)$ by -1.

Question 3

3 In this question you must show detailed reasoning.

A gardener is planning the design for a rectangular flower bed. The requirements are:

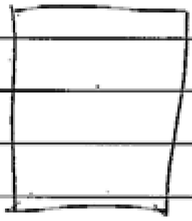
- the length of the flower bed is to be 3 m longer than the width,
- the length of the flower bed must be at least 14.5 m,
- the area of the flower bed must be less than $180\,\text{m}^2$.

The width of the flower bed is x m.

By writing down and solving appropriate inequalities in x, determine the set of possible values for the width of the flower bed. **[6]**

Exemplar 1 6 marks

3



length

$x+3$

x

width

$(x+3) \geq 14.5$

$x \geq 11.5$

$x^2+3x-180<0$

$(x+15)(x-12)<0$

$-15<x<12$

-15 11.5 12

$11.5 \leq x < 12$

Exemplar 2

6 marks

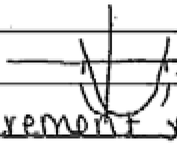
3

$x + 3 \geq 14.5$

$x(x+3) < 180$

$x \geq 11.5$ $\quad$ $x^2 + 3x - 180 < 0$

$(x-12)(x+15) < 0$



$-15 < x < 12$

As x is a measurement x cannot be < 0 so

$0 < x < 12$

$\underline{11.5 \leq x < 12}$

$\{x : 11.5 \leq x < 12\}$

Exemplar 3

5 marks

3

Let L = length of flowerbed; Let ~~w~~ = width ~~L=w+3~~ of flowerbed.

~~$+ω$~~ x

$L = w+3$

$L \geq 14.5$

Area < ~~180m~~ 180 Area = ~~$L \times W$~~ Lx

~~$A\ LW < 180$~~

x

$Lw < 180$

$L = ~~w+3~~\ x+3$

~~$(x+3)x < 180$~~ $(x+3)x < 180$ M1

~~$w^2+3x < 180$~~ $x^2+3x < 180$

$x^2+3x-180 < 0$

$\therefore -15 < x < 12$ A1

M0

x cannot be negative as ~~area mu~~ Length must equal $x+3$ $\therefore$ and ~~the greater than 14~~ $L \geq 14.5$..

$\therefore 0 < x < 12$

$L \geq 14.5 \quad L = x+3$

$\therefore 14.5 - 3 \leq x$

$\therefore 11.5 \leq x$

$\therefore$ ~~$11.5 \leq x$~~ $\therefore 11.5 \leq x < 12$

Exemplar 4

4 marks

3

$L =$ length. $\quad L = w+3. \quad \Rightarrow L = x+3.$

$W =$ width $= x$. $\quad L \geq 14.5 \quad L \geq 14.5.$

$A =$ area. $\quad A \leq 180. \quad A < 180.$

$\therefore$ since $L \geq 14.5$ and $L = x+3$

$\therefore x+3 \geq 14.5$

$\therefore \underline{x \geq 11.5}$

Area < 180 and Area $=$ Length $\times$ width.

$= (x+3) \times x$

$\therefore x(x+3) < 180.$

$\therefore x^2 + 3x < 180.$

If $x^2 + 3x < 180$, let $x^2 + 3x - 180 = 0.$

$x = \dfrac{-3 \pm \sqrt{(3)^2 - (4 \times 1 \times -180)}}{2} = \dfrac{-3 \pm \sqrt{9 + 720}}{2}$

$= \dfrac{-3 \pm \sqrt{729}}{2}.$

$= \dfrac{-3 \pm 27}{2} \quad \therefore x = -15$ or 12 m.

$\therefore x \neq -15$ as width $\neq$ a $-$ve value.

$\therefore x = 12$m $\quad \therefore$ width $= 12$ m

$\therefore$ length $= 15$ m.

$\therefore$ area $= 180$ m^2.

Exemplar 5

4 marks

3 3m longer than width ~~$W = L + 3$~~

length at least 14.5 $L \geq 14.5$

area less than 180m^2 $A < 180$

width $= x$

~~$x = L + 3$~~ $L = W + 3$

$L \geq 14.5$

$W \times L < 180$

$W + 3 \geq 14.5$

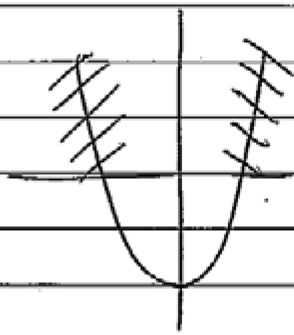
$W(W+3) < 180$

$W \geq 11.5$ $W^2 + 3W < 180$

~~$11.5(11.5+3)$~~

$W^2 + 3W - 180 < 0$

$(W+15)(W-12)$



$W = 12$ $W = 15$

$12 < W < 15$

~~$11.5 \leq W$~~

Exemplar 6

2 marks

3

Let length of flower bed $= l$ m

Let area of flower bed $= A$ m^2

& $l=3x$ $\quad l = 3x \Rightarrow x = \frac{1}{3}l$

$l \geq 14.5$

$A < 180 \quad \therefore \cancel{lw<180} \quad lx < 180$

$\Rightarrow 3x(x) < 180$

$\therefore 3x^2 < 180$

If $3x^2 = 180$, $x^2 = 60$

$\Rightarrow x = \pm 2\sqrt{15}$ but width can't be $-$ve

$\therefore x \neq -2\sqrt{15}$ so $x = 2\sqrt{15}$

When $x=0$, $0<180$ which is true

$\therefore 0 < x < 2\sqrt{15}$

$lx < 180 \Rightarrow \frac{1}{3}l(l) < 180$

$\therefore \frac{1}{3}l^2 < 180$

If $\frac{1}{3}l^2 = 180$, then $l^2 = 540$

$\Rightarrow l = \pm 6\sqrt{15}$ but length can't be $-$ve

$\therefore l \neq -6\sqrt{15}$ so $l = 6\sqrt{15}$

$l \geq 14.5$

$6\sqrt{15} < 30$

If $l=30$, $\frac{1}{3}(30^2) < 180$

$\therefore 300 < 180$ which is false

$\therefore 14.5 \leq l < 6\sqrt{15}$

Since $l = 3x$, $14.5 \leq 3x < 6\sqrt{15}$

$\therefore \frac{29}{6} \leq x < 2\sqrt{15}$

Also $0 < x < 2\sqrt{15}$

$\therefore$ overall, $\frac{29}{6} \leq x < 2\sqrt{15}$

SC B1 B1

Examiner commentary

As this was a detailed reasoning question it was expected that candidates would do just that and show sufficient reasoning so that examiners could see that a complete analytical method had been employed.

Exemplar 1 has a model answer, with the use of sketches to help avoid careless mistakes.

Exemplar 2 has given the final answer in set notation, whilst this is a content item in the reformed specification, when required for full credit it will be explicitly requested in the question.

Exemplar 3 appears to be a complete solution; however, as a detailed reasoning question, a method mark has been withheld for no justification for the solution $-15 < x < 12$.

Exemplar 4 has set up the solution nicely, but the candidate has not drawn together the results for a full detailed conclusion linking $x \geq 11.5$ with the solution to $x^2 + 3x < 180$.

Exemplar 5 has made a nice attempt at a solution but a careless mistake with the roots of the quadratic inequality has resulted in the final 2 marks dropped.

A small proportion of candidates misreading the information in the question and considered the length of the flower bed to be three times longer than its width (and not just 3 m longer than the width), as seen in Exemplar 6.

Question 4(i)(a)

4 In this question you must show detailed reasoning.

The functions f and g are defined for all real values of x by

$$f(x) = x^3 \quad \text{and} \quad g(x) = x^2 + 2.$$

(i) Write down expressions for

(a) $fg(x)$, **[1]**

Exemplar 1 1 mark

4(i)(a)	$fg(x) = (x^2+2)^3$

Exemplar 2 1 mark

4(i)(a)	$fg(x) = (x^2+2)^3$	$= (x^4+4x^2+4)(x^2+2)$	$= x^6 + 6x^4 + 12x^2 + 8$
		$= x^6 + 4x^4 + 4x^2 + 2x^4 + 8x^2 + 8$	

Examiner commentary

Nearly all candidates, such as Exemplar 1, correctly found the composite function fg(x) as $(x^2 + 2)^3$ although a number did expand the bracket in this part, as in Exemplar 2. Candidates are reminded that the number of marks available for a question or part-question are the best indicators to the amount of working and detail that is required. Another key indicator for the level of response required is the use of the phrase "Write down …"; defined in the specification Command Words section as not requiring any working or justification.

Question 4(i)(b)

(b) $\mathrm{gf}(x)$. **[1]**

Exemplar 1 — 1 mark

4(i)(b)	$gf(x) = x^6+2$

Exemplar 2 — 1 mark

4(i)(b)	$gf(x) = (x^3)^2+2 \quad = x^5+2$

Examiner commentary

Once again this was nearly always done correctly, as in Exemplar 1, with the most common errors being those minority of candidates who stated that $(x^2)^3$ was equal to either x^5 or x^8. Exemplar 2 has not been penalised at this point since this specific question rewarded the initial written response and ignored the subsequent incorrect simplification, which was penalised in part (ii).

Question 4(ii)

(ii) Hence find the values of x for which $\mathrm{fg}(x)-\mathrm{gf}(x)=24$. [6]

Exemplar 1

6 marks

4(ii)

$(x^2+2)^3-(x^6+2)=24$

$x^6+6x^4+3(x^2)(2)^2+8-x^6-2-24=0$

$6x^4+12x^2-18=0$

$x^4+2x^2-3=0$

$(x^2+3)(x^2-1)=0$

$x^2=-3$ or 1 but $x^2\geq 0 \therefore x^2\neq -3 \therefore x^2=1$

$x=\pm 1$

Exemplar 2

6 marks

4(ii)

$(x^2+2)^3-((x^3)^2+2)=24$

$(x^4+4x^2+4)(x^2+2)-x^6-2=24$

$x^6+4x^4+4x^2+2x^4+8x^2+8-x^6-2=24$

$6x^4+12x^2-18=0$

$x^4+2x^2-3=0$

let $y=x^2$

$y^2+2y-3=0$

$(y-1)(y+3)=0$

$y=1$ or -3

$y=x^2$ $\quad 1=x^2$ or $-3=x^2$

$x=\pm 1$ $\quad$ ↳ not real solution

Exemplar 3 6 marks

4(ii)	$(x^2+2)^3 - (x^6+2) = 24$
	$(2+x^2)^3$
	$= 2^3 + \binom{3}{1}\times 2^2 \times x^2 + \binom{3}{2} \times 2 \times (x^2)^2 + \binom{3}{3} \times (x^2)^3$
	$= 8 + 12x^2 + 6x^4 + x^6 - x^6 - 2 = 24$
	$= 6x^4 + 12x^2 + 6 = 24$
	$6x^4 + 12x^2 - 18 = 0$
	$x^4 + 2x^2 - 3 = 0$
	let $X = x^2$
	$X^2 + 2X - 3 = 0$
	$(X+3)(X-1) = 0$
	$X = -3 \quad X = 1$
	$x^2 = -3 \qquad x^2 = 1$
	$x = \pm\sqrt{-3} \qquad x = \pm\sqrt{1}$
	not possible.
	when $x = 1$ or when $x = -1$.

Exemplar 4 5 marks

4(ii)	$(x^2+2)^3 - (x^6+2) = 24$
	$(x^2+2)(x^4+4x^2+4) - x^6 - 2 = 24$
	$x^6 + 4x^4 + 4x^2 + 2x^4$ M1 $+ 8x^2 + 8 - x^6 - 2 = 24$ A1
	$6x^4 + 12x^2 + 6 = 24$ A1
	$6x^4 + 12x^2 - 18 = 0$
	$a = x^2$
	$6a^2 + 12a - 18 = 0$ M1
	$a^2 + 2a - 3 = 0$
	$(a-1)(a+3)$ A1
	$a = 1, -3$
	$x = 1 \quad \sqrt{-3}$ is not a real number
	$\therefore x = 1$ ^ A0

Exemplar 5

2 marks

4(ii)

$(x^2+2)^3 - \left((x^3)^2+2\right) = 24$

$(x^4+4x^2+4)(x^2+2) - x^5 - 2 = 24$

$x^6 + 4x^4 + 4x^2 + 2x^4 + 8x^2 + 8 - x^5 - 2 = 24$ M1 A1

$x^6 - x^5 + 6x^4 + 12x^2 - 18 = 0$ A0

$x = 1 \qquad x = -0.97$

Examiner commentary

It is expected at this level, that as part of the detailed reasoning, candidates should justify why $x^2+3=0$ did not provide any solutions. Exemplar 1 refers to $x^2 \geq 0$, whereas Exemplar 2 refers back to the statement in the question stem regarding *x* being real. Exemplar 3 was not penalised for stating $\sqrt{-3}$ not possible, although this misconception should not be encouraged.

