<u>3C Eccentricity</u>

Ellipse

Parabola

Hyperbola

1. Show that for $0 < e < 1$, the ellipse with focus $\mathrm{S} = (ae, 0)$ and directrix $x = \frac{a}{e}$ has equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

2.

a) Find foci of the ellipse with equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and give the equation of the directrices. Hence sketch the ellipse.

b) Find foci of the ellipse with equation $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and give the equation of the directrices. Hence sketch the ellipse.

3. If P is a point on an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $PS + PS' = 2a$

4. Show that for $e > 1$, the hyperbola with foci at $(\pm ae, 0)$ and directrices at $x = \pm\frac{a}{e}$ has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

5.

a) Sketch the hyperbola with equation $\frac{x^2}{9} - \frac{y^2}{4} = 1$, indicating the foci, directrices and equations of the asymptotes.

b) Sketch the hyperbola with equation $\frac{x^2}{16} - \frac{y^2}{25} = 1$, indicating the foci, directrices and equations of the asymptotes.

A quick note on hyperbolas and ellipses:

For hyperbolas, you don't care which of a and b are bigger. For ellipses, swapping the a and b has the effect of rotating the ellipse 90° and hence the foci/directrices too. We don't get this same rotation for hyperbolas.

Formula book p19

Conics

	Ellipse	**Parabola**	**Hyperbola**	**Rectangular Hyperbola**
Standard Form	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$y^2=4ax$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$xy=c^2$
Parametric Form	$(a\cos\theta, b\sin\theta)$	$(at^2, 2at)$	$(a\sec\theta, b\tan\theta)$ $(\pm a\cosh\theta, b\sinh\theta)$	$\left(ct, \frac{c}{t}\right)$
Eccentricity	$e<1$ $b^2=a^2(1-e^2)$	$e=1$	$e>1$ $b^2=a^2(e^2-1)$	$e=\sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x=\pm\frac{a}{e}$	$x=-a$	$x=\pm\frac{a}{e}$	$x+y=\pm\sqrt{2}c$
Asymptotes	none	none	$\frac{x}{a}=\pm\frac{y}{b}$	$x=0, y=0$

Note:

Tangent:	$ty=x+at^2$	$x+t^2y=2ct$
Normal:	$y+tx=2at+at^3$	$t^3x-ty=c(t^4-1)$