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OCR
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AS LEVEL

Exemplar Candidate Work

MATHEMATICS A

H230
For first teaching in 2017

H230/01 Summer 2018 examination series

Version 1

Contents

Introduction

These exemplar answers have been chosen from the summer 2018 examination series.

OCR is open to a wide variety of approaches and all answers are considered on their merits. These exemplars, therefore, should not be seen as the only way to answer questions but do illustrate how the mark scheme has been applied.

Please always refer to the specification https://www.ocr.org.uk/Images/308720-specification-accredited-as-level-gce-mathematics-a-h230.pdf for full details of the assessment for this qualification. These exemplar answers should also be read in conjunction with the sample assessment materials and the June 2018 Examiners' report or Report to Centres available from Interchange https://interchange.ocr.org.uk/Home.mvc/Index

The question paper, mark scheme and any resource booklet(s) will be available on the OCR website from summer 2019. Until then, they are available on OCR Interchange (school exams officers will have a login for this and are able to set up teachers with specific logins – see the following link for further information http://www.ocr.org.uk/administration/support-and-tools/interchange/managing-user-accounts/).

It is important to note that approaches to question setting and marking will remain consistent. At the same time OCR reviews all its qualifications annually and may make small adjustments to improve the performance of its assessments. We will let you know of any substantive changes.

Question 1(i)

1 In this question you must show detailed reasoning.

(i) Express $3^{\frac{7}{2}}$ in the form $a\sqrt{b}$, where a is an integer and b is a prime number. **[2]**

Exemplar 1 2 marks

1(i)

$3^{\frac{7}{2}} = \sqrt{3^7}$

$= \sqrt{3^6}\sqrt{3}$

$= 3^3\sqrt{3}$

$= 27\sqrt{3}$

Exemplar 2 0 marks

1(i)

$3^{\frac{7}{2}}$ $a\sqrt{b}$

$27\sqrt{3}$

Exemplar 3 0 marks

1(i)

$3^{\frac{7}{2}} = 46.765$

Examiner commentary

This question required detailed reasoning and many candidates answered this question well (Exemplar 1). Some candidates gave the correct answer with no working (Exemplar 2) or as a decimal rather than in the form required (Exemplar 3). These scored no marks due to the lack of detailed reasoning.

Question 1(ii)

(ii) Express $\dfrac{\sqrt{2}}{1-\sqrt{2}}$ in the form $c + d\sqrt{e}$, where c and d are integers and e is a prime number. **[3]**

Exemplar 1 — 3 marks

1(ii)

$$\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} \qquad = \frac{\sqrt{2}+2}{1-2}$$

$$= \frac{2+\sqrt{2}}{-1}$$

$$= -2-\sqrt{2}$$

Exemplar 2 — 2 marks

1(ii)

$$\frac{\sqrt{2}\,(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} \quad \text{M1} \qquad \frac{\sqrt{2}+2}{1+\sqrt{4}} \quad \text{A1}$$

$$\frac{\sqrt{2}+2}{3} \quad \text{A0}$$

Exemplar 3 — 0 marks

1(ii)

$$\frac{\sqrt{2}}{1-\sqrt{2}} \qquad \text{M0}$$

$$= -2-\sqrt{2}$$

Examiner commentary

Many candidates correctly demonstrated the detailed reasoning required (Exemplar 1). A few made a correct first step, multiplying numerator and denominator by $1 + \sqrt{2}$, but made a subsequent error (Exemplar 2). Some candidates gave the correct answer with no working (Exemplar 3). These scored no marks.

Question 2(i)

2 **(i)** The equation $x^2 + 3x + k = 0$ has repeated roots. Find the value of the constant k. **[2]**

Exemplar 1 — 2 marks

2(i)

$b^2 - 4ac = 0$

$3^2 - 4(1)(k) = 0$

$9 - 4k = 0$

$k = \frac{9}{4}$ ✓

Exemplar 2 — 1 mark

2(i)

$x^2 + 3x + k = 0$

$b^2 - 4ac = 0$ two equal roots

$3^2 - 4(1)(k) = 0$

$9 - 4k = 0$ M1

$-4k = -9$

$k = \frac{-9}{-4}$

$k = -\frac{9}{4}$ A0

Exemplar 3 — 0 marks

2(i)

$x^2 + 3x + k = 0$ has repeated roots. Find the value of the consonant k

$x^2 + 3x + k = 0$

~~fx~~ $x(3x + 3) = k$

$x = 0$

$x = -3$

$k = 0$ ~~$k =$~~

Examiner commentary

This question was answered well, particularly if using the discriminant (Exemplar 1). A few candidates started correctly but were unable to rearrange the equation to correctly find the value of *k* (Exemplar 2). Some seemed not to understand what is meant by a quadratic equation having repeated roots (Exemplar 3).

Question 2(ii)

(ii) Solve the inequality $6+x-x^2>0$. **[2]**

Exemplar 1 — 2 marks

2(ii)

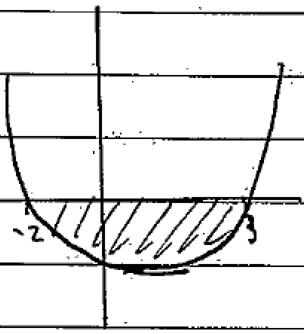
$6+x-x^2>0$

$0>x^2-x-6$

$x^2-x-6<0$

$(x-3)(x+2)<0$

$\therefore$ critical values are $x=3$, $x=-2$



$\therefore -2<x<3$

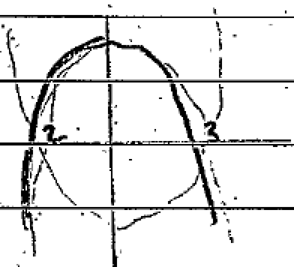
Exemplar 2 — 1 mark

2(ii)

$6+x-x^2>0$ $(x-1)$

$x^2-x-6<0$ $(x+2)(x-3)$

$x=-2$ or 3 M1



$2<x>3$ ^ A0

Exemplar 3

1 mark

2(ii)

[illegible]

$-x^2+x+6>0$

$x^2-x-6<0$

$(x-3)(x+2)<0$ M1

$x=3 \quad x=-2 <0$

$2 \quad 3^2-3-6=0$

$-2^2-2-6=-4$

$-2^2-2-6<0$

Examiner commentary

A number of candidates were able to produce well-presented correct solutions (Exemplar 1). However, many others, although able to identify key values, were unable to correctly deal with relating these to the inequality (Exemplars 2 and 3).

