3E Gradients of a Hyperbola

1. Find the equation of the tangent to the hyperbola with equation $\frac{x^2}{9} - \frac{y^2}{4} = 1$ at the point $(6, 2\sqrt{3})$

2. Prove that the equation of a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(a \cosh t\,,\ b \sinh t)$ is $\boldsymbol{ay \sinh t + ab = bx \cosh t}$

3. Show that an equation of the normal to the hyperbola with equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(\mathrm{a}\sec\theta\,, b\tan\theta)$ is $by + ax\sin\theta = (a^2 + b^2)\tan\theta$.

4. Show that the condition for the line $y = mx + c$ to be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is that m and c satisfy $b^2 + c^2 = a^2m^2$.

5. The tangent to the hyperbola with equation $\frac{x^2}{9} - \frac{y^2}{4} = 1$ at the point $(3\cosh t\,,\ 2\sinh t)$ crosses the y-axis at the point $(0, -1)$. Find the value of t.

6. The hyperbola H has equation $\frac{x^2}{36} - \frac{y^2}{9} = 1$.
The line l_1 is the tangent to H at the point $P(6\cosh t\,, 3\sinh t)$. The line l_2 passes through the origin and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point Q.
Show that the coordinates of the point Q are $\left(\frac{6\cosh t}{4\sinh^2 t + \cosh^2 t}, -\frac{12\sinh t}{4\sinh^2 t + \cosh^2 t}\right)$.