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# AS LEVEL

*Exemplar Candidate Work*

# *MATHEMATICS A*

**H230**
For first teaching in 2017

# H230/02 Summer 2018 examination series

Version 1

# Contents

# Introduction

These exemplar answers have been chosen from the summer 2018 examination series.

OCR is open to a wide variety of approaches and all answers are considered on their merits. These exemplars, therefore, should not be seen as the only way to answer questions but do illustrate how the mark scheme has been applied.

Please always refer to the specification https://www.ocr.org.uk/qualifications/as-and-a-level/mathematics-a-h230-h240-from-2017/#as-level for full details of the assessment for this qualification. These exemplar answers should also be read in conjunction with the sample assessment materials and the June 2018 Examiners' report or Report to Centres available from Interchange https://interchange.ocr.org.uk/Home.mvc/Index

The question paper, mark scheme and any resource booklet(s) will be available on the OCR website from summer 2019. Until then, they are available on OCR Interchange (school exams officers will have a login for this and are able to set up teachers with specific logins – see the following link for further information http://www.ocr.org.uk/administration/support-and-tools/interchange/managing-user-accounts/).

It is important to note that approaches to question setting and marking will remain consistent. At the same time OCR reviews all its qualifications annually and may make small adjustments to improve the performance of its assessments. We will let you know of any substantive changes.

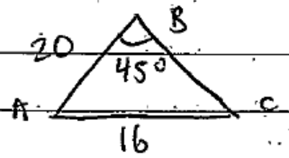
# Question 1 (i)

**1** In triangle $ABC$, $AB = 20$ cm and angle $B = 45°$.

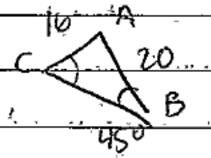
**(i)** Given that $AC = 16$ cm, find the two possible values for angle $C$, correct to 1 decimal place. **[4]**

## Exemplar 1 — 3 marks

1 (i)




$\frac{\sin C}{c} = \frac{\sin B}{b}$ $\quad$ $\frac{\sin C}{20} = \frac{\sin 45}{16}$




$\sin C = \frac{5\sqrt{2}}{8}$

$\sin^{-1} C = 62.1$

$62.1 + 45 = 107.1$

$180 - \text{ans}$

$= 72.9$

$C = 62.1°$

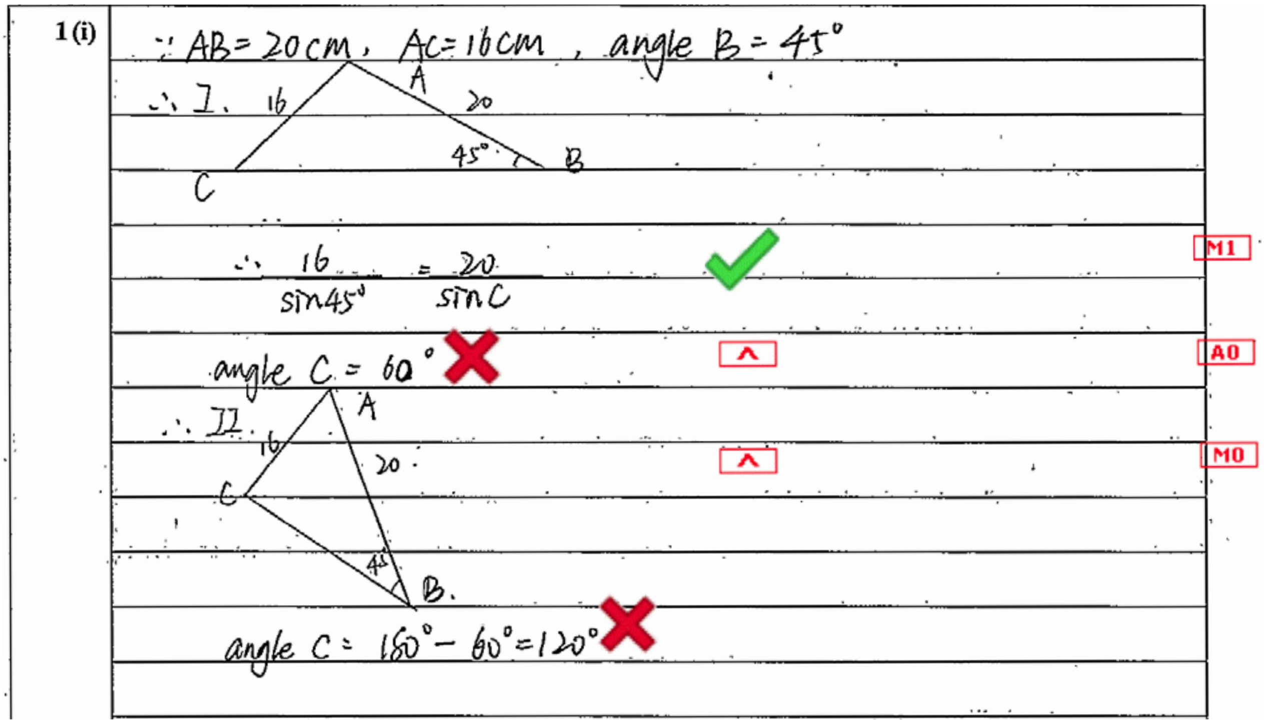
$C = 72.9°$

A0

## Examiner commentary

This solution shows a clear understanding of the Sine Rule and even uses the exact value of sin45° in the working. This was not uncommon in good attempts, although not, of course, necessary here. The request for values correct to 1 decimal place has been fulfilled, and this was needed to obtain full marks in this part. However, supplementary angles were required and this candidate has not understood this, just finding the angle *C*.

# Exemplar 2 1 mark




## Examiner commentary

This attempt correctly substitutes values into the Sine Rule, but then does not seem to use it for calculation. 60° appears, but it is unclear how it has been obtained. This candidate understands that the two angles required are supplementary but only M1 can be given for using the sine formula correctly.