Question 3(i)

3 **(i)** Solve the equation $\sin^2\theta = 0.25$ for $0° \leqslant \theta < 360°$. **[3]**

Exemplar 1 — 3 marks

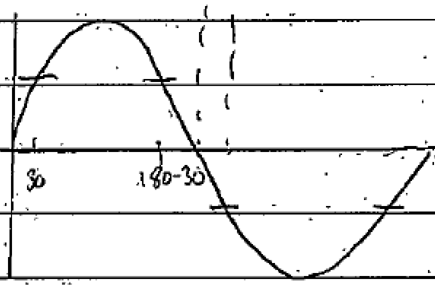
3(i)

$\sin^2\theta = 0.25$

$\sin\theta = \pm 0.5$

$\theta = \sin^{-1}(\pm 0.5)$

$= 30°, 150°, 210°, 330°$



Exemplar 2 — 2 marks

3(i)

$\sin^2\theta = 0.25$

$\therefore \sin\theta = \pm\sqrt{0.25}$ B1

$\sin\theta = \pm\frac{1}{2}$

$\therefore \sin\theta = \frac{1}{2}$ or $\sin\theta = -\frac{1}{2}$

$\theta = \sin^{-1}(\frac{1}{2})$ $\quad$ $\sin\theta = \cancel{\sin^{-1}}\ -30°$

$\theta = 30°$

BUT $\Rightarrow$ for $0° \leq \theta < 360$ $\therefore$

$\theta = 30°, \cancel{330°}, 150°, 330°$ B1

Exemplar 3

1 mark

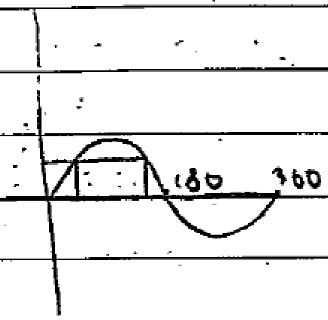
3(i)

$\sin^2\theta = 0.25$ $\quad 0 \le \theta \le 360$

$\sin\theta = \frac{1}{2}$ B0

$\theta = 30$

~~180-15θ~~ $180 - 30 = 150$



$\theta = 30°, 150°$ B1

B0

Exemplar 4

0 marks

3(i)

$\sin^2\theta = 0.25$ $\quad 0° \le \theta < 360°$

$\sin\theta^2 = 0.25$

Let $\theta^2 = u$

$\sin u = 0.25$

$u = 14.5°, 180 - 14.5$

$u = 14.5°, 165.5°$

$\therefore \sin^2\theta = 14.5°, 165.5°$

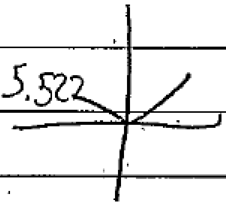
Exemplar 5

0 marks

3(i)

$\sin^2\theta = 0.25$ BO

$\sin^{-1}(0.25) = 14.477°, 165.522, 374.477, 525.522$



$\theta^2 = 14.477°, 165.522°, \cancel{165.522°}, 374.477°, 525.522°$

$\theta = 3.8°, 12.865°, 19.351°$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$1 - \cos^2\theta = 0.25$

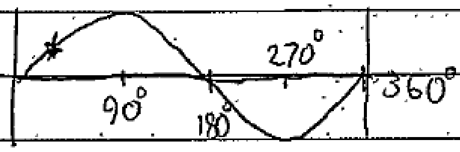
$\cos^2\theta = 0.75$

$\cos^{-1}(0.75) = 41.41$ BO BO

Exemplar 6

0 marks

3(i)



$\sin^{-1}(0.25) = 14.478$

$(14.478)^2 = 209.598$ $\quad \theta = 209.60$

Examiner commentary

Very few candidates were able to give a completely correct solution (Exemplar 1). Some only gave three rather than four solutions (Exemplar 2). Many candidates omitted $\sin\theta = -0.5$ (Exemplar 3). Others found $\sin^{-1}(0.25) = 14.5°$ (Exemplar 4). Of these, some then found $\sqrt{14.5}$ or 14.5^2 (Exemplars 5 and 6).

Question 3(ii)

(ii) In this question you must show detailed reasoning.

Solve the equation $\tan 3\phi = \sqrt{3}$ for $0° \leqslant \phi < 90°$. **[3]**

Exemplar 1 — 3 marks

3(ii)

$\tan 3\emptyset = \sqrt{3}$ $\quad 0° - 90°$

$3\emptyset = \tan^{-1}\sqrt{3} = 60$ $\quad \tan x = (x \pm 180)$

$3\emptyset = 60°, 60+180, 60+180+180, 60+180+180+180$

$60, 240, 420, 600$

$\emptyset = \frac{60}{3}, \frac{240}{3}, \frac{420}{3}, \frac{600}{3}$

$\emptyset = 20°, 80°$ $\quad \cancel{140}, \cancel{200}$

$140° > 90°$ $\quad 200° > 90°$

out of range

Exemplar 2 — 1 mark

3(ii)

$\tan^{-1}(\sqrt{3}) = 60$

$\frac{60}{3} = 20$ $\quad$ ^ $\quad$ B0

$\emptyset = 20°.$ $\quad$ B1

$\tan 3(20) = \sqrt{3}$

^ $\quad$ B0

Exemplar 3

0 marks

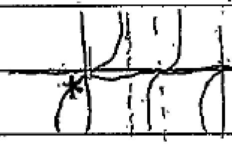
3(ii)

$\tan 3\varnothing = \sqrt{3}$ B0

$\tan^{-1}(\sqrt{3}) = 60$ B0

$(3 \times 60) = 180$ X

B0



Examiner commentary

A good number of candidates gave completely correct solutions (Exemplar 1). Most obtained "3ø" = 60°, but not all included 240° (Exemplar 2). Many then thought that they needed to multiply rather than divide by 3 (Exemplar 3).

Question 4(i)(a)

4 **(i)** It is given that $y = x^2 + 3x$.