There were a significant number of candidates that did not employ correct bracketing and mistakenly wrote $(x^2+2)^3 - x^6 + 2 = 24$ rather than $(x^2+2)^3 - (x^6+2) = 24 \Rightarrow (x^2+2)^3 - x^6 - 2 = 24$.

The majority of candidates went for expanding $(x^2+2)^3$ by writing out the bracket three times rather than using the binomial expansion of $(a+b)^n$. Exemplar 4 highlights this approach; this candidate dropped the final mark for not identifying tha $\sqrt{1} = \pm 1$.

While the majority rearranged their quartic into the form $6(x^2)^2 + 12x^2 - 18 = 0$ there were a number of candidates did not show sufficient working in solving this quartic, even though the full question required detailed reasoning. Example 5 has made an early mistake with $(x^2)^3 = x^5$ which resulted in the need to solve a complex polynomial. The two roots stated have clearly be found using calculator equation solver functionality, the "Detailed reasoning" statement should have indicated that this would not have been sufficient and provided a clue that a mistake had been made with the earlier algebra.

Question 5(i)

5 (i) Use the trapezium rule, with two strips of equal width, to show that

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \approx \frac{11}{4} - \sqrt{2}.$$

[5]

Exemplar 1

5 marks

5(i)

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \approx \frac{2}{2}(y_0+y_2+2y_1) = \frac{1}{2} + \frac{1}{2+\sqrt{4}} + 2\left(\frac{1}{2+\sqrt{2}}\right)$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{2(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})} = \frac{3}{4} + \frac{2(2-\sqrt{2})}{4-2}$$

$$= \frac{3}{4} + 2 - \sqrt{2} = \frac{11}{4} - \sqrt{2}$$

Exemplar 2

4 marks

5(i)

x	y
0	$\frac{1}{2}$
2	$\frac{1}{2+\sqrt{2}}$
4	$\frac{1}{4}$

$$= \frac{1}{2}(2)\left(\left(\frac{1}{2}+\frac{1}{4}\right) + 2\left(\frac{1}{2+\sqrt{2}}\right)\right)$$

$$= \frac{3}{4} + \frac{2}{2+\sqrt{2}}$$

$$= \frac{3(2+\sqrt{2})+8}{4(2+\sqrt{2})}$$

$$= \frac{6+3\sqrt{2}+8}{8+4\sqrt{2}} = \frac{14+3\sqrt{2}}{8+4\sqrt{2}} \times \frac{(8-4\sqrt{2})}{(8-4\sqrt{2})}$$

$$= \frac{112+24\sqrt{2}-56\sqrt{2}-24}{64-32}$$

$$= \frac{11}{4} - \sqrt{2}$$

Exemplar 3 — 2 marks

5(i)

$$\int_a^b y\,dx \approx \frac{1}{2}h\left\{(y_0+y_n)+2(y_1+y_2+\dots y_{n-1})\right\}$$

~~$h = \frac{b-a}{n}$ $n=2$ $h=$~~ $h=\frac{b-a}{n}$

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \quad n=2,\ b=4,\ a=0,\ y=\frac{1}{2+\sqrt{x}} \quad h=\frac{4-0}{2}$$

$h=2$ B1

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \approx \frac{1}{2}\times 2\left\{\left(\frac{1}{2+\sqrt{0}}+\frac{1}{2+\sqrt{2}}\right)+2\left(\frac{1}{2+\sqrt{1}}\right)\right\}$$ M0

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \approx \left(\frac{1}{2}+\frac{1}{2+\sqrt{2}}+\frac{2}{3}\right)$$

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \approx \left(\frac{1}{2}+\frac{2-\sqrt{2}}{2}+\frac{2}{3}\right)$$ M1

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \approx \frac{13}{6}-\frac{\sqrt{2}}{2}$$

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \approx \frac{11}{4}-\sqrt{2}$$ A0

Examiner commentary

Candidates found this part extremely accessible and nearly all correctly derived the given result, Exemplar 1. Candidates are reminded though that in 'show that' questions suitable working must be shown, Exemplar 2 makes a big leap from $\frac{112+24\sqrt{2}-56\sqrt{2}-24}{64-32}$ to the given answer, an additional step of collecting terms would be expected to provide a convincing mathematical argument.

Exemplar 3 highlights the positive marking approach taken, the initial mark credited for using the correct value for *h*, and although the trapezium rule has not been correctly applied this candidate has still picked up a method mark for correctly rationalising the denominator of their surd.

Question 5(ii)

(ii) Use the substitution $x = u^2$ to find the exact value of

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,\mathrm{d}x. \qquad [6]$$

Exemplar 1 — 6 marks

5(ii)

$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \qquad x = u^2 \qquad dx = 2u\,du$

$u = \sqrt{x}$

limits $2, 0 \qquad u = \sqrt{4} \qquad u = \sqrt{0}$

$I = \int_0^2 \frac{2u}{2+\sqrt{u^2}}\,du = \int_0^2 \frac{2u}{2+u}\,du$

5(ii) (continued)

$\int_0^2 \frac{2u}{2+u}\,du = \int_0^2 \frac{(2+u)2-4}{2+u}\,du$

$= \int_0^2 2 - \frac{4}{2+u}\,du$

$= \left[2u - 4\ln(2+u)\right]_0^2$

$= (4 - 4\ln 4) + 4\ln 2$

$= 4 - 8\ln 2 + 4\ln 2$

$= 4 - 4\ln 2$

Exemplar 2

2 marks

5(ii)

$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx$

$x = u^2$

$\frac{dx}{du} = 2u$

$dx = 2u\,du$

$\int_0^4 \frac{1}{2+u} \times 2u\,du$

$= \int_0^4 \frac{2u}{2+u}\,du$

$= \int_0^4 2u(2+u)^{-1}\,du$

$\int u \times \frac{dv}{dx} \ldots$

$u = 2u \qquad v = \ln(2+u)$

$\frac{du}{du} = 2 \qquad \frac{dv}{dx} = \frac{1}{2+u}$

(answer space continued on next page)

5(ii) **(continued)**

$= \left[2u\ln|2+u| - \int_0^2 \ln|2+u|\,2\,du\right]_0^2$

$= 2u\ln|2+u| -$

$\int_0^4 \ln|2+u|$

$=$

M1 A1

$\int_0^2 \frac{1}{2+u}\,2u\,du$

$\int_0^2 \frac{2u}{2+u}\,du$

$u = \frac{1}{2+u} \qquad u \times v$

$x = u^2$

$\frac{dx}{du} = 2u$

~~$dx = 2u\,du$~~

$x = u^2 \qquad x = 0$

$4 = u^2 \qquad 0 = u^2$

$2 = u \qquad 0 = u$

limits 2 and 0.

$= \left[\frac{2u}{2+u} - \int_0^2 2u - (2+u)^{-2}\,du\right]_0^2$

(minuses cancel out).

$= \frac{2u}{2+u} + \int_0^2 \frac{2u}{(2+u)^2}\,du$

M0

$u\,\frac{dv}{dx}$

$u = \frac{1}{2+u} \qquad v = 2u$

$\frac{du}{du} = (2+u)^{-1} \qquad \frac{dv}{du} = 2$

$= -(2+u)^{-2}$

Exemplar 3

2 marks

5(ii)

$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx.$ $x=u^2$ $\frac{dx}{du}=2u.$ $dx=2u\,du.$

$\int_0^4 \frac{2u}{2+\sqrt{x}}\,du.$ $x=4, u=2.$

M1 $x=0, u=0.$

$\int_0^2 \frac{2u}{2+u}\,du$ A1

$= 2\int_0^2 \frac{u}{2+u} + \frac{u}{2+u}\,du.$

$\left[2u\ln(2+u)\right]_0^2 = 4\ln 4$ M0

5(ii) (continued)

$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx.$ $x=u^2$ $\frac{dx}{du}=2u$ $dx=2u\,du.$

$\int_0^4 \frac{2u}{2+u}\,du$

$\int_0^2 \frac{2u}{2+u}\,du.$ $\cancel{2u(2+u)}^{-1}$

$\int_0^2 \frac{\cancel{2u-u+u}}{2+u}$

$\int_0^2$ $\cancel{2u-u}+2-u-2.$ $2+u \div 2u -2.$

$\int_0^2 \frac{2+u}{2+u} + \frac{u-2}{2+u}\,du$

$= \left[u + (u-2)\ln(2+u)\right]_0^2$

$= (2+0)-(0-2\ln 2)$

$= 2+2\ln 2.$

Exemplar 4

2 marks

5(ii)

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \rightarrow \int_0^2 \frac{2u}{2+u}\,du \qquad \int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$= u^2(2+u)^{-1} - \int u^2\left(-\tfrac{1}{2}(2+u)^{-2}\right)$$

$\cancel{\text{let}}$ $x = u^2$

$\cancel{\frac{dx}{dx}}\ \frac{dx}{du} = 2u$

$\cancel{dx = 2u\,du}$

$\frac{1}{2}u\,dx = du$

$$\int_0^2 2u \cdot \frac{1}{2+u}\,du$$

$\cancel{u = 2u \quad \frac{dv}{dx} = (2+u)^{-1}}$

$$= u^2(2+u)^{-1} + \frac{1}{2}\int u^2(2+u)^{-2}$$

v

$m\ \cancel{}= (2+u)^{-1} \quad \frac{dn}{du} = 2u$

$\frac{dm}{du} = -\frac{1}{2}(2+u)^{-2} \quad n = u^2$

5(ii) (continued)

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \rightarrow \int_0^2 \frac{2u}{2+u}\,du$$ A1

$x = u^2$ M1

$\frac{dx}{du} = 2u$

$dx = 2u\,du$

$p = (2+u)^{-1} \qquad \frac{dq}{du} = 2u$

$\frac{dp}{du} = -2(2+u)^{-2} \qquad q = 2$

$$= 2(2+u)^{-1} - \int 2\left(-2(2+u)^{-2}\right)du$$ M0

$$= 2(2+u)^{-1} + 4\int (2+u)^{-2}\,du$$

$$= \left[2(2+u)^{-1} - 4(2+u)^{-1}\right]_0^2$$

$$= \left[2(2+\sqrt{x})^{-1} - 4(2+\sqrt{x})^{-1}\right]_0^4$$

$$= \left[2(2+\sqrt{4})^{-1} - 4(2+\sqrt{4})^{-1}\right] - \left[2(2+\sqrt{0})^{-1} - 4(2+\sqrt{0})^{-1}\right]$$

$$= \frac{1}{2}$$

Exemplar 5

2 marks

5(ii)

$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx$ $\quad x=v^2$ $\quad \therefore v=\sqrt{x}$

$\frac{dx}{dv}=2v$ $\quad \frac{dv}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$

$\int_0^4 \frac{1}{v+2}\,dx$ $\quad dx=2v\,dv$ $\quad \frac{dv}{dx}=\frac{1}{2\sqrt{x}}$

$\int_0^4 \frac{1}{v+2}\times 2\sqrt{x}\,dv$ $\quad dv=\frac{dx}{2\sqrt{x}}$

$\int_0^4 \frac{2v}{v+2}\,dv$ $\quad dx=2\sqrt{x}\,dv$

5(ii) **(continued)**

$v=\sqrt{x}$

$\int_0^4 \frac{2v}{v+2}\,dv$ $\quad$ limits = ~~4 =~~ $v=\sqrt{4}=2$

M1 $\quad$ limits = $v=\sqrt{0}=0$

$\int_0^2 \frac{2v}{v+2}\,dv$ A1

~~$v=\ln(v+2)$ $\frac{dv}{dx}=\frac{1}{v+2}$ $\frac{du}{dx}=2$ $u=2v$~~

~~$\int u\frac{dv}{dx}\,dx = uv-\int v\frac{du}{dx}\,dx$~~

~~$\int_0^2 \frac{2v}{v+2} = 2v\ln(v+2) - \int 2\ln(v+2)$~~

$\int_0^2 \frac{2v}{v+2} = 2v\ln(v+2) - \int 2\ln(v+2)\,dv$ $\quad v=e^a-2$

$= \int 2a\,dv$ $\quad a=\ln(v+2)$ $\quad \frac{da}{dv}=\frac{1}{v+2}$

$\int 2a\times(e^a-2)\,da$ $\quad$ ~~$da=\frac{1}{v+2}$~~ $\quad dv=v+2\,da$

$\int 2ae^a-4a\,da$ M0

$= 2e^a-2a^2\,da = 2v+4-2(\ln(v+2))^2$

Examiner commentary

While nearly all candidates used the substitution correctly and re-wrote the integral as $\int_0^2 \frac{2u}{2+u}\,du$ many could not deal with the resulting improper fraction in the integrand. The most successful candidates re-wrote $\frac{2u}{2+u}$ either by using long division or realising that $\frac{2u}{2+u}=\frac{4+2u-4}{2+u}=2-\frac{4}{2+u}$, Exemplar 1. While examiners noted that some candidates employed more extreme methods (for example, further substitutions and the method of integration by parts) these were usually unsuccessful, Exemplar 2, 3, 4 and 5.