# Question 1 (ii)

**(ii)** Given instead that the area of the triangle is $75\sqrt{2}\ \text{cm}^2$, find $BC$. **[2]**

## Exemplar 1 1 mark

| | | |
|---|---|---|
| 1 (ii) | $\frac{1}{2}(ab)\sin C = 75\sqrt{2}$ | MR |
| | $\frac{1}{2}(16BC) \times 0.8839 = 75\sqrt{2}$ | M1 |
| | $16BC = 239.996$ | |
| | $\therefore BC = 14.999$ | |
| | $\Rightarrow BC \doteq 15\text{cm}$ (2sf) | A0 |

## Examiner commentary

This form of solution was marked as a MR (misread) since examiners felt it was worthy of some credit. The candidate has not understood the significance of the word 'instead' in this part and made use of 62.1°, but has used the method expected. Candidates should be clear about the meaning of 'instead' in such a context.

## Exemplar 2 0 marks

| | | |
|---|---|---|
| 1 (ii) | Area: $\frac{1}{2}ab\sin C$ | |
| | $75\sqrt{2} = \frac{1}{2} \times b \times \sin(45)$ | M0 |
| | $75\sqrt{2} = b \times \sqrt{2}/4$ | |
| | $b = 300\text{cm}$ | |

## Examiner commentary

This candidate realises that Area = $\frac{1}{2}ab\sin C$ is needed, but this is not enough alone to earn M1. The guidance in the Mark Scheme clearly indicates that it must be used correctly to earn the method mark and this is not the case here. Generally speaking formulae need to be used in a relevant way for the problem in hand before method marks are awarded.

# Question 2 (i)

**2** **(i)** The curve $y = \frac{2}{3+x}$ is translated by four units in the positive $x$-direction. State the equation of the curve after it has been translated. **[2]**

## Exemplar 1 — 1 mark

**2 (i)**

$y = \frac{2}{3+(x-4)}$

^

M1

A0

## Examiner commentary

Learners are expected to simplify algebraic and numerical expressions when giving their final answers, even if the examination question does not explicitly ask them to do so, as stipulated in the specification. This solution loses the A1 because of the lack of simplification. Note that '$y =$' was expected for A1 also.

# Question 2 (ii)

**(ii)** Describe fully the single transformation that transforms the curve $y = \dfrac{2}{3+x}$ to $y = \dfrac{5}{3+x}$. **[2]**

## Exemplar 1 — 1 mark

**2 (ii)** ~~f(x)~~ ~~translation~~

stretch in y direction. ^

B1

## Examiner commentary

This is the correct transformation but the scale factor is omitted from the description.

## Exemplar 2 — 0 marks

**2 (ii)**

$y = 2(x+3)^{-1}$ $y = 2x^{-1} + \frac{1}{6}$

$y = 5(x+3)^{-1}$ $y = 5x^{-1} + \frac{1}{15}$

~~A~~ ~~enlargement~~ ~~shift~~ Translation in the positive y direction by 3 units

## Examiner commentary

This solution starts with some very poor algebra, but it is the description which is the focus of the marking. A translation invariably scored no marks and the scale factor was incorrect.

# Question 4 (i)

**4** **(i)** Express $4x^2 - 12x + 11$ in the form $a(x+b)^2 + c$. **[3]**

## Exemplar 1 — 2 marks

4(i)

$4x^2 - 12x + 11$ $\qquad a(x+b)^2 + c$

$4\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + 11$ B1

$4\left(x - \frac{3}{2}\right)^2 + \frac{35}{4}$ B1

$4\left(x - \frac{3}{2}\right)\left(x - \frac{3}{2}\right) + \frac{35}{4}$

$x^2 - \frac{3}{2}x - \frac{3}{2}x + \frac{9}{4}$

$4\left(x^2 - 3x + \frac{9}{4}\right) + \frac{35}{4}$

$4x^2 - 12x + 9 + \frac{35}{4} =$

## Examiner commentary

This solution illustrates a reasonably common error amongst those candidates who understood the required technique.

## Exemplar 2 — 1 mark

4(i)

$4x^2 - 12x + 11$

$4[(x-6)^2 - 36] + 11$ B1

$4(x-6)^2 - 144 + 11$ B0

$4(x-6)^2 - 133$ B0

## Examiner commentary

In this solution, the processing of the 4 has been poorly executed, but it earns the first B1.

Overall candidates seemed to recognise that this question was about completing the square, but errors in the detail of the working sometimes proved costly, as here.

# Question 4 (iii)

**(iii)** Explain fully how the value of $r$ is related to the number of real roots of the equation $p(x+q)^2+r=0$ where $p$, $q$ and $r$ are real constants and $p>0$. **[2]**

## Exemplar 1 — 1 mark

4(iii)

If $r$ is a ~~negative~~ positive then there are no real roots of the equation. e.g/ in form $p(x+q)^2+r=0$
$4x^2-12x+11=0$ is $4(x-\frac{3}{2})^2+2$ . $+2 \geq 0$ so has no real roots as proved in previous question
$b^2-4ac=-32$ $-32<0$.

M1 A0

## Examiner commentary

This candidate presents an incomplete solution, just referring back to the work in (ii) to get one condition. This was enough for M1, but few candidates made this connection.

## Exemplar 2 — 0 marks

4(iii)

$b^2$ is always positive as any integer squared is always positive

4ac can depend on a and c

In this case (r)

If r ~~[illegible]~~ added with $pq^2$ (when $p>0$ and $(q)^2$ must then be a positive number) is positive aswell as a then ~~[illegible]~~ be no real roots

If r added with $pq^2$ is negative there will be real roots as it must be bigger than 0

M0

# Examiner commentary

Some candidates tried to attack this problem by considering $b^2 - 4ac$, but very few made much progress and a fully correct solution using this approach was extremely rare. This is an example of such an attempt, which is confused and does not answer the question set. The question clearly asks for how the value of r is related to the number of real roots, so this is what was looked for somehow before the method mark could be earned.

This question (as a whole) was intended to test the $a(x + b)^2 + c$ form for the quadratic, but whilst many candidates knew how to put a quadratic into this form, using it to investigate the roots of the associated quadratic equation was less well understood.