(a) Find $\dfrac{dy}{dx}$. **[2]**

Exemplar 1 2 marks

4(i)(a)	$\dfrac{dy}{dx} = 2x^1 + 3$

Exemplar 2 1 mark

4(i)(a)	$\dfrac{dy}{dx} = x + 3$

Examiner commentary

This question was answered correctly by almost all candidates (Exemplar 1 where x^1 was allowed) although some only dealt correctly with one term (Exemplar 2).

Question 4(i)(b)

(b) Find the values of x for which y is increasing. **[2]**

Exemplar 1 2 marks

4(i)(b)

$y = x^2 + 3x$

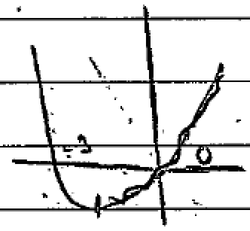
when $y = 0$, $x = 0, -3$

$\frac{dy}{dx} = 2x + 3$

$2x + 3 = 0$

$x = -\frac{3}{2}$

y is increasing for : $x > -\frac{3}{2}$ ✓



Exemplar 2 0 marks

4(i)(b)

y is increasing : $x^2 + 3x > 0$ ✗

$x(x+3) > 0$

$x < -3,\ x > 0$ y is increasing for these values.

Exemplar 3 0 marks

4(i)(b)

$2x + 3 = 0$ M0 $y = (-1.5)^2 + 3(1.5)$

$2x = -3$ $= 6.75$

$x = -1.5$ ✗

Sub. $x = -1.5$ into $x^2 + 3x$ $\boxed{y = 6.75}$ ✗

Exemplar 4

0 marks

4(i)(b)	$\frac{d^2y}{dx^2} = 2$

Examiner commentary

A good number were able to give correct well-presented solutions (Exemplar 1). A significant minority of candidates appeared not to understand what is meant by "increasing" (Exemplar 2 where $y > 0$ was found). Some found the coordinates of the stationary point, but did not recognise this as the minimum and did not proceed from this to the answer (Exemplar 3). Others seemed not to understand the question and merely stated the second differential (Exemplar 4).

Question 4(ii)

(ii) Find $\int(3-4\sqrt{x})\,dx$. **[5]**

Exemplar 1 — 5 marks

4(ii)

$\int(3-4\sqrt{x})\,dx = \int(-4x^{1/2}+3)\,dx$

$= -\frac{8}{3}x^{3/2} + 3x + c$

Exemplar 2 — 4 marks

4(ii)

$\int(~~e~~3-4\sqrt{x})\,dx$

$\int 3-4x^{\frac{1}{2}}\,dx$ M1

$= 3x - \frac{8}{3}x^{\frac{3}{2}}$

B1 M1 A1 ^ B0

Exemplar 3 — 3 marks

4(ii)

$\int(3-4\sqrt{x})\,dx = \int(3-4x^{\frac{1}{2}})\,dx$ M1

$3x\sqrt{} = \left(3x - \frac{6x^{1\frac{1}{2}}}{\cancel{1\cdot 5}}\;\cancel{6x^{1\cdot 5}} + C\right)$

B1 M1

$\int\left(3x - 4x^{1\frac{1}{2}} + C\right)$

A0 B0

Exemplar 4

2 marks

4(ii)

$\int (3 - 4\sqrt{x})\, dx$

$= 4x^{\frac{1}{2}}$ M1

$= 2x^{-\frac{1}{2}} + c$

^

B0 M0 A0 B1

Exemplar 5

1 mark

4(ii)

$\int -4\sqrt{x}^{2} + 3\, dx$

$\frac{-4\sqrt{x}^{2}}{2} + 3x$ M0

$-2\frac{\sqrt{x}^{2}}{2} + 3x$ B0

B1 ^

M0 A0

Examiner commentary

This question was answered correctly by many candidates (Exemplar 1). A few candidates omitted "+ *c*" (Exemplar 2).

A few obtained $x^{\frac{3}{2}}$ correctly, but with an incorrect coefficient (Exemplar 3, where the B mark was also lost due to the inclusion of the integral sign in the final answer). A few correctly recognised the index for root x but then differentiated rather than integrating (Exemplar 4, although they did also recognise the need for "+ *c*"). Some "integrated" $4\sqrt{x}$ to give $4\frac{\left(\sqrt{x}\right)^{2}}{2}$ (Exemplar 5, although they did correctly integrate the 3 term).

Question 5

5 N is an integer that is not divisible by 3. Prove that N^2 is of the form $3p+1$, where p is an integer. **[5]**

Exemplar 1 — 5 marks

5 Let $N = 3n+1$; M1

$(3n+1)^2$

$= 9n^2 + 6n + 1$ M1

$= 3(3n^2+2n)+1 = 3p+1$, where $p = 3n^2+2n$

A1

~~Let N = 3n+2:~~

~~$(3n+2)^2$~~

~~$= 9n^2+12n+4$~~

~~$= 3(3n^2+4n) +1 = 3p+1$, where $p = 3n^2+4n$~~

~~These are the only two ways types possibilities for an integer not divisible by~~

Let $N = 3n-1$

$(3n-1)^2$

$= 9n^2 - 6n + 1$ A1

$= 3(3n^2-2n)+1 = 3p+1$, where $p = 3n^2-2n$

Since these are the only two possibilities for N not divisible by 3, N^2 must be of the form $3p+1$; where p is an integer, by exhaustion. QED B1

Exemplar 2

0 marks

5 N isn't divisible by 3

$N = 5$ not divisible by 3

$N^2 = 25$

$3(8) + 1 =$

$24 + 1 = 25$

$2^2 = 4$ $3(1) + 1$ The three times table

$4^2 = 16$ $3(5) + 1$ is is one away from

$5^2 = 25$ $3(8) + 1$ all square numbers not

$7^2 = 49$ $3(16) + 1$ divisible by 3.

$8^2 = 64$ $3(21) + 1$

Therefore $3e + 1$ means

that it covers all square numbers

as e could be any number from

0 to infinity.

Examiner commentary

This question tests "proof by exhaustion" as included in paragraph 1.01a of the specification. This method of proof involves either considering all possible values or all possible categories of values. This question tests the latter. Although some candidates were able to present an excellent proof (Exemplar 1), it was not well answered on the whole. Many candidates merely verified the result in a few numerical cases (Exemplar 2). These scored no marks.