Question 5(iii)

(iii) Using your answers to parts **(i)** and **(ii)**, show that

$$\ln 2 \approx k + \frac{\sqrt{2}}{4},$$

where k is a rational number to be determined. **[2]**

Exemplar 1 — 2 marks

5(iii)

$4 - 4\ln 2 \approx \frac{11}{4} - \sqrt{2}$

$4\ln 2 \approx 4 - \left(\frac{11}{4} - \sqrt{2}\right)$

$4\ln 2 \approx \frac{5}{4} + \sqrt{2}$ $\quad \ln 2 \approx \frac{5}{16} + \frac{\sqrt{2}}{4}$

Exemplar 2 — 1 mark

5(iii)

$4\ln 4 \approx \frac{11}{4} - \sqrt{2}$ M1 $\quad 2 + 2\ln 2 = \frac{11}{4} - \sqrt{2}$

$8\ln 2 \approx \frac{11}{4} - \sqrt{2}$

$\ln 2 \approx \frac{11}{32} - \frac{\sqrt{2}}{8}$.

Exemplar 3 — 0 marks

5(iii)

$\ln 2 \approx k + \frac{\sqrt{2}}{4}$

$\ln 2 - \frac{\sqrt{2}}{4} \approx k$

$k \approx 0.340$ (3 sf).

Examiner commentary

The majority of candidates, such as Exemplar 1, who were successful in part (ii) correctly determining that $k = \frac{5}{16}$.

Of those who had struggled with part (ii), most left this part blank, but there was the opportunity of gaining the method mark by equating their answers to parts (i) and (ii), Exemplar 2.

There was no marks credited for those candidates that simply found a numerical value for k from the given equation, Exemplar 3

Question 6(i)

6 It is given that the angle θ satisfies the equation $\sin\left(2\theta+\frac{1}{4}\pi\right) = 3\cos\left(2\theta+\frac{1}{4}\pi\right)$.

(i) Show that $\tan 2\theta = \frac{1}{2}$. [3]

Exemplar 1 3 marks

6(i)

$\sin 2\theta \cos\frac{\pi}{4} + \cos 2\theta \sin\frac{\pi}{4} = 3\left(\cos 2\theta \cos\frac{\pi}{4} - \sin 2\theta \sin\frac{\pi}{4}\right)$

$\frac{1}{\sqrt{2}}\sin 2\theta + \frac{1}{\sqrt{2}}\cos 2\theta = \frac{3}{\sqrt{2}}\cos 2\theta - \frac{3}{\sqrt{2}}\sin 2\theta$

$\tan 2\theta + 1 = 3 - 3\tan 2\theta$

$4\tan 2\theta = 2$

$\tan 2\theta = 1/2$

Exemplar 2 3 marks

6(i)

$\sin(2\theta + \frac{1}{4}\pi) = 3\cos(2\theta + \frac{1}{4}\pi)$ $\quad \tan 2\theta = \frac{2\tan^2\theta}{1-\tan^2\theta}$

$\frac{\sin(2\theta+\frac{1}{4}\pi)}{\cos(2\theta+\frac{1}{4}\pi)} = 3$ $\quad \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan(2\theta + \frac{1}{4}\pi) = 3$

$\frac{\tan 2\theta + \tan\frac{1}{4}\pi}{1 - \tan 2\theta \tan\frac{1}{4}\pi} = 3$ $\quad \tan\frac{1}{4}\pi = 1$

$\frac{\tan 2\theta + 1}{1 - \tan 2\theta} = 3$

$\tan 2\theta + 1 = 3(1 - \tan 2\theta)$

$\tan 2\theta + 1 = 3 - 3\tan 2\theta$

$4\tan 2\theta = 2$

$\tan 2\theta = \frac{1}{2}$.

Exemplar 3

1 mark

6(i)	$\sin(2\theta+\frac{\pi}{4}) = 3\cos(2\theta+\frac{\pi}{4})$
	$\sin 2\theta\cos\frac{\pi}{4} + \cos 2\theta\sin\frac{\pi}{4} = 3\cos 2\theta\, 3\cos\frac{\pi}{4} - 3\sin\frac{\pi}{4}\, 3\sin 2\theta$ M1
	$\frac{\sqrt{2}}{2}\sin 2\theta + \frac{\sqrt{2}}{2}\cos 2\theta = \frac{9\sqrt{2}}{2}\cos 2\theta - \frac{9\sqrt{2}}{2}\sin 2\theta$
	$\sqrt{2}\sin 2\theta + \sqrt{2}\cos 2\theta - 9\sqrt{2}\cos 2\theta - 9\sqrt{2}\sin 2\theta = 0$ $\quad \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$
	$\sqrt{2}(\sin 2\theta + \cos 2\theta) - 9\sqrt{2}(\cos 2\theta + \sin 2\theta) = 0$
	$\sin 2\theta + \cos 2\theta - 9\cos 2\theta - 9\sin 2\theta = 0$
	$-8\sin 2\theta - 8\cos 2\theta = 0$ A0
	$-8\sin 2\theta = 8\cos 2\theta$
	$\frac{-8\sin 2\theta}{8\cos 2\theta} = 1$
	$-1\tan 2\theta = 1$
	$\tan 2\theta = -1$ A0

Exemplar 4

0 mark

6(i)	$\frac{\sin(2\theta+\frac{1}{4}\pi)}{3\cos(2\theta+\frac{1}{4}\pi)} = \frac{1}{3}\tan(2\theta+\frac{1}{4}\pi)$
	$\cancel{\frac{\sin(2\theta+\frac{1}{4}\pi)}{\cos(2\theta+\frac{1}{4}\pi)} = \tan(2\theta + \frac{1}{4}\pi) = 0}$
	~~∴ They have all been translated through~~ [illegible]
	~~$\therefore \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$~~ [illegible]

6(i)	$\frac{1}{3}\tan(2\theta+\frac{\pi}{4}) = 0$ B0
	$\tan(2\theta+\frac{\pi}{4}) = 0$
	$2\theta + \frac{\pi}{4} = 0$
	$2\theta = -\frac{\pi}{4}$
	$\theta = -\frac{\pi}{2}$

Examiner commentary

Candidates were equally split in how to tackle this part. Approximately half expanding the brackets (using the correct compound-angle formulae as in Exemplar 1) while the other half re-wrote as tan $\left(2\theta + \frac{\pi}{4}\right) = 3$ before expanding, as in Exemplar 2. Both approaches proved equally successful in obtaining the expected result.

Errors were commonly related to careless algebraic manipulation. Exemplar 3 incorporates a careless sign error. Exemplar 4 involved multiplying every function within the bracket by 3, rather than both terms.

Exemplar 4 shows a careless cross-multiplying error followed by ignoring the instructions of the question and the answer given to work toward.

Question 6(ii)

(ii) Hence find, in surd form, the exact value of $\tan\theta$, given that θ is an obtuse angle. [5]

Exemplar 1 — 5 marks

6(ii)

$$\frac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta = \frac{1}{2}$$

$$2\tan\theta = \frac{1-\tan^2\theta}{2}$$

$$\frac{1}{2}\tan^2\theta + 2\tan\theta - \frac{1}{2} = 0$$

$$\tan^2\theta + 4\tan\theta - 1 = 0$$

$$\tan\theta = \frac{-4 \pm \sqrt{16-4(1)(-1)}}{2(1)}$$

$$\tan\theta = -2 \pm \sqrt{5}$$

Cannot be $-2+\sqrt{5}$ as acute or reflex

$\therefore \tan\theta = -2-\sqrt{5}$

Exemplar 2 — 3 marks

6(ii)

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = 1/2$$

$2\tan\theta = 1/2 - 1/2\tan^2\theta$

$4\tan\theta = 1 - \tan^2\theta$

$\tan^2\theta + 4\tan\theta - 1 = 0$

$\tan\theta = -2+\sqrt{5}$ $\tan\theta = -2-\sqrt{5}$

~~$\theta \approx 0.13$~~ ~~$\theta = -1.338$~~

Exemplar 3 2 marks

6(ii)	$\frac{1}{2} = \frac{2\tan\theta}{1-\tan^2\theta}$ M1 $1-\tan^2\theta = 4\tan\theta$ M1 $\tan^2\theta + 4\tan\theta - 1 = 0$ $(\tan\theta+2)^2 - 4 = 1.$ ~~(tan θ)(tan θ)= 0~~ $(\tan\theta+2)^2 = 5$ $\tan\theta = 2\pm\sqrt{5}$ ~~but θ.~~ but $\frac{\pi}{2} < \theta < \pi$ $\therefore \tan\theta < 0$ $\therefore \tan\theta = 2-\sqrt{5}$

Exemplar 4 0 marks

6(ii)	$\cancel{\ }\ 2\theta < \cancel{\pi}$ $\tan 2\theta = 1/2$ $\frac{\pi}{2} < \theta < \pi$ $2\theta = \arctan 1/2$ $= 0.46365$ $[x+\pi]$ $= 3.6052$ $\theta = 1.8026^c$ $= 103.28°$ degrees $= 103\ \cancel{.28}°$ radians $= 1.80^c$

Examiner commentary

Many candidates did not read the question carefully and, as in Exemplar 4, began their response by writing $2\theta = \tan^{-1}\left(\frac{1}{2}\right) \Rightarrow \theta = \ldots$ even though the question specifically asked for the exact value of tanθ. Of those candidates that used the correct double-angle formula for tan2θ many derived the correct three-term quadratic in tan with most correctly stating that tanθ $-2\pm\sqrt{5}$. However, a significant proportion ended their response here and did not go on to determine the exact value of tanθ given that θ is an obtuse angle, as in Exemplar 2. A full solution needed the explicit realisation that since $-2+\sqrt{5}>0$, $\tan\theta = -2+\sqrt{5}$, would not give an obtuse angle and therefore the only valid solution was tan$\theta = -2-\sqrt{5}$, a model answer seen in Exemplar 1.

Careless manipulation of algebra terms, such as in Exemplar 3, prevented a significant minority of candidates gaining full credit.

Question 7

7 The gradient of the curve $y = f(x)$ is given by the differential equation

$$(2x-1)^3\frac{dy}{dx}+4y^2=0$$

and the curve passes through the point $(1,\ 1)$. By solving this differential equation show that

$$f(x)=\frac{ax^2-ax+1}{bx^2-bx+1},$$

where a and b are integers to be determined. **[9]**

Exemplar 1

9 marks

7

$(2x-1)^3\frac{dy}{dx} = -4y^2$

$\int \frac{dy}{-4y^2} = \int \frac{dx}{(2x-1)^3}$

$-\frac{1}{4}\int y^{-2}\,dy = \int (2x-1)^{-3}\,dx$

$-\frac{1}{4}(-y^{-1}) = \frac{(2x-1)^{-2}}{(-2)(2)} + c$

$\frac{1}{4y} = \frac{1}{-4(2x-1)^2} + c$

When $y=1$, $x=1$ $\therefore \frac{1}{4(1)} = \frac{1}{-4(2(1)-1)^2} + c$

$\frac{1}{4} = \frac{1}{-4} + c \qquad c = \frac{1}{2}$

$\therefore \frac{1}{4y} = \frac{1}{-4(2x-1)^2} + \frac{1}{2}$

$\frac{1}{y} = 2 - \frac{1}{(2x-1)^2}$

$\frac{1}{y} = \frac{2(2x-1)^2-1}{4x^2-4x+1}$

$y = \frac{4x^2-4x+1}{2(4x^2-4x+1)-1}$

$y = \frac{4x^2-4x+1}{8x^2-8x+2-1}$

$y = \frac{4x^2-4x+1}{8x^2-8x+1}$

Exemplar 2

7 marks

7 $(2x-1)^3 \frac{dy}{dx} + 4y^2 = 0 \quad (1,1)$

$\cancel{dy} \int \cancel{(2x-1)^3}$

$\int 4y^2\,dy = \int \frac{1}{(2x-1)^3}\,dx \qquad \int (2x-1)^{-3} = \frac{(2x-1)^{-2}}{-2\times 2}$