# Question 5

**5 In this question you must show detailed reasoning.**

The line $x+5y=k$ is a tangent to the curve $x^2-4y=10$. Find the value of the constant $k$. **[5]**

## Exemplar 1 — 5 marks

5

$x + 5y = k$ $\qquad$ $x = k - 5y$

$5y = k - x$

$y = \frac{k}{5} - \frac{x}{5}$ $\qquad$ $(k-5y)^2 - 4y = 10$

$k^2 - 10ky + 25y^2 - 4y = 10$

$\cancel{k}\ 25y^2 - 4y - 10ky + k^2 - 10 = 0$

$\cancel{4}$

$a = 25$ $\qquad$ $b^2 - 4ac = 0$

$b = -4 - 10k$ $\qquad$ $(-4-10k)^2 - 4 \times 25 \times (k^2-10) = 0$

$c = k^2 - 10$

$(-4-10k)(-4-10k) - 100(k^2-10) = 0$

$16 + 40k + 40k + 100k^2 - 100k^2 + 1000 = 0$

$16 + 80k + 1000 = 0$

$80k + 1016 = 0$

$80k = -1016$

$k = -12.7$

## Exemplar 2 5 marks

5 $x + 5y = k \qquad x^2 - 4y = 10$

$5y = k - x \qquad\qquad 4y = x^2 - 10$

$y = \frac{k}{5} - \frac{x}{5} \qquad\qquad y = \frac{x^2}{4} - 2.5$

gradient $= \frac{x}{5} = \frac{1}{5}x \qquad\qquad \frac{dy}{dx} = \frac{2x}{4} = 0.5x$

$\frac{dy}{dx} = -\frac{1}{5} \qquad\qquad 0.5 \times x = -\frac{1}{5}$

$x = -\frac{2}{5}$ where the lines meet

$\left(\frac{-2}{5}\right)^2 - 4y = 10$

$\frac{4}{25} - 4y = 10 \qquad -4y = \frac{246}{25} \qquad y = -2.46$

$\frac{-2}{5} + 5(-2.46) = k \qquad k = -12.7$

## Examiner commentary

The two examples above are excellent examples of the two methods shown in the Mark Scheme. Both are fully correct and show sufficient working to satisfy the request for detailed reasoning.

## Exemplar 3 2 marks

5

$x^2 - 4y = 10$

$x^2 - 10 = 4y$

$y = \frac{1}{4}x^2 - \frac{5}{2}$

$\frac{dy}{dx} = \frac{1}{2}x$

$\Rightarrow$ gradient of tangent $= \frac{1}{2}x$

$x = k - 5y$

sub into curve equation

$(k-5y)^2 - 4y = 10$

$k^2 - 10ky + 25y^2 - 4y = 10$

$25y^2 + (-4-10k)y + k^2 - 10 = 0$ M1 A1

$5y = k$ $\qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{k - x}{5}$

$\cancel{y = -(-4-10k) \pm \sqrt{(-4-10k)^2 - 4 \times 2}}$

$y = \frac{-(-4-10k) \pm \sqrt{(-4-10k)^2 - 4 \times 25 \times (k^2+10)}}{50}$

$y = \frac{4+10k \pm \sqrt{16 + 80k + 100k^2 - 100k^2 + 1000}}{50}$

$y = \frac{4+10k \pm \sqrt{80k + 1016}}{50}$ M0

$y =$

## Examiner commentary

This attempt contains elements of both methods. Neither is complete, but learners should be aware that what is considered to be the last (complete) attempt is the one which will be credited, even when this gains fewer marks. The work highlighted in blue has been seen but is not credited. The work which does gain credit does not earn the second M1 for the discriminant – its significance is not evident here.

# Exemplar 4

**1 mark**

5

$x+5y=k \qquad x^2-4y=10 \qquad x^2 = 10+4y$

$y = \frac{-x}{5} + \frac{k}{5} \qquad y = \frac{x^2}{4} \cancel{4x} - \frac{10}{4}$

$\frac{x^2}{4} - \frac{10}{4} = \frac{-x}{5} + \frac{k}{5}$

$\frac{5x^2}{4} - \frac{50}{4} = -x + k$ M1

$5x^2 - 50 = -4x + 4k$

$5x^2 + 4x - 50 = 4k$

$5x^2 + 4x - 50 + 4k$ A0

$k = \frac{5}{4}x^2 + x - \frac{25}{2}$

$5y = \frac{5}{4}x^2 + x - x - \frac{25}{2}$

$5y = \frac{5}{4}x^2 - \frac{25}{2}$

## Examiner commentary

Although some were able to produce some level of detailed reasoning, it was clear that some struggled. The substitution is made, but the subsequent rearrangement is not accurate or incomplete and the candidate does not seem to know where to go next.

# Question 6 (iii)

**6** A pan of water is heated until it reaches 100 °C. Once the water reaches 100 °C, the heat is switched off and the temperature $T$ °C of the water decreases. The temperature of the water is modelled by the equation

$$T = 25 + ae^{-kt},$$

where $t$ denotes the time, in minutes, after the heat is switched off and $a$ and $k$ are positive constants.

When the heat is switched off, the initial rate of decrease of the temperature of the water is 15 °C per minute.

**(iii)** Calculate the value of $k$. **[3]**

## Exemplar 1 — 3 marks

6 (iii)

$\frac{dT}{dt} = -k\cancel{t}ae^{-kt}$

when $t=0$,

$-kae^{-kt} = -15$

$-k \times 75 \times e^{-k\times 0} = -15$

$-k \times 75 \times 1 = -15$

$\cancel{-k}\ -75k = -15$

$k = \frac{-15}{-75}$

$k = \frac{1}{5} = 0.2$

## Exemplar 2 — 2 marks

6 (iii)

$T = 25 + ae^{-kt}$

$\frac{dT}{dt} = -k\cancel{t}75e^{-kt}$

$15 = -k\cancel{t}75e^{-k0}$ BOD B1

$15 = -k\,75$ M1

$\frac{15}{-75} = -\frac{1}{5}$

$k = -\frac{1}{5}$ A0

## Examiner commentary

Exemplar 1 is a rare example of a completely correct solution. This is followed by a good effort, which failed to use -15.

# Exemplar 3

## 0 marks

**6(iii)**

15°C per minute

$2^3 = 8$

$\log_2 8 = 3$

$T = 25 + 100e^{-k1}$ ✗

$85 = 25 + 100e^{-k}$ B0

$60 = 100e^{-k}$

$\frac{3}{5} = e^{-k}$ $\quad \ln\left(\frac{3}{5}\right) = \cancel{\ln}$ $\quad \log_e(3 \div 5) = \log_e e^{-k}$ M0

$\ln\left(\frac{3}{5}\right) = -k \ln$

$\ln\left(\frac{3}{5}\right) = -k$

$-k = -0.511$ ✗

$k = 0.511$

# Examiner commentary

This approach was seen quite frequently. $a$ has been given the value $100$ in (i) – also quite common – and since the rate of decrease is given as 15°C per minute $T = 85$ when $t = 1$ has been used in the given equation. The significance of 'initial rate of decrease' has not been realised – it has been incorrectly assumed to be a constant rate of decrease. A derivative attempt was needed for credit in this part.