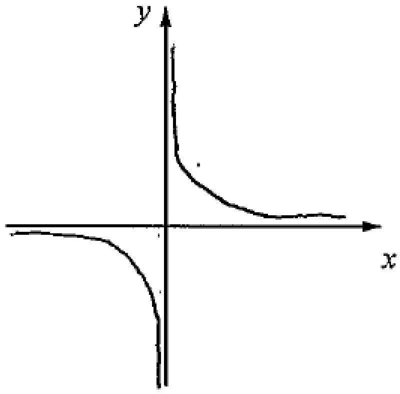
Question 6(i)

6 Sketch the following curves.

(i) $y = \frac{2}{x}$ **[2]**

Exemplar 1 — 2 marks

6(i)



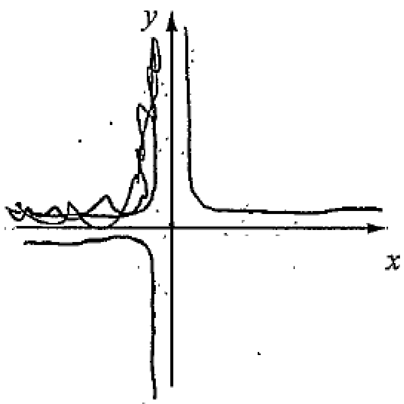
B1

B1

asymptotes ~~at~~ on the x and y axis

Exemplar 2 — 1 mark

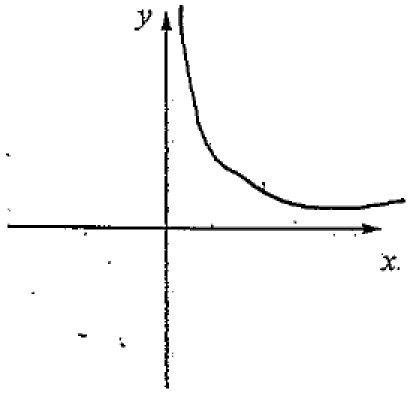
6(i)



Exemplar 3

0 marks

6(i)



Examiner commentary

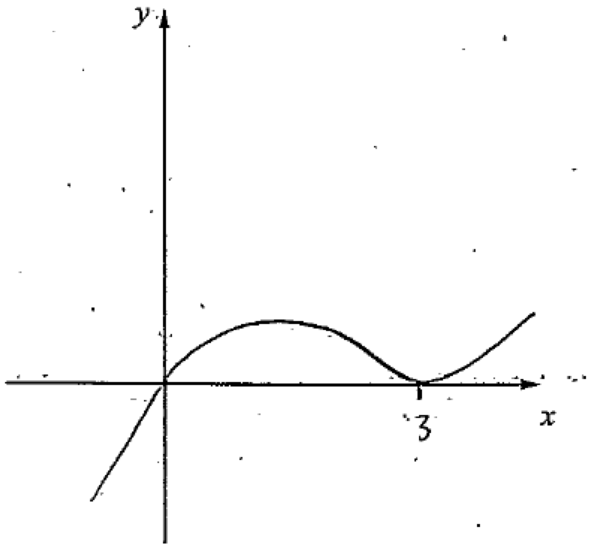
Most candidates knew what the graph looked like and a good number were able to gain full marks (Exemplar 1) but many sketched it carelessly, with one or more "tails" moving slightly away from the axis instead of towards it (Exemplar 2, where the crossed out part is ignored). Some candidates drew the curve in only one quadrant (Exemplar 3 where the curve is also carelessly sketched so scores no marks).

Question 6(ii)

(ii) $y = x^3 - 6x^2 + 9x$ **[5]**

Exemplar 1 5 marks

6(ii)



If $y = x^3 - 6x^2 + 9x$,

when $x = 0$, $y = 0$

when $y = 0 \Rightarrow$ $x^3 - 6x^2 + 9x = 0$

$x(x^2 - 6x + 9) = 0$

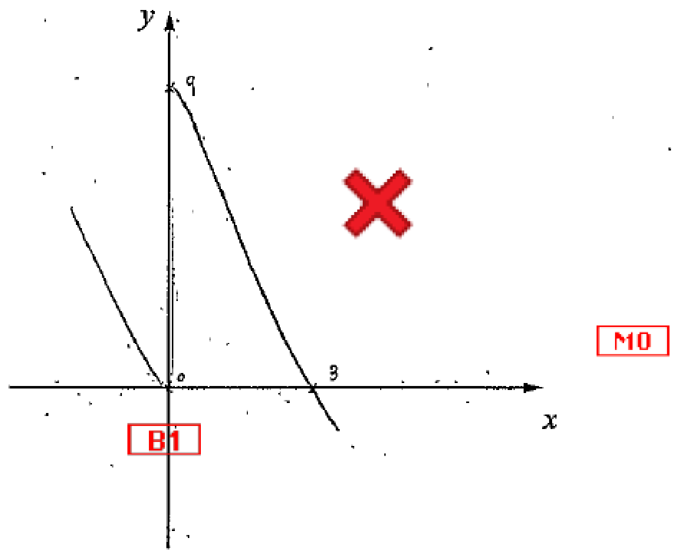
$x(x-3)^2 = 0$

$\therefore$ single root at $x = 0$, double root at $x = 3$

Exemplar 2

2 marks

6(ii)



$y = (x)(x-3)(x-3)$ M1

$y = (-3)(-3)$

$= 9.$

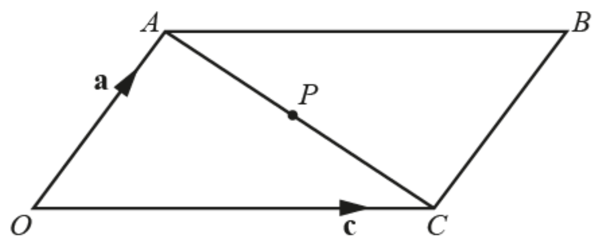
$x = 0, 3, 3$

Examiner commentary

Many were able to draw this curve correctly (Exemplar 1).Some candidates factorised correctly but then drew the curve touching the *x*-axis at $x = 0$ instead of $x = 3$ (Exemplar 2).

Question 7(i)(a)

7 $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. P is the midpoint of AC.



(i) Find the following in terms of **a** and **c**, simplifying your answers.