$\frac{4}{3}y^3 = -\frac{1}{4(2x-1)^2} + c \qquad 4y^3 = -\frac{3}{4(2x-1)^2} + c$

$y^3 = -\frac{3}{16(2x-1)^2} + c$

$\frac{4}{3} = -\frac{1}{4} + c \qquad c = \frac{19}{12} \qquad y = \sqrt[3]{\frac{3}{16(2x-1)^2}} + c$

$1 = \sqrt[3]{\frac{3}{16}} + c$

$\frac{4}{3}y^3 = -\frac{1}{4(2x-1)^2} + \frac{19}{12} \qquad 1 = -\frac{3}{16} + c \qquad c = \frac{19}{16}$

$\cancel{16y^2 = -\frac{3}{(2x-1)^2}} + 19$

$\cancel{16y^2(2x-1)^2 = -3 + 19(2x-1)^2}$

$\cancel{16y^2(2x-1)^2 = -3 + 19(4x^2-4x+1)}$

$\cancel{16y^2(2x-1)^2 = -3 + 76x^2 - 76x + 19}$

$\cancel{16y(2x-1)^2 = 76x^2 - 76x + 16}$

$\cancel{16y} = y(2x-1)^2 = \frac{19}{4}x^2 - \frac{19}{4}x + 1$

$\cancel{y = \frac{\frac{19}{4}x^2 - \frac{19}{4}x + 1}{4x^2 - 4x + 1}}$

$\cancel{64y^3(2x-1)^2 = -12 + 57(4x^2-4x+1)}$

$\cancel{64y^3(2x-1)^2 = -12 + 228x^2 - 228x + 57}$

$\cancel{64y^3(2x-1)^2 = 228x^2 - 228x + 45}$

$\cancel{\frac{4}{3}y^3 = -\frac{1}{4(2x-1)^2} + \frac{19}{16}}$

$\cancel{\frac{64}{3}y^3 = -\frac{4}{(2x-1)^2} + 19}$

$\cancel{64y^3 = -\frac{12}{(2x-1)^2} + 57}$

$\cancel{64y^3(2x-1)^2 = -12 + 57(2x-1)^2}$

Exemplar 3

6 marks

7

$\frac{dy}{dx} = -\frac{4y^2}{(2x-1)^3}$ $\qquad \frac{1}{(2x-1)^3}$ $\qquad u = 2x-1$

$y^{-2} \quad (2x-1)$

$-\frac{1}{4y^2}\,dy = \frac{1}{(2x-1)^3}\,dx$

$\int -\frac{1}{4y^2}\,dy = \int \frac{1}{(2x-1)^3}\,dx$ M1

$= \frac{1}{4}\,y^{-2} = (2x-1)^{-3}\,dx$

$\frac{1}{4}y^{-1} = \frac{1}{2}(2x-1)^{-2} + C$ A1 M1 A0

$\frac{1}{4y} = \frac{1}{2(2x-1)^2} + C$

when $x=1$ $y=1$ M1 $C = -\frac{1}{4}$

I4 throw

$\frac{1}{4} = \frac{1}{2} + C$

$\frac{1}{4y} = \frac{1}{2(2x-1)^2} - \frac{1}{4}$ A0

$\frac{1}{4y} = \frac{2}{4(2x-1)^2} - \frac{(2x-1)^2}{4(2x-1)^2}$

$\frac{1}{4y} = \frac{2-(2x-1)^2}{4(2x-1)^2}$ M1

M1

$4y = \frac{4(2x-1)^2}{2-(2x-1)^2}$

Exemplar 4

4 marks

7

$u = 4\!\!\!/\,2y$

$y = f(x)$ is given by $v = 2x-1$

$(2x-1)^3 \frac{dy}{dx} + 4y^2 = 0$ the curve passes through $(1,1)$.

$\therefore (2x-1)^3 \frac{dy}{dx} = -4y^2$

$(2x-1)^3 \cancel{= -4y} \frac{1}{dx} = -4y^2 \frac{1}{dy}$

$\int -\frac{1}{4y^2}\, dy = \int \frac{1}{(2x-1)^3}\, dx$ M1

$\int -(4y^2)^{-1}\, dy = \int \cancel{\frac{1}{(2x}} \, (2x-1)^{-3}\, dx$

$\int -(\cancel{4}u)^{-2}\, dy = \int v^{-3}\, dx$

$\frac{dv}{dx} = 2 \qquad dx = \frac{1}{2}\, dv$

$\frac{du}{dx} = 2 \qquad dx = \frac{1}{2}\, du$

$\int -2u^{-2}\, dy = \int \frac{1}{2} v^{-3}\, dx$

$2u^{-1} = \cancel{-v^{-2}+c} - \frac{1}{4} v^{-2} + c$

$\frac{2}{\cancel{u}} \cancel{\ldots}\ 2(2y)^{-1} = -\frac{1}{4}(2x-1)^{-2} + c$ A1 BOD M1 A1

7 **(continued)**

$\frac{2}{2y} = -\frac{1}{4(2x-1)^2} + c$

$\frac{1}{y} = -\frac{1}{4(2x-1)^2} + c$

$y = 4(2x-1)^2 + c$ M0

$Y = \cancel{\tfrac{1}{2}}\, 4(4x^2 - 4x + 1) + c$

$Y = 16x^2 - 16x + 4 + c$

$1 = 16 - 16 + \cancel{4} + c$

$1 = \cancel{4} + c$

$\cancel{c = 1 - 4 = -3}$ $c = 1$

$Y = 16x^2 - 16x + 1$

$\cancel{32x^2 - 32}$

$\cancel{\therefore Y =}$

$Y = \dfrac{ax^2 - ax + 1}{bx^2 - bx + 1} = \dfrac{16x^2 - 16x + 1}{0 \times b - 0 \times b + 1} = \dfrac{16x^2 - 16x + 1}{1}$

where $a = 16$ and $b = 0$.

Exemplar 5

4 marks

7

$(2x-1)^3 \frac{dy}{dx} = -4y^2$

$\int \frac{1}{4y^2}\,dy = \int \frac{1}{(2x-1)^3}\,dx$ M1

A1 $-\frac{1}{4y} = \int \frac{1}{(2x-1)^3}\,dx$

$u = 2x-1 \qquad \frac{u+1}{2} = x \qquad \frac{dx}{du} = \frac{1}{2}$

$dx = \frac{1}{2}\,du$

$\int \frac{1}{2u^3}\,du = \frac{1}{-u^2}$

$-\frac{1}{4y} = \frac{1}{-(2x-1)^2} + c$ M1 A0

$(1,1) \qquad -\frac{1}{4} = -1 + c$ M1 $c = \frac{3}{4}$

$-\frac{1}{4y} = \frac{1}{-(2x-1)^2} + \frac{3}{4}$ A0

$4y = (2x-1)^2 - \frac{4}{3}$ M0 M0

$y = \frac{(2x-1)^2 - \frac{4}{3}}{4}$

(answer space continued on next page)

Examiner commentary

The responses to this final question in the pure section were mixed with examiners reporting a mixture of excellent responses followed by those that struggled with both the integration and the resulting algebraic manipulation required to obtain the answer in the required form.

Exemplar 1 shows a fully correct solution.

While most correctly separated the variables and wrote $-\frac{1}{4}\int\frac{dy}{y^2}=\int\frac{dx}{(2x-1)^3}$ many candidates had issues with the placement of the fraction on the left-hand side, in Exemplar 2 this became $\int 4y^2$, with examiners reporting that others frequently had $-4\int\frac{dy}{y^2}=\int\frac{dx}{(2x-1)^3}$.

Some candidates made careless errors that cost accuracy marks.

In Exemplar 3 a careless omission of a negative sign, where $\frac{1}{2}(2x-1)^{-2}$ should have been $\frac{1}{-2}(2x-1)^{-2}$.

Exemplar 4, shows a careless application of notation when substitutions used, which led to further errors.

Exemplar 5 shows another careless omission in the denominator after integrating using substitution.

Question 8(i)

8 In this question $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ denote unit vectors which are horizontal and vertically upwards respectively.

A particle of mass 5 kg, initially at rest at the point with position vector $\begin{pmatrix}2\\45\end{pmatrix}$ m, is acted on by gravity and also by two forces $\begin{pmatrix}15\\-8\end{pmatrix}$ N and $\begin{pmatrix}-7\\-2\end{pmatrix}$ N.

(i) Find the acceleration vector of the particle. **[3]**

Exemplar 1 — 3 marks

8(i)

Gravitational force = mass × $\begin{pmatrix}0\\9.8\end{pmatrix}$ = $5 \times \begin{pmatrix}0\\-9.8\end{pmatrix} = \begin{pmatrix}0\\-49\end{pmatrix}$

Resultant force = $\begin{pmatrix}15\\-8\end{pmatrix} + \begin{pmatrix}-7\\-2\end{pmatrix} + \begin{pmatrix}0\\-49\end{pmatrix} = \begin{pmatrix}8\\-59\end{pmatrix}$

~~Reco~~ $F = ma$ $\quad \therefore a = \frac{F}{m}$

∴ Acceleration of the particle = Resultant Force ÷ mass

$\therefore \underline{a} = \begin{pmatrix}8\\-59\end{pmatrix} \div 5$

$\underline{a} = \begin{pmatrix}1.6\\-11.8\end{pmatrix}$

Exemplar 2 — 3 marks

8(i)

$F = ma \qquad a = \begin{pmatrix}3\\-\frac{8}{5}\end{pmatrix} + \begin{pmatrix}-\frac{7}{5}\\-\frac{2}{5}\end{pmatrix} + \begin{pmatrix}0\\-\frac{49}{5}\end{pmatrix}$

$a = \begin{pmatrix}\frac{8}{5}\\-\frac{59}{5}\end{pmatrix} ms^{-2}$

Exemplar 3

2 marks

8(i) $N,II(\rightarrow): 15-7=5a_x \qquad a_x = \frac{8}{5} = 1.6$

M1

$N,II(\uparrow): -8-2-9.8 = 5a_y \qquad a_y = \frac{-99}{25} = -3.96$

B1

A0

$\underline{a} = \begin{pmatrix} 1.6 \\ -3.96 \end{pmatrix} ms^{-2}$

Exemplar 4

1 mark

8 (i) $F = ma$

$\begin{pmatrix} 8 \\ -20 \end{pmatrix} = 5\begin{pmatrix} \underline{a} \end{pmatrix} \qquad \underline{a} = \begin{pmatrix} 8 \\ -20 \end{pmatrix} \div 5 \qquad \underline{a} = \begin{pmatrix} 8/5 \\ -4 \end{pmatrix}$

B0 M1 BOD A0

Exemplar 5

1 mark

8(i) $\begin{pmatrix} 15 \\ -8 \end{pmatrix} + \begin{pmatrix} -7 \\ -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 5g \end{pmatrix} = 5a$ M1

B0

$\begin{pmatrix} 8 \\ 39 \end{pmatrix} = 5a$

$\begin{pmatrix} 8/5 \\ 9.8 \end{pmatrix} = a \qquad \underline{a} = \begin{pmatrix} 1.6 \\ 9.8 \end{pmatrix} ms^{-2}$

A0

Examiner commentary

Exemplar 1 and 2 show clear solutions with full credit awarded.

A significant number of candidates wrote that $\begin{pmatrix} 15 \\ -8 \end{pmatrix} + \begin{pmatrix} -7 \\ -2 \end{pmatrix} + \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} = 5\mathbf{a}$ therefore forgetting to multiply the gravity term by the mass of the particle, as in Exemplar 3. Others forgot to include force due to gravity, as in Exemplar 4. Exemplar 5 highlights a common issue of not taking into consideration the direction of each force.