# Question 6 (iv)

**(iv)** Find the time taken for the temperature of the water to drop from 100 °C to 45 °C. **[3]**

## Exemplar 1 3 marks

**6 (iv)**

$45 = 25 + 75e^{-\frac{1}{5}t}$

$20 = 75e^{-\frac{1}{5}t}$

$\frac{4}{15} = e^{-\frac{1}{5}t}$

$\ln \frac{4}{15} = \ln e^{-\frac{1}{5}t}$

$\ln \frac{4}{15} = -\frac{1}{5}t.$

$t = -5\left(\ln \frac{4}{15}\right)$

$t = 6.61$ minutes (3sf).

## Exemplar 2 2 marks

**6 (iv)**

$45 = 25 + 75e^{-2.01t}$

$\frac{20}{75} = e^{-2.01t}$ M1

$\ln\left(\frac{20}{75}\right) = \ln e^{-2.01t}$

M1

$\ln\left(\frac{20}{75}\right) = -2.01t \ln e$

$\ln\left(\frac{20}{75}\right) = -2.01t$

$t = 0.6575\ldots$ A0

$t = 0.658$

## Examiner commentary

Exemplar 1 shows a rare correct solution. Note that units were required in the final answer enabling this to be allocated AO 3.2a. Candidates should be made aware of the potential for this, especially in modelling questions.

In Exemplar 2, although the value of $k$ is incorrect, the methods are sound.

## Exemplar 3 1 mark

| | |
|---|---|
| 6(iv) | $T = 25 + 100e^{-0.01918t}$ |
| | $55 = 25 + 100e^{-0.01918t}$ M0 |
| | $30 = 100e^{-0.01918t}$ |
| | $0.3 = e^{-0.01918t}$ M1 |
| | $\ln(0.3) = -0.01918t$ |
| | $t = \dfrac{\ln(0.3)}{-0.01918}$ A0 |
| | $=$ ~~83.4 minutes~~ or ~~84 minutes~~ 62.8 minutes or 63 minutes ✗ |

## Examiner commentary

Another candidate with $a = 100$, and an incorrect value for $k$. In this case instead of using $T = 45$ we see $T = 100 - 45$, hence M0, but the following M1 was allowed.

# Question 7 (i)

**7** **(i)** Show that the equation

$$2\sin x \tan x = \cos x + 5$$

can be expressed in the form

$$3\cos^2 x + 5\cos x - 2 = 0.$$

**[3]**

## Exemplar 1 — 2 marks

**7 (i)**

$2\sin x \times \tan x = \cos x + 5$

~~tan~~ $\frac{\sin x}{\cos x}$ $\quad \sin^2 x + \cos^2 x = 1$

$2\sin x \times \frac{\sin x}{\cos x} = \cos x + 5$ **M1**

$2\sin^2 x = \cos^2 x + 5\cos x$

$2(1 - \cos^2 x)$

$2 - 2\cos^2 x = \cos^2 x + 5\cos x$ **M1**

$0 = 3\cos^2 x + 5\cos - 2$ **A0**

## Examiner commentary

The question gives the form of the equation we want to see and examiners expected to see fully accurate working leading to this before awarding A1. Here cos, not $\cos x$ loses this mark. Candidates should know that 'Answer Given (AG)' will be marked strictly like this without 'Benefit of Doubt (BoD)'.

A few also lost this mark by omitting '=0' in their answer, or writing $\cos x^2$.

# Question 7 (ii)

**(ii)** Hence solve the equation

$$2\sin 2\theta \tan 2\theta = \cos 2\theta + 5,$$

giving all values of $\theta$ between 0° and 180°, correct to 1 decimal place. **[5]**

## Exemplar 1 4 marks

**7 (ii)**

$2\sin 2\theta \tan 2\theta = \cos 2\theta + 5$ $\qquad 0 < \theta < 180$

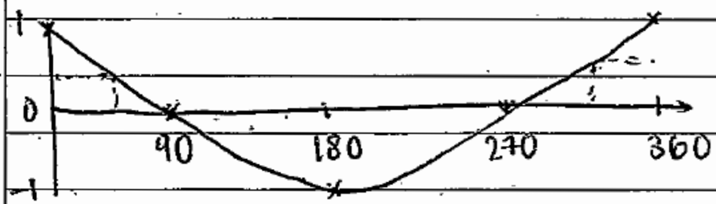
$3\cos^2 2\theta + 5\cos 2\theta - 2 = 0$ $\qquad 0 < 2\theta < 360$

$(3\cos 2\theta - 1)(\cos 2\theta + 2)$

$3\cos 2\theta = 1$ $\qquad \cos 2\theta = -2$

$\cos 2\theta = \frac{1}{3}$ $\qquad 2\theta =$ no answer

$2\theta = 70.5$




$360 - 70.5 = 289.5$ $\qquad 70.5°, 289.5° = 2\theta$

$35.3°, 144.8° = \theta$ $\qquad$ A0

## Examiner commentary

This question requested values correct to 1 decimal place and this was expected to be accurately done for full marks. 144.7° was expected. This is a good solution which has lost the final A1 because of premature approximation.