(a) $\overrightarrow{AC}$ **[1]**

Exemplar 1 — 1 mark

7(i)(a)	$\overrightarrow{AC} = -\underset{\sim}{a} + \underset{\sim}{c}$
	$= \underset{\sim}{c} \cancel{+} - \underset{\sim}{a}$

Examiner commentary

Almost all candidates answered this correctly (Exemplar 1).

Question 7(i)(b)

(b) $\overrightarrow{OP}$ **[2]**

Exemplar 1 — 2 marks

7(i)(b)

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= \underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{c} - \underset{\sim}{a})$

$= \underset{\sim}{a} + \frac{1}{2}\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}$

$= \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{c}$ ✓

Exemplar 2 — 1 mark

7(i)(b)

$\overrightarrow{OP} = a + \frac{1}{2}(c-a)$

$\overrightarrow{OP} = a + \frac{c-a}{2}$ M1

$\overrightarrow{OP} = 2a + c - a$ A0

Exemplar 3 — 0 marks

7(i)(b)

$\overrightarrow{OP} = \frac{1}{2}(-a+c)$ M0

$= -\frac{a}{2} + \frac{c}{2}$

Examiner commentary

Many answered this correctly (Exemplar 1). A few gave correct initial statements but did not simplify these appropriately (Exemplar 2). Others seemed unaware of the difference between position vectors and direction vectors, simply giving half the vector AC (Exemplar 3).

Question 7(ii)

(ii) Hence prove that the diagonals of a parallelogram bisect one another. **[4]**

Exemplar 1 — 4 marks

7(ii)

Point P is the midpoint of AC as given.

$\overrightarrow{OB} = \underset{\sim}{a} + \underset{\sim}{c}$ M1

$\overrightarrow{OP} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{c}) = \frac{1}{2}(\overrightarrow{OB})$ A1

Thus P is also the midpoint of diagonal $\overrightarrow{OB}$. A1

Since P is the midpoint of both lines, the lines intersect at each other's midpoint, so they bisect. B1

Exemplar 2 — 2 marks

7(ii)

~~To prove this, I must prove that ABCO is a parallelogram.~~

$\overrightarrow{OP} = \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{c}$

$\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

$\overrightarrow{BO} = -\underset{\sim}{c} - \underset{\sim}{a}$

$\overrightarrow{BP} = \overrightarrow{BA} + \overrightarrow{AP}$

$= -\underset{\sim}{c} + \frac{1}{2}\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}$ M1

$= -\frac{1}{2}\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}$

$\overrightarrow{BP} = -\overrightarrow{OP}$ A1 and $|\overrightarrow{BO}| = |\overrightarrow{AC}|$

Hence diagonals cross. ^ A0 B0

Exemplar 3 0 marks

7(ii)

~~Assump~~ $\vec{AC} = c - a$

$\vec{BO} = -c + a$ M0

~~[illegible]~~ they're perpendicular?

Exemplar 4 0 marks

7(ii)

$\vec{OP}$ = reciprical of $\vec{AC}$

~~$a + \frac{(c-a)}{2} = (c-a)$~~

~~$c + \frac{(c-a)}{2} = 0$~~

$2\cancel{c} + c - a$

$3c - a = 0$

~~$3c$~~ ~~$2a$~~

Exemplar 5

0 marks

7(ii)

$\overline{AO}$ and $\overline{BC}$ are parallel.

so $\overline{AO} = \overline{BC}$

$\overline{AB}$ and $\overline{OC}$ are parallel.

so $\overline{AB} = \overline{OC}$

$\overline{AC}$ Bisects Δ AOC to Δ ABC.

$\overline{AC}$ Bisects this parallelogram.

Examiner commentary

Although some were able to give a clear and concise proof (Exemplar 1), it was clear that many candidates had not significantly met the concept of proof using vectors. A number of the more able of these were able to make some correct statements but did not develop these into a complete proof (Exemplar 2). Some did not make a correct start and misstated the vector *OB* (Exemplar 3). However, perhaps the majority of candidates did not know how to start answering this question at all (Exemplar 4 and 5).

Question 8

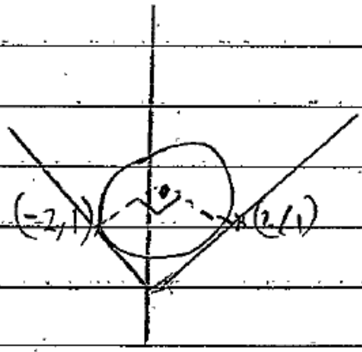
8 **In this question you must show detailed reasoning.**

The lines $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$ are tangents to a circle at (2, 1) and (−2, 1) respectively. Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. **[6]**

Exemplar 1

6 marks

8



~~Since the gradients of the tangents are perpendicular~~

$m_1 = -2$:

$y - 1 = -2(x-2)$

$y - 1 = -2x + 4$

(1) $y = -2x + 5$

$m_2 = 2$

$y - 1 = 2(x+2)$

$y = 2x + 5$

$2x + 5 = -2x + 5$

$x = 0$ } at centre

$\therefore$ $y = 5$

radius $= \sqrt{(0-2)^2 + (5-1)^2}$

$= 2\sqrt{5}$

$(x-0)^2 + (y-5)^2 = 20$

$x^2 + y^2 - 10y + 5 = 0$

$(a = 0)$

Exemplar 2

0 marks

8

$y = \frac{1}{2}x \quad , \quad y = -\frac{1}{2}x \qquad (2, 1) \ \& \ (-2, 1)$

$(x-a)^2 + (y+b)^2 = r^2$

$y = \frac{1}{2}x \qquad\qquad y = -\frac{1}{2}x$

$\frac{1}{2} \times m_\perp = -1 \qquad\qquad -\frac{1}{2} \times m_\perp = -1$

$m_\perp = \frac{-1}{\frac{1}{2}} \qquad\qquad m_\perp = \frac{-1}{-\frac{1}{2}}$