Question 8(ii)

(ii) Find the position vector of the particle after 10 seconds. [3]

Exemplar 1 — 3 marks

8(ii)

$v = \begin{pmatrix} \frac{8}{5}t \\ -\frac{59}{5}t \end{pmatrix} + c \quad ms^{-1}$ when $t=0$ $v=0$ $v = \begin{pmatrix} \frac{8}{5}t \\ -\frac{59}{5}t \end{pmatrix}$

$s = \begin{pmatrix} \frac{4}{5}t^2 \\ -\frac{59}{10}t^2 \end{pmatrix} + c \quad m$ when $t=0$ $s = \begin{pmatrix} 2 \\ 45 \end{pmatrix}$

$s = \begin{pmatrix} \frac{4}{5}t^2 + 2 \\ -\frac{59}{10}t^2 + 45 \end{pmatrix} m$ $t=10$ $s = \begin{pmatrix} 82 \\ -545 \end{pmatrix} m$

Exemplar 2 — 2 marks

8(ii)

$a = \frac{d^2s}{dt^2} \Rightarrow \iint a \, dt \, dt = s + c$

$\begin{pmatrix} 1.6 \\ -3.96 \end{pmatrix}$

$v = \begin{pmatrix} 1.6t \\ -3.96t \end{pmatrix}$ $s = \frac{1}{2}\begin{pmatrix} 1.6t^2 \\ -3.96t^2 \end{pmatrix} + \begin{pmatrix} 2 \\ 45 \end{pmatrix}$ M1 A1 FT

$t=10$ $s = \begin{pmatrix} 82 \\ -153 \end{pmatrix}$ A0

Exemplar 3 1 mark

8(ii)

$$s = ut + \frac{1}{2}at^2$$

$$= \frac{1}{2}\begin{pmatrix}1.6\\-11.8\end{pmatrix}100 = \begin{pmatrix}80\\-2360\end{pmatrix}$$

$$\begin{pmatrix}80\\-2360\end{pmatrix} + \begin{pmatrix}2\\45\end{pmatrix} = \begin{pmatrix}82\\-2315\end{pmatrix}$$

Examiner commentary

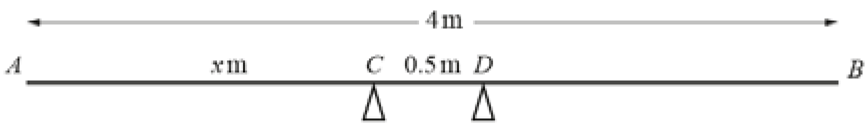
The majority of candidates correctly applied the vector form of $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ in an attempt to find the position vector of the particle after 10 seconds, as in Exemplar 1.

Generally, only the errors mentioned in part (i) that stopped candidates from scoring full marks in this part, as in Exemplar 2.

There were a small minority that had problems using vector notation, as in Exemplar 3.

Question 9(i)

9 A uniform plank AB has weight 100 N and length 4 m. The plank rests horizontally in equilibrium on two smooth supports C and D, where $AC = x$ m and $CD = 0.5$ m (see diagram).



The magnitude of the reaction of the support on the plank at C is 75 N. Modelling the plank as a rigid rod, find

(i) the magnitude of the reaction of the support on the plank at D, **[1]**

Exemplar 1 — 1 mark

9(i)	Sum↑ = Sum↓ ⇒ $100 = 75 + F$
	$F = 25$

Examiner commentary

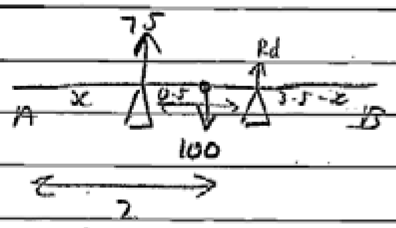
Nearly all candidates correctly stated that the magnitude of the reaction of the support on the plank at D was 25 N. No specific command word has been used to explicitly require any working, although a clear concise mathematical argument should always be encouraged.

Question 9(ii)

(ii) the value of x. [3]

Exemplar 1 — 3 marks

9(ii)



$\overset{\curvearrowleft}{A}: 2 \times 100 = 75x + (x + 0.5)25$

$200 = 75x + 25x + 12.5$

$187.5 = 100x$

$1.875\text{m} = x$

~~R(↑) 75 + Rd = 100~~

~~Rd = 25 N.~~

Exemplar 2 — 3 marks

9(ii)

$\overset{\curvearrowleft}{M}(C): (2 - x)(100) = 0.5(25)$

$200 - 100x = 12.5$

$\frac{200 - 12.5}{100} = x$ ~~$x =$~~

$x = 1.875\text{ m}$

Exemplar 3 — 0 marks

9(ii)

~~[illegible]~~ $m = f \times d.$

~~[illegible]~~ $75 = 100(2 - x)$

M0

~~[illegible]~~ $75 = 200 - 100x$

~~[illegible]~~ $100x = 125$

$x = 1.25\text{m}$

Exemplar 4 — 0 marks

9(ii)

Moments around D

$x + 0.54(3.5 - x)$

$4 - x\ (100) = x$

$75 \times 0.5 = 35 - x$

$37.5 = 35 - x$

$|x| = 2.5\text{m}$

$\cancel{75x = 4x(25)}$

$\cancel{75x = 4 - 75x}$

$\cancel{100x = 4 \quad x = 25}$

Exemplar 5 — 0 marks

9(ii)

$C = (2 - x) \times 100$

$\therefore C = 200 - 100x$

$75 = 200 - 100x$ M0

$100x = 125$

$x = 125 \div 100$

$x = 1.25\text{m}$

Examiner commentary

The most common point on the plank to take moment about in this part was *A* and most who did so were successful in finding *x*, as in Exemplar 1. Of those that considered moments about a different point, while some were successful, as in Exemplar 2, too often examiners noted that candidates did not include all the required terms, such as in Exemplar 3. Exemplar 4 shows an attempt to take moments about D, however the right hand side is not clearly a moment of force times distance. A similar issue with units is seen in Exemplar 5; here the candidate has confused a force at C with the moment of the force at C about an point (which has not been explicitly defined).

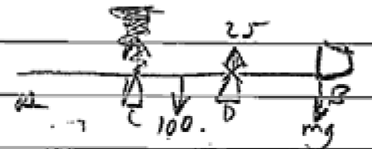
Question 9(iii)

A stone block, which is modelled as a particle, is now placed at the end of the plank at B and the plank is on the point of tilting about D.

(iii) Find the weight of the stone block. **[3]**

Exemplar 1 — 3 marks

9(iii)



$D\circlearrowleft$: $1.625mg$ ~~...~~ $= 0.375 \times 100$

$1.625mg = 37.5$

$mg = \frac{300}{13}$ N

Exemplar 2 — 0 marks

9(iii)

moments about B: A   M0

CW: $W \times \frac{13}{8}$ ACW: 0.5×7.5

$100 \times \frac{3}{2}$ $\frac{13}{8}W + \frac{47.5}{8} = 20 \ldots \frac{1125}{8}$

$W \times \frac{13}{8} + 12\ \frac{7.5}{2} = \frac{7.5}{2}$ $W = 173$ N

CW: $W \times \frac{13}{8}$ ACW: 2×100

$\frac{19}{8} \times 2.5$ $7.5 \times \frac{15}{2}$

Exemplar 3 — 1 mark

9(iii)

$R_C = 0$ $R_D = 100 + W$ W = weight of block

$-x = 0.125$ m from C - centre of mass

$0.125 \times 100 = 1.625(W)$ M1   A0

$W = 7.69$ N $= \frac{100}{13}$ N   A0

Exemplar 4 — 0 marks

9(iii)

$M(D)$: $W = 100(0.375) = 37.5$ N   M0

Exemplar 5 0 marks

9(iii)	
	$\cancel{D}$ D to B = 4 − 1.25 − 0.5 = 2.25 m
	C = D + 2.25W
	$\cancel{100}$ $\cancel{75}$ 75 = 25 + 2.25W M0
	2.25W = 50
	W = 22.23 N

Examiner commentary

Many candidates struggled with this part and did not realise that if the plank was about to tilt about *D* then this was the point to take moments about as the reaction at *C* is therefore zero in this limiting case. Candidates are strongly advised when taking moments to make it clear to the examiners which point (in this case on the plank) they are taking moments about, as seen in Exemplar 1 where a helpful diagram has been sketched.

A number of candidates attempted to take moments about another point together with resolve forces vertically; these attempts were usually unsuccessful, as in Exemplar 2. There were also those candidates that mixed forces and moments, such as in exemplars 3, 4 and 5.

Question 9(iv)(a)

(iv) Explain the limitation of modelling

(a) the stone block as a particle, [1]

Exemplar 1 1 mark

9(iv)(a)	The stone's weight may not actually act perfectly on the end of the plank. B1 BOD

Exemplar 2 1 mark

9(iv)(a)	As a particle, it acts through a single point which doesn't account for any imbalances in its centre of mass

Exemplar 3 0 marks

9(iv)(a)	Block can't be a particle as may be too large and have friction if modelled as a particle it doesn't account for this.

Exemplar 4 0 marks

9(iv)(a)	~~Assuming no width which cannot be true~~ Assuming the stone is not affected by air resistance, which is highly unlikely.

Examiner commentary

It was pleasing that candidates were well prepared for this type of question with many correctly stating that the modelling the stone as a particle assumes that the weight of the block acts exactly at *B*, and therefore the block's dimensions (or the distribution of the mass of the block) has not been taken into account, exemplars 1 and 2. A number of candidates incorrectly stated that modelling the block as a particle implied that the block had negligible mass, or mistakenly introduced issues of friction or air resistance, as in Exemplar 3 and 4, which are unrelated to the modelling of a particle in a statics problem.

Question 9(iv)(b)

(b) the plank as a rigid rod. **[1]**

Exemplar 1 — 1 mark

9(iv)(b)	The plank may actually bend, which this model doesn't take into account.

Exemplar 2 — 1 mark

9(iv)(b)	The ~~Rod~~ plank would bend, causing tension which absorbs some energy.

Exemplar 3 — 0 marks

9(iv)(b)	The rod ~~ca~~ potentially slide off pivot.

Exemplar 4 — 0 marks

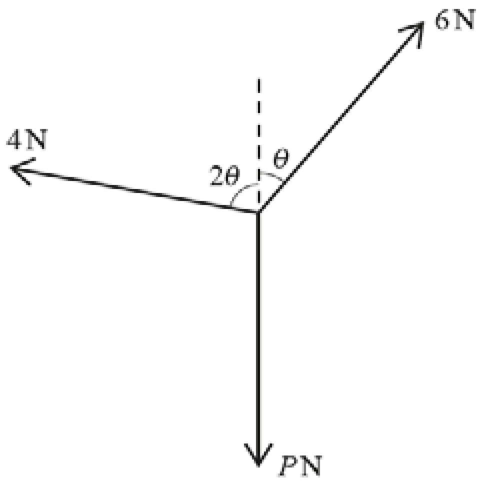
9(iv)(b)	Friction of plank might effect to result.

Examiner commentary

This part was also answered extremely well, with many correctly stating that modelling the block as a rigid rod assumes that the plank remains in a straight line and does not bend, Exemplar 1. Exemplar 2 takes this further into considering energy from Further Maths/Physics content, going beyond the A Level Maths specification is fine, but care should be taken not to get into concepts that may not be fully understood. A number of candidates, however, assumed that friction and/or air resistance had some relevance to this part, care needs to be taken to focus the answer in the context of the question. Whilst the idea of the rod sliding may well be true, as mentioned in Exemplar 3, this is not appropriate in the context of the plank being rigid. Similarly, whilst friction would have an impact on whether the rod slipped or pivoted, as mentioned in Exemplar 4, but this is not related to the rigid rod model.

Question 10(i)

10 Three forces, of magnitudes 4 N, 6 N and P N, act at a point in the directions shown in the diagram.



The forces are in equilibrium.

(i) Show that $\theta = 41.4°$, correct to 3 significant figures. **[4]**

Exemplar 1 — 4 marks

10(i)

$R(\rightarrow)$: $4\sin 2\theta = 6\sin\theta$

$8\sin\theta\cos\theta = 6\sin\theta$

$\cos\theta = \frac{6}{8} = \frac{3}{4}$

$\theta = 41.4°$ (3s.f.)

Exemplar 2 — 4 marks

10(i)

Horizontal $6\sin\theta + 4\sin 2\theta$

Vertical $6\cos\theta + 4\cos 2\theta - P$

$6\sin\theta = 4\sin 2\theta$

$6\sin\theta = 8\cos\theta\sin\theta$

$0 = 2\sin\theta(4\cos\theta - 3)$

$\sin\theta = 0$ or $\arcsin 0 = 0°, 180°, 360°\ldots$

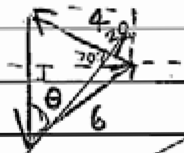
$\cos\theta = \frac{3}{4}$ $\arccos\frac{3}{4} = 41.40962211\ldots°$

$\therefore \theta = 41.4°$

Exemplar 3

1 mark

10(i)



- - - - - - - $= 4\cos 70$

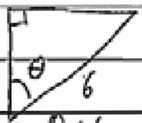
$\frac{\sin\theta}{4\cos 70} = \frac{\sin 90}{6}$

$\sin\theta = \left(\frac{\sin 90}{6} \times 4\cos 70\right)$

$\theta = \sin^{-1}\left(\frac{\sin 90}{6}\right) \times 4\cos 70)$

$\theta =$

$\frac{4\sin 20}{4\cos 70}$



S O H

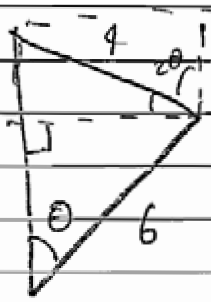
$\sin\theta = \frac{4\cos 70}{6}$

$\theta = \sin^{-1}\left(\frac{4\cos 70}{6}\right)$: $\theta =$

continued in extra space.