# Exemplar 2

**2 marks**

**7 (ii)**

let $\cos 2\theta = x$

$3x^2 + 5x - 2$

$(3x - 1)(x + 2)$ M1

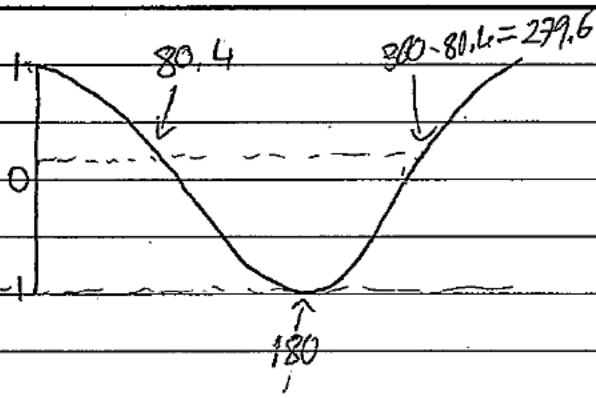
$x = \frac{1}{3}$ or $x = -2$

$\cos 2\theta = \frac{1}{3}$ or $-2$ A1

$\cos\theta = \frac{1}{6}$ or $-1$ M0

$\cos^{-1}\left(\frac{1}{6}\right) = 80.405$

$\cos^{-1}(-1) = 180$




$\theta = 80.4,\ 180,\ \cancel{279.6}$

all values of $\theta$ between 0 and 180

$\theta = 80.4,\ \cancel{180}$

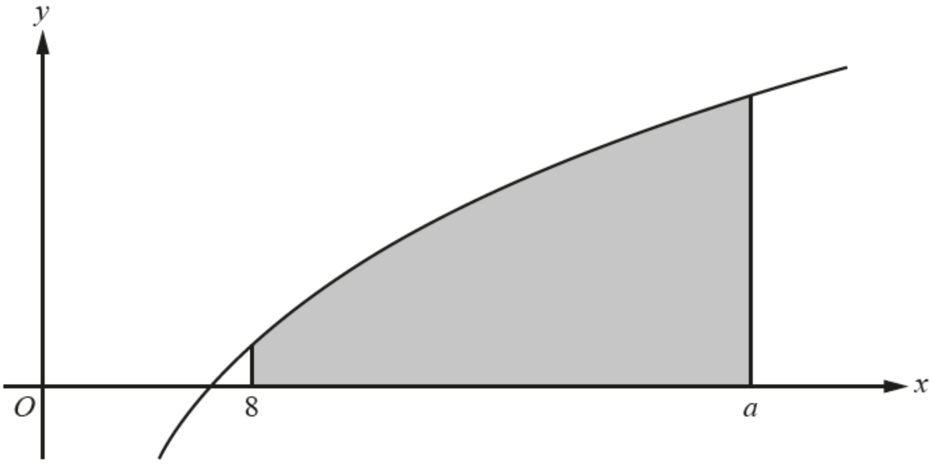
## Examiner commentary

Many candidates recognised the quadratic and managed to solve it accurately. This exemplar shows a lack of understanding of the correct order of operations to find $\theta$ and so gains only the first two marks.

# Question 8

**8 In this question you must show detailed reasoning.**

The diagram shows part of the graph of $y = 2x^{\frac{1}{3}} - \frac{7}{x^{\frac{1}{3}}}$. The shaded region is enclosed by the curve, the $x$-axis and the lines $x = 8$ and $x = a$, where $a > 8$.




Given that the area of the shaded region is 45 square units, find the value of $a$. **[9]**

## Exemplar 1 — 8 marks

8

$y = 2x^{\frac{1}{3}} - 7x^{-\frac{1}{3}}$

$\therefore \int_8^a (2x^{\frac{1}{3}} - 7x^{-\frac{1}{3}}) = \left(\frac{3}{2}x^{\frac{4}{3}} - \frac{21}{2}x^{\frac{2}{3}}\right)_8^a$

$= \cancel{\left(\frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}}\right) = 18}$

$\because$ the area of the shaded region is 45

$= \left(\frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}}\right) + 18$

$\therefore \frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}} + 18 = 45$

$\frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}} = 27.$

# Exemplar 1 continued

$y = 2x^{\frac{1}{3}} - 7x^{-\frac{1}{3}}$ M1

$\therefore \int_8^a (2x^{\frac{1}{3}} - 7x^{-\frac{1}{3}}) = \left(\frac{3}{2}x^{\frac{4}{3}} - \frac{21}{2}x^{\frac{2}{3}}\right)_8^a$ A1 A1

$= \left(\frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}}\right) + 18$ BOD M1 A1

$\because$ shaded region = 45

$\therefore \frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}} + 18 = 45$ M1

$\frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}} = 27.$

assume $a^{\frac{2}{3}} = x$

$\frac{3}{2}x^2 - \frac{21}{2}x - 27 = 0$ M0

$x_1 = 9 \quad x_2 = -2.$ SC

$\therefore a^{\frac{2}{3}} = 9$ or $a^{\frac{2}{3}} = -2$ (x) B1

$a_1 = 27$

$\because a > 0$

$\therefore a = 27$ B1

## Examiner commentary

This good solution was quite common. The question requested detailed reasoning, so examiners felt that there should be sufficient evidence of how the quadratic equation had been solved to gain M1. A special case (SC) in the Mark Scheme allowed for a possible 8/9 as here.

## Exemplar 2 5 marks

8 $\cancel{x = 2x^{1/3}}$

$y = 2x^{\frac{1}{3}} - 7x^{-\frac{1}{3}}$

$\int 2x^{\frac{1}{3}} - 7x^{\frac{1}{3}} \, dx.$ M1

$= \frac{3}{2}x^{\frac{4}{3}} - \frac{21}{2}x^{\frac{2}{3}} + c$ A1 A1

$\left[\frac{3}{2}x^{\frac{4}{3}} - \frac{21}{2}x^{\frac{2}{3}}\right]_8^a$ ✓

$\left[\frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}}\right] - \left[\frac{3}{2}8^{\frac{4}{3}} - \frac{21}{2}8^{\frac{2}{3}}\right]$ M1

$\left[\frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}}\right] - \left[18\right] = 45$ A0 M1

$\frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}} - 18 = 45$

$3a^{\frac{4}{3}} - 21a^{\frac{2}{3}} - 63 = 0$

$\cancel{a} \quad a^{\frac{4}{3}} - 7a^{\frac{2}{3}} - 21 = 0$

$\cancel{a} \quad a^{\frac{4}{3}} - 7a^{\frac{2}{3}} = 21$

$\cancel{a^{4/3} - 7a a^{2/3} = 21}$

$\cancel{a}^2 = 3$

$\cancel{a}$

### Examiner commentary

This candidate makes a sign error and then is unable to deal with the 'hidden' quadratic. The integration work is sound though.