$m_\perp = -2 \qquad\qquad m_\perp = 2.$

$(x-2)^2 + (y+1)^2 = r^2 \quad -①$

$(x+2)^2 + (y+1)^2 = r^2 \quad -②$

① $\Rightarrow x^2 - 4x + 4 + y^2 + 2y + 1 = r^2$

② $\Rightarrow x^2 + 4x + 4 + y^2 + 2y + 1 = r^2$

$\Rightarrow x^2 + y^2 - 4x + 2y + 1 = 0$

Exemplar 3 — 0 marks

8

midpoint $= \left(\frac{2+(-2)}{2}\right), \left(\frac{1+1}{2}\right)$

$(0, 1)$ (radius)

gradient $= \frac{1-1}{-2-2} = 0$

$y - y_1 = m(x - x_1)$

$\frac{1}{2}x - 1 = 0(x - 0)$ M0

$\frac{1}{2}x = 1$

$x = \frac{1}{\frac{1}{2}}$ M0

$x = 2$

$y = \frac{1}{2}(2)$

$y = 1$

$y = mx + c$

$x^2 + y^2 + 4x + 4y + 0 = 0$ M0

Examiner commentary

Some produced clear and succinct solutions (Exemplar 1) but many candidates did not draw a sketch, thereby making it extremely difficult to make a reasonable attempt. Some did not consider one or both normals and, therefore, were unable to make any significant progress (Exemplar 2 where they recognised the need to find the gradients of the normals but did nothing with these). Many found the midpoint of the line joining (2, 1) and (–2, 1) believing this to be significant but appeared to have little knowledge of the standard equation of a circle in terms of the centre coordinates and the radius (Exemplar 3).

Question 9(i)

9 Jo is investigating the popularity of a certain band amongst students at her school. She decides to survey a sample of 100 students.

(i) State an advantage of using a stratified sample rather than a simple random sample. **[1]**

Exemplar 1 — 1 mark

9(i)	A stratified sample better represents the proportions of different sub-groups within the population. B1

Exemplar 2 — 0 marks

9(i)	using stratified sampling could get more accurate results from the students as ~~[illegible]~~ Jo may not be asking the correct range ~~of~~ of audience therefore obtaining results that will be mis leading. ^

Exemplar 3 — 0 marks

9(i)	It is easier and cheaper.

Exemplar 4 — 0 marks

9(i)	A stratified sample is representative of the whole population. ^

Examiner commentary

Few completely correct answers were seen (Exemplar 1). Many incorrect answers were seen, such as "It is more accurate"(Exemplar 2) or "Not biased" or "It's easier" (Exemplar 3). Many inadequate answers were also seen, such as "It is representative" (Exemplar 4).

Question 9(ii)

(ii) Explain whether it would be reasonable for Jo to use her results to draw conclusions about all students in the UK. **[1]**

Exemplar 1 — 1 mark

9(ii)	No, because it is only a small sample and biased due to the sample only coming from her particular school therefore it does not reflect other schools/ students in the UK.

Exemplar 2 — 1 mark

9(ii)	It would not be resonable because the results are only from her school / area so doesnt reflect the UK. B1 Students elsewhere in the UK might not like the band.

Examiner commentary

A reason why the school might not be representative was required for the mark (Exemplar 1 and 2 illustrate the variety of acceptable responses).

Question 10(i)

10 The probability distribution of a random variable X is given in the table.

x	0	2	4	6
$\mathrm{P}(X=x)$	$\frac{3}{8}$	$\frac{5}{16}$	$4p$	p

(i) Find the value of p. **[2]**

Exemplar 1 — 2 marks

10(i)

$\frac{3}{8} + \frac{5}{16} + 4p + p = 1$

$\frac{11}{16} + 4p + p = 1$

$5p = 1 - \frac{11}{16}$

$5p = \frac{5}{16}$ $\quad p = \frac{1}{16}$

Exemplar 2 — 0 marks

10(i)

$\therefore 16 = 3 + 5 + 4p + p$

M0

$8 = 5p$

$p = 1.6$

Examiner commentary

This question was answered correctly by most candidates (Exemplar 1). Some gave an indication that they knew that they needed to use Σp but they did not give a correct initial statement (Exemplar 2).

Question 10(ii)

(ii) Two values of X are chosen at random. Find the probability that the product of these values is 0. **[3]**

Exemplar 1 3 marks

10(ii)

Product is 0 when one or the other is 0.

$P(\text{product} = 0) = \cancel{0\%}\ 1 - (P(x \neq 0))^2$

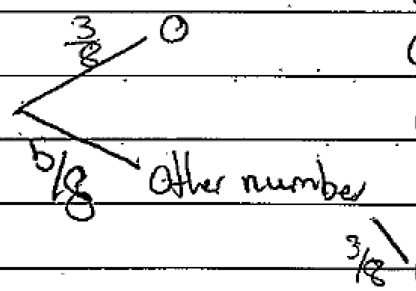
$= 1 - \left(\frac{5}{8}\right)^2$

$= 1 - \frac{25}{64}$

$= 0.609$

Exemplar 2 3 marks

10(ii)



0,2

0,0

0,4

0,6

$\frac{3}{8} + \left(\frac{5}{8} \times \frac{3}{8}\right)$

$= 0.609$

Exemplar 3

1 mark

10(ii)

$P(\text{choose } 0 \text{ then } 2) + P(\text{choose } 0 \text{ then } 4) + P(\text{choose } 0 \text{ then } 6)$

$= \frac{15}{128} + \frac{3}{32} + \frac{3}{128}$

$= \frac{15}{64}$ M1

$\frac{15}{64} \times 2 = \frac{15}{32}$ ^ M0

This is because they can pick the values of X either the 0 then the other number, or the other number and then the 0.

Ans: $\frac{15}{32}$ ✗

Exemplar 4

0 marks

10(ii)

$0 = 3/18$ $2 = \frac{5}{18}$ $4 = \frac{4}{18}$ $6 = \frac{1}{18}$

$= 6/18$

	0	2	4	6
0	0	0	0	0
2	0	4	8	12
4	0	8	16	24
6	0	12	24	36

12 out comes

6 ways to get 0

$p = \frac{6}{12} = \frac{1}{2}$ ✗

Examiner commentary

A number of correct methods to gain full marks were seen (Exemplars 1 and 2) but most candidates scored only one mark because they omitted one or two of the possible routes to obtaining a product of 0 (Exemplar 3). Some ignored the given probabilities and assumed that all outcomes were equally likely (Exemplar 4 where they also make the mistake of concluding that there are only twelve possible outcomes of which six result in a product of 0).

Question 11(i)

11 The probability that Janice sees a kingfisher on any particular day is 0.3. She notes the number, X, of days in a week on which she sees a kingfisher.

(i) State one necessary condition for X to have a binomial distribution. **[1]**

Exemplar 1 — 1 mark

11(i) The sightings must each be independent from one another.