---

(10) i)

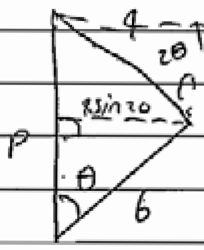


- - - - $= 4\sin 2\theta$ $\sin 2\theta = \sin^2\theta + \cos^2\theta$

$\frac{\sin\theta}{4\sin 2\theta} = \frac{\sin 90}{6}$ B1

$\sin\theta = \frac{1}{6} \times 4\sin 2\theta$

$\sin\theta = \frac{2}{3}(\sin^2\theta + \cos^2\theta)$ M0



$4\sin 2\theta$

~~$\frac{4\sin\theta}{70}$ =~~

Exemplar 4 — 0 marks

10(i)	$\cancel{4\cos 2\theta = 6\sin\theta}$ $\quad 4\cos 2\theta = 6\sin\theta$ B0
	$\cancel{6\sin\theta - 4\cos 2\theta = 0}$ $\quad 4(\cos 2\theta) = 6\sin\theta$
	$\cancel{3\sin\theta - 2\cos 2\theta = 0}$ $\quad 4(1 - \sin^2\theta) = 6\sin\theta$ M0
	$4 - 4\sin^2\theta = 6\sin\theta$
	$4\sin^2\theta + 6\sin\theta - 4 = 0$ $\quad \dfrac{\sin\theta}{6\sin\theta} = \dfrac{\sin 90}{6}$
	$\sin\theta = \frac{1}{2}$, $\cancel{\sin\theta = -2}$
	$\theta = 30°, 150°$ $\quad \sin\theta = \dfrac{6\sin\theta}{6}$
	$\cancel{4\sin^2\theta + 6\sin\theta - 4}$ $\quad \cancel{\sin\theta = 4}$ $\quad 4\sin\theta = -2$
	$\sin\theta(4\sin\theta + 6) = 4$ $\quad \sin\theta = -\frac{1}{2}$

Examiner commentary

For those candidates who correctly resolve horizontally and stated that $4\sin 2\theta = 6\sin\theta$ most went on to show with sufficient detail the required given answer, Exemplar 1. The most common errors, as expected, were those that did not deal correctly with the two different angles, Exemplar 3 or those that had the standard confusion with sine and cosine when resolving forces, Exemplar 4.

Question 10(ii)

(ii) Hence find the value of P. **[2]**

Exemplar 1 — 2 marks

10(ii)	R(↑): $P = 4\cos 2\theta + 6\cos\theta = 5N$.

Exemplar 2 — 1 mark

10(ii)	$P = 4\sin(82.8) + 6\cos(41.4)$ M1
	$P = 8.47N$ (3sf)

Examiner commentary

This part was answered extremely well with the majority of candidates correctly resolving vertically and obtaining $P = 4\cos 2\theta + 6\cos\theta \Rightarrow P = 5$, Exemplar 1. Exemplar 2 highlights the importance of showing working, the method is correct but for some reason the calculator was not used correctly.

Question 10(iii)(a)

The force of magnitude 4 N is now removed and the force of magnitude 6 N is replaced by a force of magnitude 3 N acting in the same direction.

(iii) Find

(a) the magnitude of the resultant of the two remaining forces. **[3]**

Exemplar 1 — 3 marks

10(iii)(a)

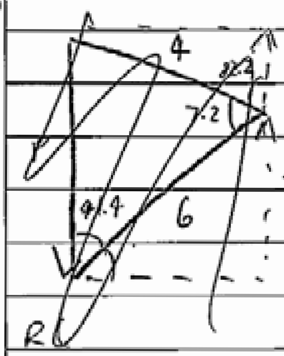
horizontally: $F_x = 3\sin\theta = 1.98\ldots N$

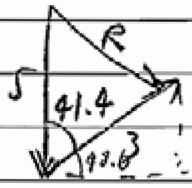
vertically: $F_y = \cancel{4\cos\theta}\ 5 - 3\cos\theta = \frac{11}{4} N$

$\therefore$ magnitude $= \sqrt{(3\sin\theta)^2 + \left(\frac{11}{4}\right)^2} = 3.39 N$ (3 s.f.)

Exemplar 2 — 3 marks

10(iii)(a)





~~R(↓): 5 − 4~~

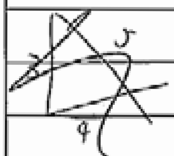
$R^2 = 5^2 + 3^2 - 2bc\cos A$

$a^2 = b^2 + c^2 - 2bc\cos A$

$R^2 = 34 - 30\cos 41.4$

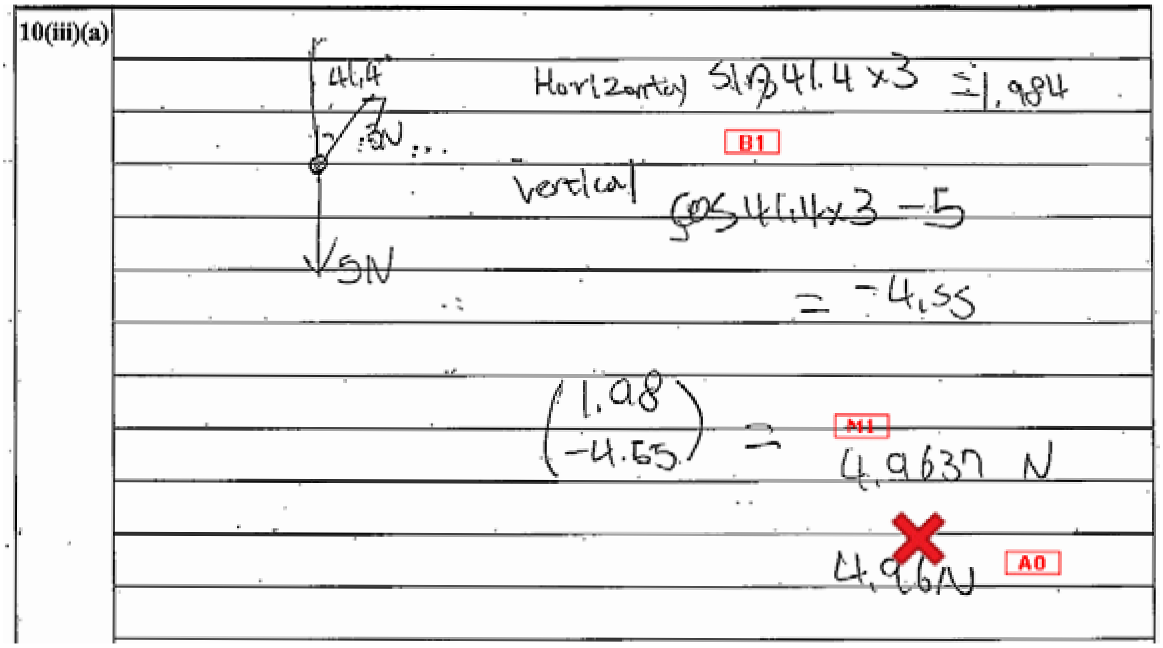
$R^2 = 11.497$

$R = 3.39 N$



Exemplar 3

2 marks



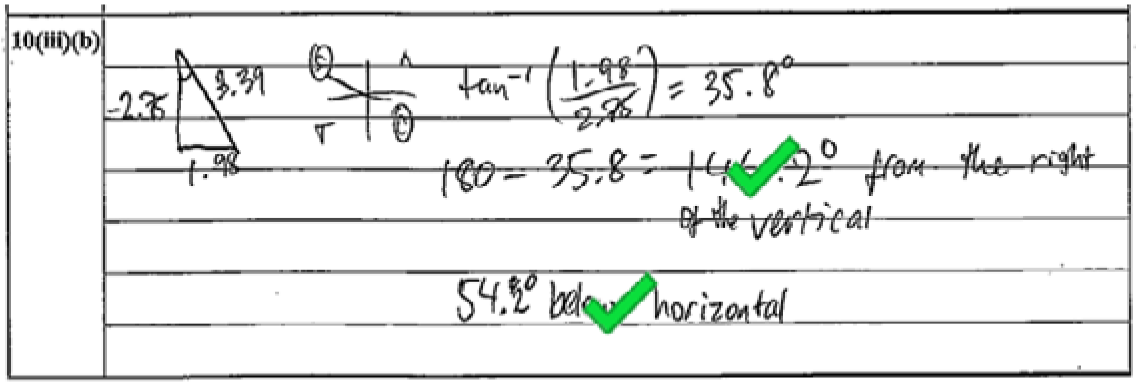
Examiner commentary

While nearly all candidates understood the method for finding the magnitude of the resultant of the two remaining forces it was generally those candidates who resolved horizontally and vertically and then applied Pythagoras who were the more successful, Exemplar 1. Examiners noted that those candidates who attempted to form a triangle of forces and then use the cosine rule were generally not as successful making errors with the correct placement of the required lengths (representing the magnitudes) on the sides of the triangle, Exemplar 2 shows this method carried through to a successful conclusion. Careless data entry was responsible for some of the dropped marks, such as in Exemplar 3.

Question 10(iii)(b)

(b) the direction of the resultant of the two remaining forces. **[2]**

Exemplar 1 — 2 marks



10(iii)(b)

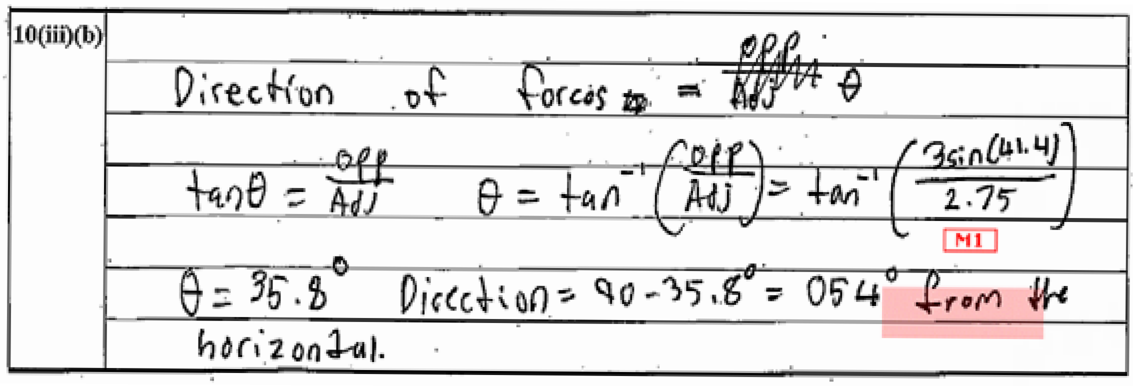
2.75, 3.39, 1.98

$\tan^{-1}\left(\frac{1.98}{2.75}\right) = 35.8°$

~~$180 - 35.8 = 144.2°$ from the right of the vertical~~

$54.2°$ below horizontal

Exemplar 2 — 1 mark



10(iii)(b)

Direction of forces = $\frac{Opp}{Adj}$ θ

$\tan\theta = \frac{Opp}{Adj}$ $\quad\theta = \tan^{-1}\left(\frac{Opp}{Adj}\right) = \tan^{-1}\left(\frac{3\sin(41.4)}{2.75}\right)$ M1

$\theta = 35.8°$ Direction $= 90 - 35.8° = 054°$ from the horizontal.

Examiner commentary

When giving the direction of a force candidates are reminded that giving just the angle is insufficient. Candidates are strongly advised to draw a diagram (with an arrow on the resultant and/or the components) and to state that their angle is either 'below/above the horizontal' or 'to the upward/downward vertical'. Exemplar 1 demonstrates a clear answer, whereas Exemplar 2 highlights that an ambiguous description of the direction will not gain credit.

Question 11(i)

11 The velocity $v\,\mathrm{m\,s^{-1}}$ of a car at time $t\,\mathrm{s}$, during the first 20 s of its journey, is given by $v = kt + 0.03t^2$, where k is a constant. When $t = 20$ the acceleration of the car is $1.3\,\mathrm{m\,s^{-2}}$. For $t > 20$ the car continues its journey with constant acceleration $1.3\,\mathrm{m\,s^{-2}}$ until its speed reaches $25\,\mathrm{m\,s^{-1}}$.