# Question 9

**9** In this question the horizontal unit vectors **i** and **j** are in the directions east and north respectively.

A model ship of mass 2 kg is moving so that its acceleration vector $\mathbf{a}\,\mathrm{m\,s^{-2}}$ at time $t$ seconds is given by $\mathbf{a} = 3(2t-5)\mathbf{i}+4\mathbf{j}$. When $t = T$, the magnitude of the horizontal force acting on the ship is 10 N.

Find the possible values of $T$. **[4]**

## Exemplar 1 4 marks

**9**

(2a) $F = ma$

$10 = 2(3(2t-5)\mathbf{i} + 4\mathbf{j})$

$10 = 2\begin{pmatrix} 6t-15 \\ 4 \end{pmatrix}$

$10 = \begin{pmatrix} 12t-30 \\ 8 \end{pmatrix}$

$10 = \sqrt{(12t-30)^2 + 64}$

$100 = 144t^2 - 720t + 900 + 64$

$144t^2 - 720t + 864 = 0$

$t^2 - 5t + 6 = 0$

$(t-2)(t-3) = 0$

$t = 2$ or $t = 3$

$\therefore\ T = 2$ or $T = 3$

## Examiner commentary

Candidates found this question difficult. Here is a correct solution, which gained full credit, but which starts with the false equation $10 = 2(3(2t - 5)\mathbf{i} + 4\mathbf{j})$.

However, the correct method is recovered, but many candidates started along such lines and manufactured incorrect equations in $t$ and thus scored M0. $T$ or $t$ was allowed in the working here, and although units were not required to gain the final A1 it is no bad thing for candidates to be drilled to include them.

## Exemplar 2 2 marks

9 $2kg$

$a = 3(2t-5)i + 4j.$

$F = m\ a$

$10 = 2(3(2T-5)i + 4j))$

$10 = 6(2T-5)i + 8j$

$5 = 2(2T-5)i + 4j$ M1

$5 = \sqrt{(4T-10)^2 + 4^2}$

$5 = \sqrt{16T^2 - 80T + 100 + 16}$ A0

$\cancel{16T^2 - 40T + \ldots = 0}$

$\cancel{16T^2 - 80T + 111 = 0}$

$5 = \sqrt{16T^2 - 80T + 116}.$

$25 = 16T^2 - 80T + 116.$

$16T^2 - 80T + 91 = 0.$ M1

$T = 3.25$ or $T = 1.75$ A0

## Examiner commentary

The same false equation as in Exemplar 1 appears, but although the correct methods then follow this solution is marred by numerical inaccuracy. This was not unusual.

# Exemplar 3

## 2 marks

9

$F = ma$

$10 = 2a$

$5 = a$

$5 = 3(2T-5)\hat{i} + 4\hat{j}$

$5 = (6T-15)\hat{i} + 4\hat{j}$

$5^2 = 4^2 + 3^2$

$5 = (6T-15)\hat{i} + 4\hat{j}$ M1

what $= 3$ A0

$5 = 3\hat{i} + 4\hat{j}$ M1

$6T - 15 = 3$ A0

$6T = 18$

$T = 3$

## Examiner commentary

This slightly unusual approach was seen occasionally. A false equation again appears, but this solution would have led to the correct values of $T$ had $\pm 3$ been used.

## Exemplar 4 0 marks

9 $a = 3(2T - 5)\mathbf{i} + 4\mathbf{j}$

$a = (6T - 15)\mathbf{i} + 4\mathbf{j}$

$F = ma$

$10 = 2(6T - 15) + 4$

$10 = 12T - 30 + 8$

$32 = 12T$

$T = \frac{32}{12}$

$T = \frac{8}{3}$

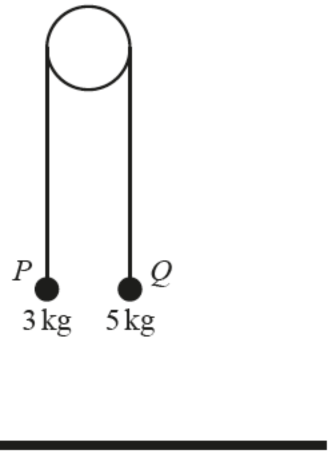
M0

## Examiner commentary

Here the significance of **i** and **j** is not realised and so there is no use of Pythagoras. This lack of appreciation of vectors was prevalent. Use of the equation $10 = 2(6T - 15)$, or some such, also appeared in many solutions.

# Question 10 (i)

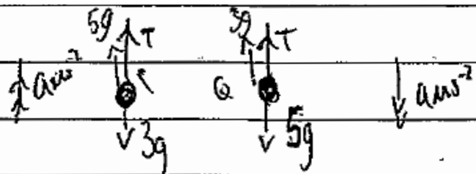
**10** Particles $P$ and $Q$, of masses 3 kg and 5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is held at rest with the string taut. The hanging parts of the string are vertical and $P$ and $Q$ are above a horizontal plane (see diagram).




**(i)** Find the tension in the string immediately after the particles are released. **[4]**

## Exemplar 1 — 0 marks

**10 (i)**




Q: $F - T - F_p = ma$

$5g - T - 3g = 5a$

$T = 2g - 5a$

P: $T + F_Q + F_p = 3a$

$T + 5g - 3g = 3a$

$T + 2g = 3a$

$a = \frac{5g - T - 3g}{5}$

$T = 2g - 5\left(\frac{5g - T - 3g}{5}\right)$

T2h

$T = 2g - 5g - T - 3g$

$2T = -6$

$T = \frac{-6}{2} = -3$

$T = 3N$

M0

## Examiner commentary

This is a standard pulley question, usually done well by those candidates who understood Newton's Second Law. A sensible use of this was vital for credit in this question.

Here an extra term has appeared in both equations leading to M0.

# Exemplar 2

## 0 marks

**10 (i)**

$F = ma$

$a = 0$ as they are released

$T - 3g = ma$

$T - 3g = 0 \times 3$

$T = 3g$

$\therefore T = 29.4\text{N}$

M0

## Examiner commentary

This is an attempt using $a = 0$. Examiners also saw a significant number who used $a = 9.8$. These attempts were invariably brief and, given that this question is allocated 4 marks, perhaps candidates should have been aware that a fuller response was required.

# Question 10 (ii)

After descending 2.5 m, $Q$ strikes the plane and is immediately brought to rest. It is given that $P$ does not reach the pulley in the subsequent motion.