B1

Exemplar 2 — 1 mark

11(i) The probability of seeing a king fisher must not change.

B1

Exemplar 3 — 0 marks

11(i) The events of the random variable X must be independent from one another.

B0

Exemplar 4 — 0 marks

11(i) X is a constant

Examiner commentary

Only a few candidates were able to give correct solutions in context (Exemplars 1 and 2) whereas many gave no context (Exemplar 3) and thus did not gain the mark. Many candidates appeared to have little understanding of what was meant by the question, despite this type of question having been asked frequently in paper 4732 (Exemplar 4).

Question 11(ii)

Assume now that X has a binomial distribution.

(ii) Find the probability that, in a week, Janice sees a kingfisher on exactly 2 days. **[1]**

Exemplar 1

1 mark

11(ii)	$X \sim B(7, 0.3)$
	$p(X = 2)$
	$= {}^7C_2 \times 0.3^2 \times 0.7^5$
	$= 0.3176$

Exemplar 2

1 mark

11(ii)	$X \sim B(7, 0.3)$
	$P(x = 2) = \cancel{0.3177}$ 0.3176523
	$= \cancel{0.318}$ 0.318 (3sf)

Examiner commentary

Most candidates understood how to calculate a binomial probability. Some used the formula (Exemplar 1), while others used the calculator function, giving only the answer (Exemplar 2). This is acceptable because "detailed reasoning" is not asked for in this question.

Question 11(iii)

Each week Janice notes the number of days on which she sees a kingfisher.

(iii) Find the probability that Janice sees a kingfisher on exactly 2 days in a week during at least 4 of 6 randomly chosen weeks. **[3]**

Exemplar 1 — 3 marks

11(iii)

$X \sim B(6, 0.318)$

$P(x \geq 4) = \cancel{{}^6C_4 \times 0.318^4 \times (1-0.318)^2}$

$1 - P(x \leq 3)$

$= 1 - 0.9146$

$= 0.0854$

Exemplar 2 — 1 mark

11(iii)

~~P At least twice on~~

P(twice on at least 4 out of 6 weeks) = 1 − P(twice on three weeks)

P(twice on three weeks] = 0.3177×3 M1

$= 0.9530$ (4sf) M0

$1 - 0.9530 = 0.04704$ (4sf)

Exemplar 3 1 mark

11(iii)	
	$X \sim B(\cancel{\#}28, 0.3)$
	$P(X \leq \cancel{\#}2) = 0.9712045$
	$P(X = \cancel{6}) = \cancel{0.99} \quad 0.9997813$
	$0.9997813 - 0.9712045$
	$= 0.0285768$
	$\Rightarrow 0.0286$
	$X \sim B(\cancel{20}7, 0.3)$
	$\cancel{P(X \leq 2)}\ P(x=2) = 0.318$
	$P(X \leq 4) = 0.9712045$
	$0.9712045 - 0.3176523$ M1 M0
	$= 0.6535522$
	$= 0.654.$

Exemplar 4 0 marks

11(iii)	
	0.15

Examiner commentary

Many candidates understood that they needed to use their answer from part (ii) (Exemplar 1) but many did not do so correctly (Exemplars 2 and 3). Low ability candidates tended to show little or no working (Exemplar 4).

Question 12

12 It is known that 20% of plants of a certain type suffer from a fungal disease, when grown under normal conditions. Some plants of this type are grown using a new method. A random sample of 250 of these plants is chosen, and it is found that 36 suffer from the disease. Test, at the 2% significance level, whether there is evidence that the new method reduces the proportion of plants which suffer from the disease. **[7]**

Exemplar 1

7 marks

12 ~~X~B~~

$H_0: p = 0.2$ where p is the proportion of plants with

$H_1: p < 0.2$ the disease in the underlying population.

$X \sim B(250, 0.2)$

$P(X \leq 36) = 0.01389$

For a one-tailed test, critical value $= 0.02$

$0.01389 < 0.02$

$\therefore$ There is enough evidence to reject H_0:

the evidence suggests that ~~there~~ the new method might

reduce the population of plants suffering from

the disease.

Exemplar 2

6 marks

12 Let X be r.v. "suffer from fungal disease"

~~X~B(280, 0.2)~~ sig. level = 0.02

$H_0: p = 0.2$ B1

$H_1: p < 0.2$ ^ B0

Under H_0: $X \sim B(250, 0.2)$ M1

$P(X \leq 36) = 0.01389$ (4sf) A1 A1

$0.01389 < 0.02$, hence reject H_0. M1

Therefore there is enough evidence at the 2% significance level to reject the null hypothesis. This means there is enough evidence that the new method of growing plants reduces the proportion of plants that suffer from the disease and hence we have enough evidence to accept the alternate hypothesis. A1

Exemplar 3

2 marks

12

Hypothesis Testing

H_0 : $p = \frac{1}{5}$ plants suffering from fungal disease.

H_1 : $p < \frac{1}{5}$ plants suffer from fungal disease.

B1 B0

Significance level at ~~5~~ 2%

$X \sim B(250, 0.2)$ M1

$P(X = 36) = 0.00505051193417$ A0

A0

$0.02 > 0.00505051193417$ M0 A0

∴ H_1 is not rejected, there is sufficient evidence that the new ~~fertiliser~~ method reduces the portion of plants that suffer from the disease.

Exemplar 4

3 marks

12

2%

$X \sim B(250, \frac{1}{5})$ where x = number of plants with disease

$H_0: p = \frac{1}{5}$

$H_1: p < \frac{1}{5}$ B1 ^ B0

$P(X \leq 36) = 0.01389371934$ M1 A1

$\frac{36}{250} \times 100 = 14.4$ 1 tailed test

~~0.013893719~~ A0

$0.0134 < 0.2$ so H_0 is not accepted M0

There is insufficient evidence to accept H_0.

Theres insufficient evidence to suggest that the new method doesnt reduce disease

Theres sufficient evidence to suggest that the new method does reduce the proportion of plants with disease.

Exemplar 5

0 marks

12 H_0 : 0·

H_1 : 0·2 B0 B0

x = 36

n = 250

p = 0.2

$P(X \leq 0.2)$

$1 - P(X \leq 0.2)$ M0 M0

p = 0·981

Insufficient evidence that new method reduces the proportion.