(i) Find the value of k. **[3]**

Exemplar 1 — 3 marks

11(i)	~~$a = k = 0.0$~~ $a = k + 0.06t$
	When $t = 20$, $a = 1.3$ ∴ $1.3 = k + 0.06(20)$ $k = 0.1$

Exemplar 2 — 2 marks

11(i)	$v = kt + 0.03t^2$
	$a = k + 0.06t$ $t = 20$ B1
	$1.3 = k + 0.06(20)$ $k = 1.08$ (3sf) M1 A0

Exemplar 3 — 0 marks

11(i)	~~$v = u + at$~~ $v = kt + 0.03t^2$ $\frac{v}{20} = 1.3$
	∴ ~~$kt = v$~~ and ~~$0.03t = a$~~
	$v = 20k + 12$ ~~$0.03t = 1.3$~~ $v = 26\ \mathrm{ms^{-1}}$
	~~$25 = 25k + 1.3$~~ $kt +$ 1.3
	$26 = 20k + 12$ $20k = 14$ $k = 0.7$

Examiner commentary

Nearly all candidates correctly differentiated the expression for *v* and correctly obtained the value of *k* as 0.1, Exemplar 1. Arithmetic or algebraic manipulation errors cost marks for a few candidates, Exemplar 2. A small minority used constant acceleration equation, Exemplar 3.

Question 11(ii)

(ii) Find the total distance the car has travelled when its speed reaches $25\,\mathrm{m\,s^{-1}}$. [7]

Exemplar 1

7 marks

11(ii)

First! $s = \frac{0.1}{2}t^2 + 0.01t^3 + c$

When $t=0$, $s=0$ ∴ $c=0$

$s = 0.05t^2 + 0.01t^3$

When $t=20$, $s = 0.05(20)^2 + 0.01(20)^3 = 100\,m$

~~Then~~ When $t=20$, $v = 0.1(20) + 0.03(400) = 14\,ms^{-1}$

Then: $u=14$, $a=1.3$, $v=25$; $v^2 = u^2 + 2as$

$s = \frac{v^2 - u^2}{2a} = \frac{25^2 - 14^2}{2(1.3)} = 165\,m$

$100 + 165 = 265$

Exemplar 2

6 marks

11(ii)

$s = 0.05t^2 + 0.01t^3$

$0 = \cancel{t}\,0.05t^2 + 0.01t^3$ $t=0$

$-0.05 = 0.01t$ $t = -5$ seconds.

$\int_0^{20} 0.1t + 0.03t^2\,dt = \left[0.05t^2 + 0.01t^3\right]_0^{20}$

$100 - 0 = 100\,m$

$v = 0.1(20) + 0.03(20)^2 = 14\,ms^{-1}$

$s = ?$
$u = 14$
$v = 25$
$a = 1.3$
$t = ?$

$v^2 = u^2 + 2as$

$625 = 196 + 2.3s$ M1 BOD

$s = 186.52\,m$

$100 + 186.52 = 286.52\,m$ A0

~~$v = u + at$~~

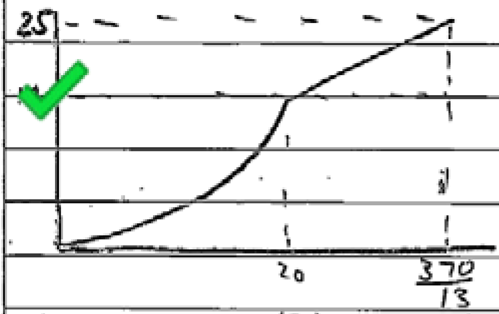
~~$25 = 14 + 1.3t$~~

Exemplar 3

5 marks

11(ii)

B1



$\frac{25-14}{t-20} = 1.3$

$11 = 1.3t - 26$

$37 = 1.3t$

$\frac{370}{13} = t$

First 20 seconds:

distance $=$ $\int_0^{20} 0.1t + 0.03t^2$ M1

$v = 0.1t + 0.03t^2$
when $t = 20$
$v = 14$ A1

$= \left[\frac{0.1t^2}{2} + \frac{0.03t^3}{3}\right]_0^{20}$

$= \left[0.05t^2 + 0.01t^3\right]_0^{20}$

$= [20 + 80] - [0]$ B1

$= 100\text{m.}$ M1

Area of triangle.

$\frac{1}{2}\left(11 \times \frac{370}{13} - 20\right)$

$= \frac{605}{13}$ M0

^

$100 + \frac{605}{13} = \frac{1905}{13}\text{ m.}$ A0

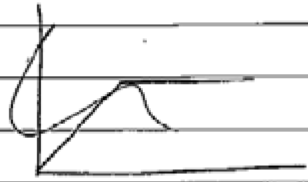
Exemplar 4 5 marks

11(ii)

~~$v = \frac{1}{10}t + 0.03t^2$~~

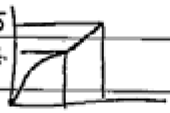
~~$25 = 0.1t + 0.03t^2$~~

~~$250 = t + 0.3t^2$~~



$0.3t^2 + t - 250 = 0$

~~$(t \quad)(t \quad) = 0$~~



25
14

$a = 1.3$

$\int a = v = 1.3t$

$25 = 1.3t \quad t = \frac{250}{13}$

$v = 2 + 12 = 14$

$\int_0^{20} \frac{1}{10}t + 0.03t^2 \, dt$

$\left[\frac{1}{20}t^2 + 0.01t^3\right]_0^{20} + \frac{1}{2} \times \frac{250}{13}(14 + 25)$

M1 A1 B1 M0 B1

$= 20 + 80 + 375$

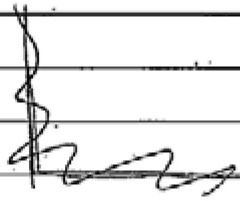
M1 A0

$= $ ~~475m~~ $\quad$ 475m

Exemplar 5 4 marks

11(ii)

$s = \int \frac{13}{12}t + 0.03t^2 \, dt$



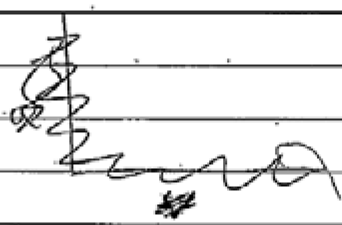
$s = \frac{13}{24}t^2 + 0.01t^3$

M1 A1 FT B0

when
$t = 20$

$\frac{13}{24}(20)^2 + 0.01(20^3) = \frac{890}{3}\text{m} \approx 296.7\text{m}$

M1



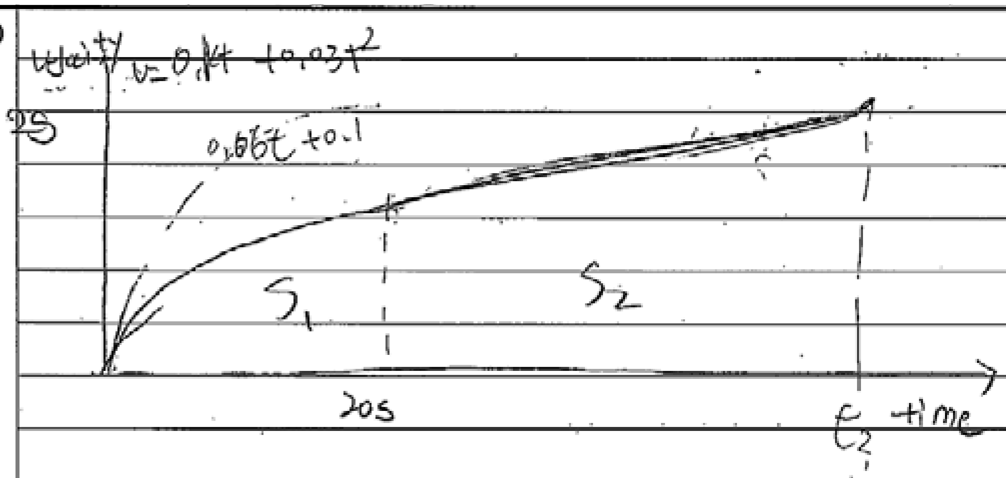
v when $t = 20$

$v = \frac{13}{12}(20) + 0.03(20)^2 = \frac{101}{3} \approx 33.7\text{ms}^{-1}$

B1 FT

Exemplar 6 2 marks

11(ii)



When $t=20$ $v = 2 + 0.03 \times 400$

$= 2 + 12 = 14$ ✓

B1

$14 = u$ $v = u + at$ t = time taken to get 25m/s

$25 = v$ $25 = 14 + 1.3t$

$t = 8.46$

$S_1 = ut + \frac{1}{2}at^2$

M0

$= 0 + \frac{1}{2}(0.06t + 0.1)t^2 = 260$ m

$S_2 = \left(\frac{u+v}{2}\right)t = \left(\frac{25+14}{2}\right) \times 8.46$ M1

$= 164.97$

$164.97 + 260 = 424.97$ m

Examiner commentary

This part was answered extremely well with many candidates correctly finding the total distance that car had travelled when its speed had reached 25 ms^{-1}. Many correctly realised that the problem had to be split into two calculations:

First use integration to find an expression for the displacement in terms of *t* which they could then use to find that the distance travelled by the car in the first 20 seconds was 100 m.

Then use the SUVAT equations to work out that the remaining distance travelled when the speed increased from 14 to 25 was 165 m.

Exemplar 1 shows a clear method and correctly calculated the total distance as 265 m.

Exemplar 2 shows a clear method, unfortunately a substitution error, followed by unrecorded manipulation and data entry errors resulted in an incorrect value for the second distance.

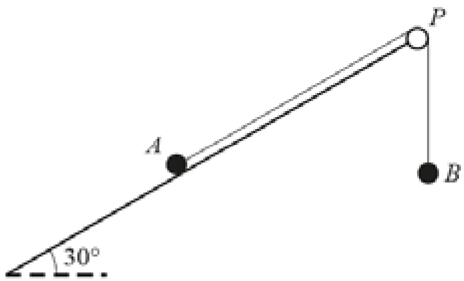
The method attempted in Exemplar 3 should have worked fine; this candidate ignored their own sketched graph and took the area under the graph for the second part of the journey to be found using a triangle, rather than the obvious trapezium (or triangle plus rectangle). Exemplar 4 also attempted the use the area under the curve method, here the mistake was to find the time the car the car would have taken to accelerate from rest to 25 ms^{-1}, rather than from 14 ms^{-1} to 25 ms^{-1}.

However, many ignored the constant of integration that would arise from the corresponding indefinite integral; even though this constant was zero it is mathematically incorrect to simply ignore it (and for full marks candidates either had to consider this displacement expressed as a definite integral or explain why the constant was zero). This is seen in Exemplar 5; other errors made by this candidate highlight the benefit of recording a clear mathematical argument that help the examiner award partial credit for method when mistakes have been made.

Exemplar 6 shows confusion about when to the SUVAT formulae, incorrectly attempting to use these to find the distance travelled during the variable acceleration section.

Question 12(i)(a)

12 One end of a light inextensible string is attached to a particle A of mass m kg. The other end of the string is attached to a second particle B of mass λm kg, where λ is a constant. Particle A is in contact with a rough plane inclined at 30° to the horizontal. The string is taut and passes over a small smooth pulley P at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane. The particle B hangs freely below P (see diagram).



The coefficient of friction between A and the plane is μ.

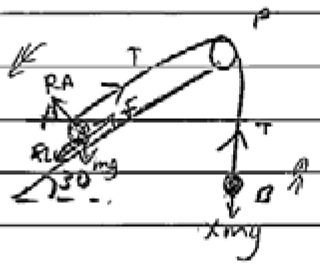
(i) It is given that A is on the point of moving down the plane.

(a) Find the exact value of μ when $\lambda = \frac{1}{4}$. **[7]**

Exemplar 1

7 marks

12(i)(a)



[Considering A].