**(ii)** Find the distance travelled by $P$ between the instant when $Q$ strikes the plane and the instant when the string becomes taut again. **[4]**

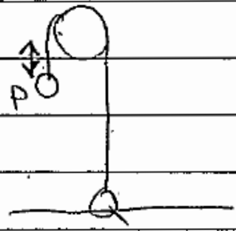
## Exemplar 1 — 2 marks

**10 (ii)**

~~$V = u + at$~~

~~$V = 0 + 2.5 \times 2.45$~~

~~$V = 6.125\ ms^{-1}$~~




$s$ 2.5
$u$ 0
$v$ ?
$a$ 2.45
$t$ —

$V^2 = u^2 + 2as$

$V^2 = 0 + 2 \times 2.45 \times 2.5$ B1

$V^2 = 12.25$ M1

$V = 3.5\ ms^{-1}$

$s$ ?
$u$ ~~6.125~~ 3.5
$v$ 0
$a$ 2.45
$t$ —

$V^2 = u^2 + 2as$

$0 = 3.5^2 + 2 \times 2.45 \times S$ M0

$0 = 12.25 + 4.9S$

$\frac{-12.25}{4.9} = S$

$S = -2.5\ m$ down

So distance travelled is $2.5\ m$

## Examiner commentary

This question was often found challenging and it does depend on some progress being made with 10 (i). This candidate has the correct work for the downward motion of $Q$, but then does not realise that the next part is motion under gravity.

# Exemplar 2 1 mark

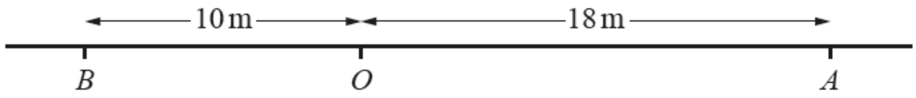
| | |
|---|---|
| 10 (ii) | $s = 2{\cdot}5 \quad a = 2{\cdot}45 \quad v = 0 \quad t = t$ **B1** |
| | $s = vt - \frac{1}{2}at^2$ $\qquad 2{\cdot}5\text{m}$ |
| | $2{\cdot}5 = 0 - \frac{1}{2} \times 2{\cdot}45 \times t^2$ |
| | $2{\cdot}5 = 1{\cdot}225t^2$ |
| | $\frac{100}{49} = t^2$ **M0** |
| | $\frac{10}{7} = t$ |
| | $\frac{10}{7} \quad a = 2{\cdot}45 \quad s = s \quad u = 0 \quad v = v \quad t = \frac{10}{7}$ |
| | $s = ut + \frac{1}{2}at^2 \qquad v = u + at$ |
| | $s = 0 + 2{\cdot}5 \qquad v = 0 + -\frac{7}{2}$ |
| | $2{\cdot}5\text{m} \qquad 0 = u \qquad v = -3{\cdot}5\text{ms}^{-1}$ |
| | $v = -3{\cdot}5 \qquad s = s$ |
| | $v^2 = u^2 + 2as \qquad a = -2{\cdot}45$ |
| | $12{\cdot}25 = 0 + -4{\cdot}9s \qquad s = -2{\cdot}5$ |

## Examiner commentary

In this exemplar the correct acceleration has been found in 10 (i), but the solution uses $v = 0$ not $u = 0$, perhaps because the question says $Q$ is 'immediately brought to rest'. An inconvenient negative sign has been ignored. Once again there is no work with motion under gravity.

# Question 11 (i)(a)

**11**




A particle $P$ is moving along a straight line with constant acceleration. Initially the particle is at $O$. After 9 s, $P$ is at a point $A$, where $OA = 18$ m (see diagram) and the velocity of $P$ at $A$ is $8\,\text{m}\,\text{s}^{-1}$ in the direction $\overrightarrow{OA}$.

**(i)** **(a)** Show that the initial speed of $P$ is $4\,\text{m}\,\text{s}^{-1}$. **[2]**

## Exemplar 1 — 1 mark

| | | | |
|---|---|---|---|
| **11 (i)(a)** | $s = 18$ | $s = \frac{u+v}{2} \times t$ | |
| | $U = u$ | | |
| | $V = 8$ | $18 = \frac{u+8}{2} \times 9$ | |
| | $A$ | | |
| | $T = 9$ | $4 = u + 8$ ✓ | M1 |
| | | $u = 4\text{ms}^{-1}$ ✗ | A0 |
| | | | |

## Examiner commentary

Most candidates seemed to know what was required here. The examiners expected to see $u = -4$ in the working, although the explanation for an initial speed of 4 was often less than we would have liked. Here, no doubt because 4 is in the question, the minus has not appeared.

# Question 11 (i)(b)

**(b)** Find the acceleration of $P$. **[2]**

## Exemplar 1

**1 mark**

| | | | |
|---|---|---|---|
| 11 (i)(b) | $S = 18$ | | |
| | $U = 4$ | $v = u + at$ | |
| | $V = 8$ | $8 = 4 + 9a$ | M1 |
| | $A = 9$ | $4 = 9a$ | |
| | $T = 9$ | $a = 4/9\ ms^{-2}$ or $\underline{0.44 ms^{-2}}$ | A0 |

## Examiner commentary

This solution was very common, even when -4 had appeared in 11 (i)(a).

# Question 11 (ii)

$B$ is a point on the line such that $OB = 10\,\text{m}$, as shown in the diagram.

**(ii)** Show that $P$ is never at point $B$. **[4]**

## Exemplar 1 — 2 marks

**11 (ii)**

$s = 10 \quad a = \frac{4}{3}$

$u = -4 \quad v = 8v$

~~$v^2 = u^2 + 2as$~~

~~$64 = 16 + 2 \times \frac{4}{3} \times 10$~~

~~$64 = 16 + \frac{80}{3}$~~

$64 \neq 42\cdot7$

~~So doesn't work for $\overrightarrow{OB}$~~

If accelerating and moving in $\overrightarrow{OA}$ at $8\,\text{ms}^{-1}$ cannot return to B unless rebounds off an object and changes direction

$v^2 = u^2 + 2as$ **M1**

$v^2 = 16 + 2 \times \frac{4}{3} \times 10$ **M1**

$v^2 = 16 + \frac{80}{3}$

$v^2 = 42\cdot7$ **A0**

$\sqrt{\ } = \pm 6\cdot53$

$-6\cdot53 \quad \overrightarrow{OB}$

**ISW**

$v = u + at$

$-6\cdot53 = -4 + \frac{4}{3}t$

$-2\cdot53 = \frac{4}{3}t$

$-1\cdot90 = t$

t cannot be negative so never reaches B

## Examiner commentary

This part of Question 11 proved difficult for candidates. It required understanding and accurate use of signs in this context. In this exemplar, 10 is used rather than -10. Note the use of the ISW (Ignore subsequent working).