Examiner commentary

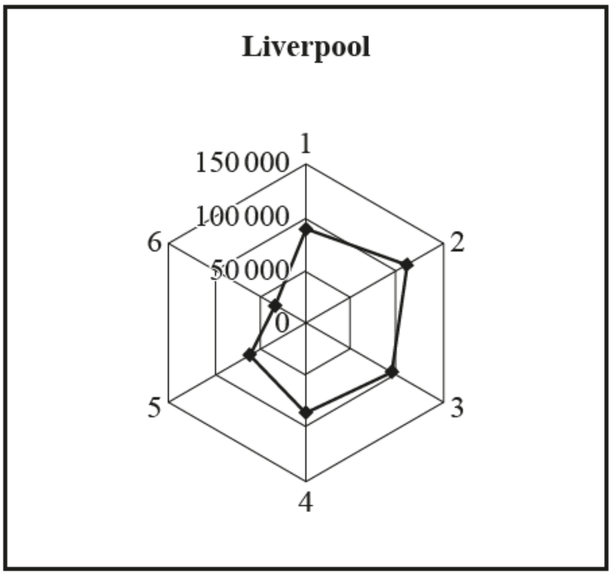
Many candidates had clearly been well prepared for a hypothesis test question (Exemplar 1). However, even these often made errors. Examples of such errors were as follows.

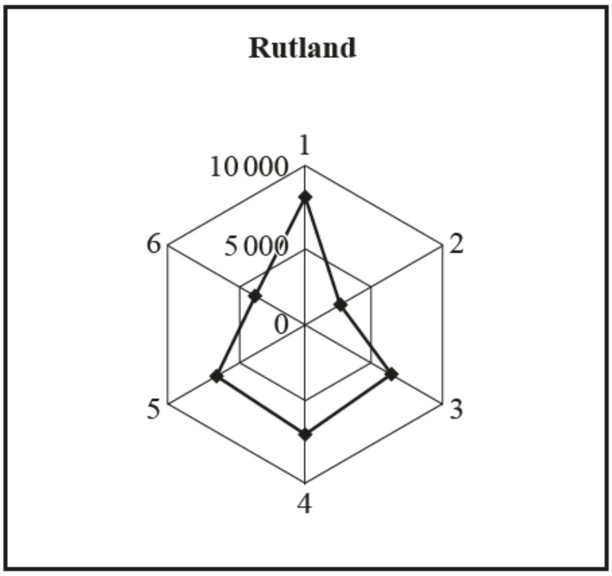
- Failure to define "*p*" in the hypotheses (Exemplar 2)
- Finding $P(X = 36)$ instead of $P(X \leq 36)$ (Exemplar 3, where they also did not define "*p*")
- Comparing the probability with 0.2 instead of 0.02 (Exemplar 4, where they also did not define "*p*")

Some candidates appeared hardly to have met the concept of a hypothesis test (Exemplar 5).

Question 13(i)

13 The radar diagrams illustrate some population figures from the 2011 census results.





Each radius represents an age group, as follows:

Radius	1	2	3	4	5	6
Age group	0–17	18–29	30–44	45–59	60–74	75+

The distance of each dot from the centre represents the number of people in the relevant age group.

(i) The scales on the two diagrams are different. State an advantage and a disadvantage of using different scales in order to make comparisons between the ages of people in these two Local Authorities. **[2]**

Exemplar 1 — 2 marks

13(i) **Advantage:** There are many more people in Liverpool, hence using Rutland's scale would make Liverpool's diagram B1 too big. Using different scales allows for the data to appear presentable and of the right size.

Disadvantage: We cannot directly compare easily from the different scales. Some people may misinterpret that there are two different scales and hence misinterpret the size of the ~~[illegible]~~ populations. B1

Exemplar 2

0 marks

13(i)	**Advantage:**
	Accurate results are shown on both results tables
	Disadvantage: Harder to compare the two graphs to each other

Examiner commentary

Many candidates gave correct answers here, although frequently they used far more words than were required (Exemplar 1). Some gave inadequate answers (Exemplar 2).

Question 13(ii)

(ii) Approximately how many people aged 45 to 59 were there in Liverpool? **[1]**

Exemplar 1 — 1 mark

13(ii)	90,000 people

Exemplar 2 — 0 marks

13(ii)	~~40000 people~~ 30 000 people

Examiner commentary

Almost all candidates gave an acceptable answer to this question (Exemplar 1) although some gave answers which demonstrated misinterpretation of the question (Exemplar 2).

Question 13(iii)

(iii) State the main two differences between the age profiles of the two Local Authorities. **[2]**

Exemplar 1 — 2 marks

13(iii)
- Rutland has a much smaller proportion of group 2 (18-29). B1
- Rutland has a greater proportion of group 1 (0-17). B1

Exemplar 2 — 1 mark

13(iii) Theres ~~are more~~ a higher proportion of young people in liverpool compared to rutford (a higher proportion are under 89) B1 B0

Exemplar 3 — 0 marks

13(iii) There are alot more 18-29 in Liverpool than in Rutland.
There are more 0-17 in Rutland than in Liverpool ✗

Examiner commentary

A few were able to give clear and concise differences (Exemplar 1). Others gave only one difference rather than two as required (Exemplar 2). Answers that compared numbers (as opposed to proportions) of people in the two areas were not accepted in this part (Exemplar 3).

Question 13(iv)

(iv) James makes the following claim.

"Assuming that there are no significant movements of population either into or out of the two regions, the 2021 census results are likely to show an increase in the number of children in Liverpool and a decrease in the number of children in Rutland."

Use the radar diagrams to give a justification for this claim. **[2]**

Exemplar 1 — 2 marks

13(iv) Since Liverpool has so many more young adults ~~for~~ (group 2), they are more likely to have children. B1 Rutland is also likely to show a decrease since there are many children currently, so due to the lower number of young adults, the current proportion seems unlikely to be maintained. B1

Exemplar 2 — 1 mark

13(iv) Yes, because as liverpool have more young adult that want to have children while Rutland have lesser adult as it data shows more 30-74 group age and should already been married.

B1

B0

Examiner commentary

Most candidates recognised the key point here, which was that children are being born during the 10 years from 2011 (Exemplar 1). However, some only stated that Liverpool had a large proportion of people in the potential childbearing group, and did not state that Rutland has a small proportion in this group (Exemplar 2).

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