R(↙ //) :

$mg\sin 30 = F + T$

R(⊥):

$R = mg\cos 30$

Consider B. (equilibrium)

$T = \frac{1}{4}mg$

$F = mg\sin 30 - \frac{1}{4}mg.$

$F = \mu R$

$mg\sin 30 - \frac{1}{4}mg = \mu\, mg\cos 30$

$\sin 30 - \frac{1}{4} = \mu\cos 30$

$\frac{\sin 30 - \frac{1}{4}}{\cos 30} = \mu$

$\frac{\sqrt{3}}{6} = \mu$

Exemplar 2

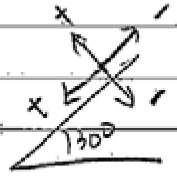
7 marks

12(i)(a)

force

Horizontal to plane

$mg\sin 30 - \mu\, mg\cos 30 - \frac{1}{4} mg = 0$



$m\cos 30$

$\frac{1}{2}m - \frac{\sqrt{3}}{2}\mu m - \frac{1}{4}m = 0$

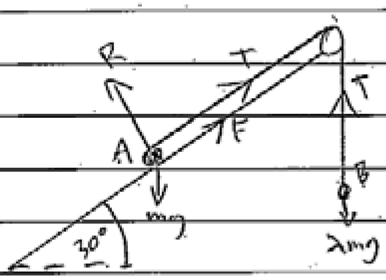
$\frac{1}{4} = \frac{\sqrt{3}}{2}\mu$

$\frac{\sqrt{3}}{6} = \mu$

Exemplar 3

6 marks

12(i)(a)



R(perpen plane): $R = mg\cos 30°$ B1 [1]

R(parallel plane): $F + T = mg\sin 30°$ [2] M1 A1

R(↑ B): $T = \frac{1}{4}mg$ B1 [3]

$F = \mu R$ [4]

[2] − [3]: $F = mg\sin 30° - \frac{1}{4}mg$

$F = mg\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{2}mg$ [5]

[1]: $R = \frac{\sqrt{3}}{2}mg$

[1] & [5] into [4]: $\frac{1}{2}mg = \mu\left(\frac{\sqrt{3}}{2}mg\right)$ M1 M1

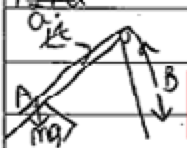
$\mu = \frac{\sqrt{3}}{3}$ A0

Exemplar 4

5 marks

12ia

~~12ia~~. $R = mg\cos 30 + 1.4 mg\cos(30)$



$1.4g$. MR B0

(A) $mg\sin 30 - t - (\mu mg\cos 30 + \cancel{1.4g\cos 30}\ 1.4g\cos 30) = ma$.

(B) $t - 1.4mg = 1.4ma$.

$mg\sin 30 - t - \mu mg\cos 30 = ma$. B0 M1 $t - 1.4mg = 1.4ma$.

$mg\sin 30 - \mu mg\cos 30 - ma = 1.4mg + 1.4ma$.

$g\sin 30 - \mu g\cos 30 - a = 1.4g + 1.4a$.

M1 A1 $4.9 - \mu g\cos 30 = 13.72 + 2.4a$. $a = 0$ as not moving.

$\mu g\cos 30 = -8.82$. M1 MR

$\mu = 9/5$ A0

Exemplar 5

3 marks

12(i)(a)

$\frac{1}{4}Mg$ ^ $F_r = \mu R$

$R = mg\cos 30°$ B1 $mg\sin 30 = mg\cos 30\,\mu$ M1

$F_r = mg\sin 30°$ $\mu = \tan 30$

$mg\sin 30 = \lambda mg$ $mg\sin 30 = \frac{1}{4}mg$

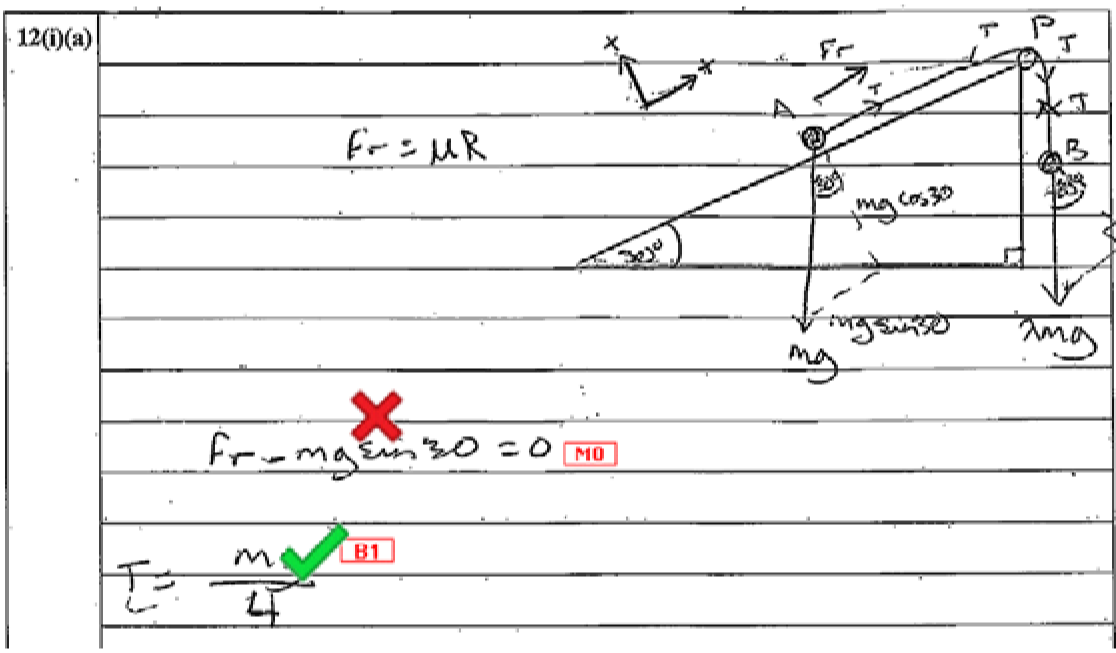
$\sin 30$ $mg\sin 30 - \frac{1}{4}mg = 0$

$F_r = mg\sin 30 + \lambda mg$ M1 A0 B0 ^ $mg(\sin 30 - \frac{1}{4}) = 0$

$mg = \cancel{0}$

^ M0 A0

Exemplar 6 — 1 mark



Examiner commentary

This question was answered extremely well with most candidates setting out their working clearly and explaining what they were doing at each step. Candidates are strongly advised to deal with each particle separately and many correctly resolved vertically for B, and resolved both parallel and perpendicular to the plane for *A* using $F \le \mu R$ to find the value of μ when $\lambda = \frac{1}{4}$, Exemplar 1.

Exemplar 2 shows a nice touch to avoid directional issues by defining clearly the positive direction in a sketch.

Exemplar 3 shows the benefits of setting out a clear solution, a careless arithmetic error can be spotted, meaning that only 1 accuracy mark was lost.

A common issue on mechanics papers has traditionally been with sine/cosine confusions, Exemplar 4 shows that this year was no different. Clear working means that the examiner can still award method marks.

Exemplar 5 highlights the issue of missed forces, a clear diagram can help avoid this, although Exemplar 6 shows that this is no guarantee.

Question 12(i)(b)

(b) Show that the value of λ must be less than $\frac{1}{2}$. **[2]**

Exemplar 1 — 2 marks

12(i)(b)

$R(\uparrow)$: $T = \lambda mg$ [1]

R(parallel plane): $T + F = \cancel{\frac{1}{2}mg}$ $mg\sin 30°$ [2]

[2]−[1]: $F = mg(\frac{1}{2} - \lambda)$ ✓

$mg = \frac{F}{\frac{1}{2} - \lambda}$

but $mg > 0$ & $F \geq 0$

$\therefore \frac{1}{2} - \lambda \geq 0$ $\lambda \leq \frac{1}{2}$

but $\frac{1}{2} - \lambda \neq 0$ ✓ $\therefore$ $\lambda < \frac{1}{2}$ BOD

Exemplar 2 — 1 mark

12(i)(b)

~~If λ is more than $\frac{1}{2}$~~ It move upword (by) friction. A0

$g m \sin 30 - \frac{\sqrt{3}}{6} g m \cos 30 - \frac{1}{2}mg$ M1 BOD

It doesn't make sense.

$= \frac{1}{2}m - \frac{1}{2}m - \frac{\sqrt{3}}{6} m \cos 30$

Two force canceled

Exemplar 3 — 1 mark

12(i)(b)

For the particle A to be moving

$mg\sin 30 > \mu mg\cos 30 + \lambda mg$

$\sin 30 > \frac{\cos 30}{2\sqrt{3}} + \lambda$ $\frac{1}{2} > \frac{1}{4} + \lambda$

$\Rightarrow$ $\frac{1}{2} - \frac{1}{4} > \lambda$ $\Rightarrow \lambda < \frac{1}{2}$

12 i b

For object to be moving $F_s \leftarrow\ > F_s \rightarrow$

$F_r = \mu R$ } as to movement $mg\sin 30 > F_r + \lambda mg$

$F_r = mg\mu\cos 30$

$mg\sin 30 > mg\mu\cos 30 + \lambda mg$ M1

$\sin 30 - \mu\cos 30 > \lambda$ if we neglect friction

$\lambda < \sin 30 - \frac{\sqrt{3}}{6}\cos 30$ $\lambda < \sin 30$ A0

$< \frac{1}{4}$ $\lambda < \frac{1}{2}$ as required

$\frac{1}{2}\mu + \mu\sin 30 - \cos 30$

$\mu\left(\frac{1}{2} - \sin 30\right) < \cos 30$

μ $\qquad \frac{1}{2}\mu$

$\sin 30 > \mu\cos 30 - \frac{1}{2}$

$\sin 30 + \frac{1}{2} > \mu\cos 30$

$\mu > \dfrac{\sin 30 + \frac{1}{2}}{\cos 30}$

Examiner commentary

This part differentiated well with very few candidates showing clearly why $\lambda < \frac{1}{2}$. Only the most able realised that resolving parallel to the plane and then considering the fact that frictional force had to be positive was the easiest way of tackling this part, Exemplar 1.

Exemplar 2 has started with the given answer. In general, candidates are reminded that 'show that' questions are just that; the information in the question should be used to show a given result and therefore the result (or a limiting case of it) should not be used in an attempt to show that the given result is indeed true.

Exemplar 3 has talked about 'neglecting friction' rather than discussing when the friction would be zero, or that the friction could never be negative.

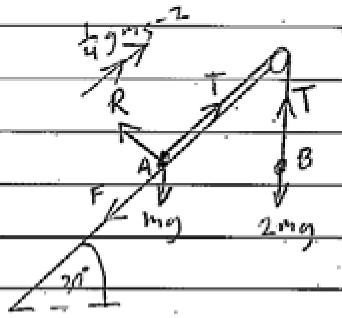
Question 12(ii)

(ii) Given instead that $\lambda = 2$ and that the acceleration of A is $\frac{1}{4}g\,\mathrm{m\,s^{-2}}$, find the exact value of μ. **[5]**

Exemplar 1

5 marks

12(ii)



R(para plane): $R = mg\cos 30°$ [1]

N, II (perp plane): $T - F - mg\sin 30° = \frac{1}{4}mg$ [2]

N, II (↑, B): $2mg - T = \frac{1}{2}mg$ [3]

$F = \mu R$ [4]

[2] + [3]: $2mg - F - \frac{1}{2}mg = \frac{3}{4}mg$

$F = mg(2 - \frac{1}{2} - \frac{3}{4}) = \frac{3}{4}mg$ [5]

[1]: $R = \frac{\sqrt{3}}{2}mg$

[1] & [5] into [4]: $\frac{3}{4}mg = \mu(\frac{\sqrt{3}}{2}mg)$

$\mu = \frac{3/4}{\sqrt{3}/2} = \frac{\sqrt{3}}{2}$

Examiner commentary

Like part (i)(b) this part differentiated well with many candidate unaware that if the value of λ was now greater than a half then particle A would move up the plane. Exemplar 1, shows that with clear diagrams, including arrows showing the direction of motion and/or acceleration can help to reduce the risk of sign errors when identifying the direction that any frictional force will act.

The small print

We'd like to know your view on the resources we produce. By clicking on the 'Like' or 'Dislike' button you can help us to ensure that our resources work for you. When the email template pops up please add additional comments if you wish and then just click 'Send'. Thank you.

Whether you already offer OCR qualifications, are new to OCR, or are considering switching from your current provider/awarding organisation, you can request more information by completing the Expression of Interest form which can be found here: www.ocr.org.uk/expression-of-interest

OCR acknowledges the use of the following content:
Square down and Square up: alexwhite/Shutterstock.com

Please get in touch if you want to discuss the accessibility of resources we offer to support delivery of our qualifications: resources.feedback@ocr.org.uk

Looking for a resource?

There is now a quick and easy search tool to help find **free** resources for your qualification:

www.ocr.org.uk/i-want-to/find-resources/

www.ocr.org.uk

OCR Customer Contact Centre

General qualifications

Telephone 01223 553998
Facsimile 01223 552627
Email general.qualifications@ocr.org.uk

OCR is part of Cambridge Assessment, a department of the University of Cambridge. *For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored.*

Cambridge Assessment

CERTIFIED ISO 9001

UKAS MANAGEMENT SYSTEMS 001