# Exemplar 2

## 2 marks

11 (ii)

$S = -10$ $\quad s = ut + \frac{1}{2}at^2$ M1

$V = 4$ $\quad -10 = 4t + \frac{1}{2}(4/9)t^2$ A0

$V$ $\quad \frac{2}{9}t^2 + 4t + 10 = 0$

$A = 4/9 \text{ ms}^{-2}$ $\quad t = -3\text{s}$ or $t = -15\text{s}$ M1

$T = t$ A0

Time is not vector ∴ P can never be at point B

## Examiner commentary

The methods are fine here – the candidate is using their $u$ and $a$ in a correct way.

# Question 11 (iii)

A second particle $Q$ moves along the same straight line, but has variable acceleration. Initially $Q$ is at $O$, and the displacement of $Q$ from $O$ at time $t$ seconds is given by

$$x = at^3 + bt^2 + ct,$$

where $a$, $b$ and $c$ are constants.

It is given that

- the velocity and acceleration of $Q$ at the point $O$ are the same as those of $P$ at $O$,
- $Q$ reaches the point $A$ when $t = 6$.

**(iii)** Find the velocity of $Q$ at $A$. **[5]**

## Exemplar 1 — 4 marks

**11 (iii)**

$x = at^3 + bt^2 + ct$ displacement

$vQ = vP$

$aQ = aP$ Q reaches A at 6

$Qv$ at O $= 4\,ms^{-1}$

$Qa$ at O $= \frac{4}{3}\,ms^{-2}$

$\frac{dx}{dt} = 3at^2 + 2bt + c$ velocity ✓ M1

at $0 = t$ $= 4$

so $c = 4$

$\frac{d^2x}{dt^2} = 6at + 2b$

at $0 = t$ $\cancel{a = \frac{4}{3}}$ acc. $= \frac{4}{3}$ M1

$2b = \frac{4}{3}$ $b = \frac{2}{3}$ ✓ FT A1

$x = at^3 + bt^2 + ct$

$18 = \cancel{6} \cdot 216a + 36b + 6c$

$216a + 24 + 24$

$-30 = 216a$

$\frac{-5}{36} = a$ ✓ FT A1

vel $= 3at^2 + 2bt + c$

at $A = 108a + 12b + c$

$= -15 + 8 + 4$

$Qv$ at $A = -3\,ms^{-1}$ ✗ A0

## Examiner commentary

This candidate understands this part very well, but is using 4 rather than -4 as the value of $u$. The working from this value is accurate and earns ft marks, losing just the final answer mark.

## Exemplar 2 2 marks

**11 (iii)**

velocity at initial ~~speed~~ = 4 m/s

$v = 3at^2 + bt + c = 4$ M1

when $t=0$

$c=4$

acceleration $= \frac{4}{3}$ m/s² M1

$a = 6at + b$ when $t = 0$.

$\therefore 6at + b = \frac{4}{3}$ $b = \frac{4}{3}$ FT FT A0

$\because t=6$ the distance $= 18$

$18 = a\times 6^3 + b\times 6^2 + c\times 6$

$216a + 36b + 24 = 18$

$\therefore 216a + 36b = -6$.

~~$\because t=6$ $s=18$ $a=\frac{4}{3}$ $u=4$.~~

~~$\therefore v = 4 + \frac{4}{3}\times 6$~~

~~$= 12$ m/s.~~

~~$\therefore 6a\times 6 + b =$~~

~~$3a\times 6^2 + b\times 6 + 4 = 12$.~~

~~$108a + 6b = -8$~~

~~$216a + 36b = -6$~~

~~$\therefore a = 0.1$~~

~~$b =$~~

$\because b = \frac{4}{3}$

$\therefore 216a + 48 = -6$

$\therefore a = -0.25$ A0

$\therefore x = -0.25t^3 + \frac{4}{3}t^2 + 4t$.

$v = 3at^2 + bt + c$

$\therefore$ when $t = 6$,

$v = 3\times(-0.25)\times 6^2 + \frac{4}{3}\times 6 + 4$

$= 15$ m/s. A0

## Examiner commentary

A careless error in differentiation has proved costly here for a candidate who has a clear understanding of this question.

# Exemplar 3

**1 mark**

**11 (iii)**

$x = at^3 + bt^2 + ct$

$\frac{dx}{dt} = 3at^2 + 2bt + c$ When $t = 6$ M1

$V = 108a + 12b + c$

$A = 12a + 2b.$

$\frac{d^2x}{dt^2}$ $A = 6at + 2b$

Velocity of $p$ at O

$S = 0$

~~v=~~

$V = v$

$u = 4$

$4 = 108a + 12b + c$ When $t = 0$ $v = 0$ M0

$0.44 = 12a + 2b$ $\therefore c = 0$

A0

$4 = 108a + 12b$

$0.44 = 12a + 2b$

$\frac{0.44 - 12a}{2} = b$

$4 = 108a + 12\left(\frac{0.44 - 12a}{2}\right)$

A0

$4 = 108a + \frac{5.28 - 144a}{2}$ $4 = \frac{1450}{17} \times 108 + 12b$

$b = \frac{[illegible]}{51}$ A0

$8 = 216a + 5.28 - 144a$

$72a = 2.72$ $V = 4\,ms^{-1}$

$a = \frac{1450}{17}$

# Examiner commentary

Many candidates only gained the first M1 for differentiating $x$.

In this case, instead of using their $u$ and a with $t = 0$, they have been used with $t = 6$ and $t = 0$, $v = 0$ has produced $c = 0$ to enable a value for the other unknowns to be calculated. Such confusion was not uncommon.

Some candidates incorrectly tried to use constant acceleration formulae.