12B First Principles

1. A point A with coordinates (4,16) lies on the curve with equation $y = x^2$. At point A the curve has gradient g.

a) Show that $g = \lim_{\delta x \to 0} (8 + \delta x)$

b) Deduce the value of g

12C General Rules

1. Find $f'(x)$ for each of the expressions for $f(x)$ to the right.
a) $f(x) = x^6$

b) $f(x) = x^{\frac{1}{2}}$

c) $f(x) = x^{-2}$

d) $f(x) = x^2 \times x^3$

e) $f(x) = \frac{x}{x^5}$

f) $f(x) = 7x^3$

g) $f(x) = -4x^{\frac{1}{2}}$

h) $f(x) = \sqrt{36x^3}$

12D Multiple terms

1. Find $\frac{dy}{dx}$ when $y = x^2 - 6x - 4$

2. Find the gradient of the curve $y = x^3 + x^2$ when $x = 2$

3. Find the gradient of the curve $y = \frac{1}{3}x^{\frac{1}{2}} + 4x^2$ at the point $\left(1, \frac{13}{3}\right)$

12E Preparing for Calculus

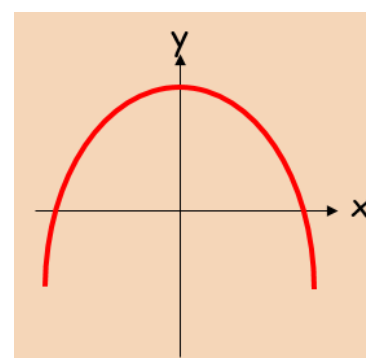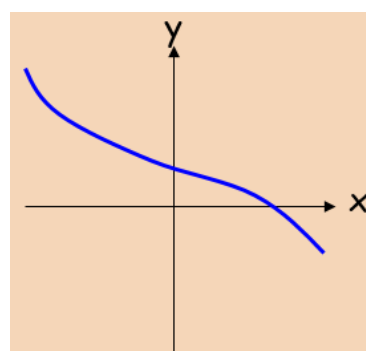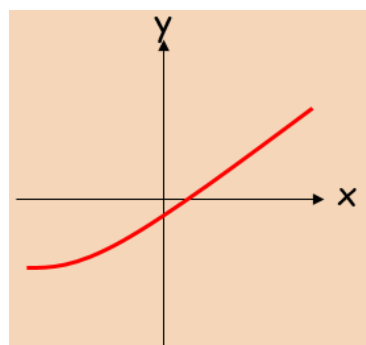1. Find $\frac{dy}{dx}$ when $y = \frac{1}{4\sqrt{x}}$

2. Find the gradient of the curve $y = \frac{x-2}{x^2}$ at the point (2,0)

12F Tangents & Normals

1. Find the equation of the underlined tangent to the curve $y = x^3 - 3x^2 + 2x - 1$, at the point (3,5).

2. Find the equation of the normal to the curve $y = 8 - 3\sqrt{x}$ at the point where $x = 4$.

12G Increasing & Decreasing Functions

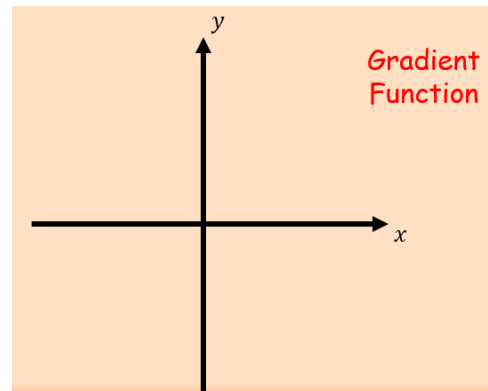

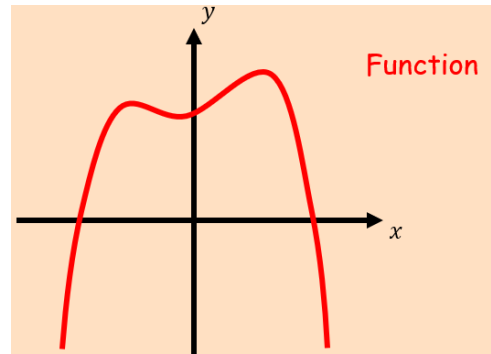1. Show that the function ;

$$f(x) = x^3 + 24x + 3$$

is an increasing function.

2. Find the range of values where:
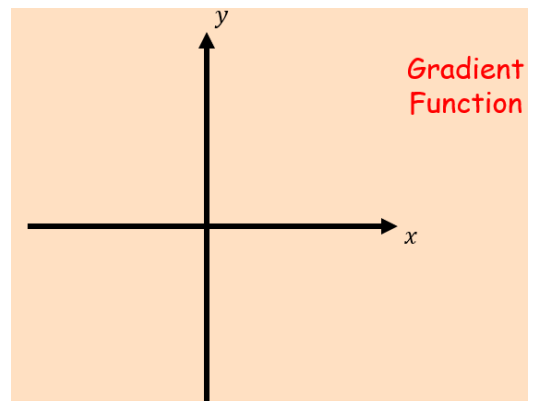$$f(x) = x^3 + 3x^2 - 9x$$
is a decreasing function.

12J Graphing the Gradient Function
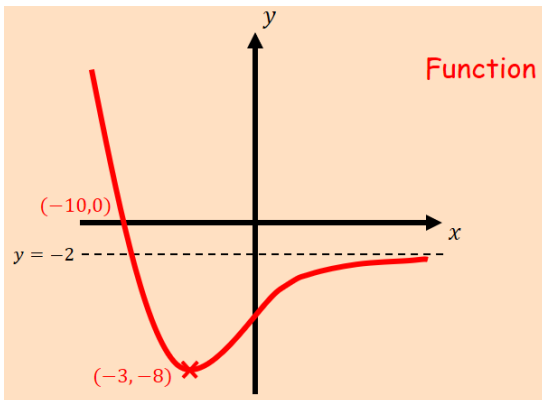

Function

Gradient Function

1. The diagram shows the curve with equation $y = f(x)$. It has an asymptote at $y = -2$, a turning point at $(-3, -8)$ and it cuts the x-axis at $(-10, 0)$

a) Sketch the graph of $y = f'(x)$

Function

(−10,0)

$y = -2$

(−3,−8) ✘

Gradient Function

b) State the equation of the asymptote of $y = f'(x)$
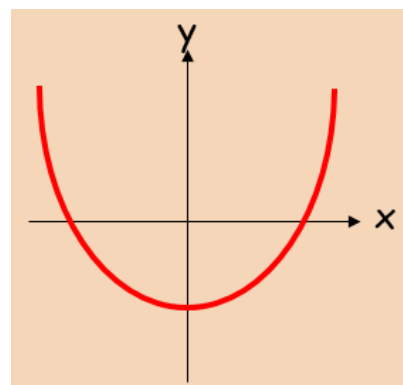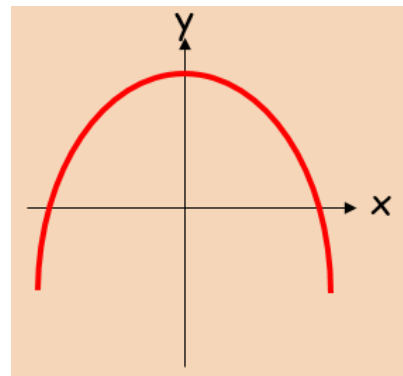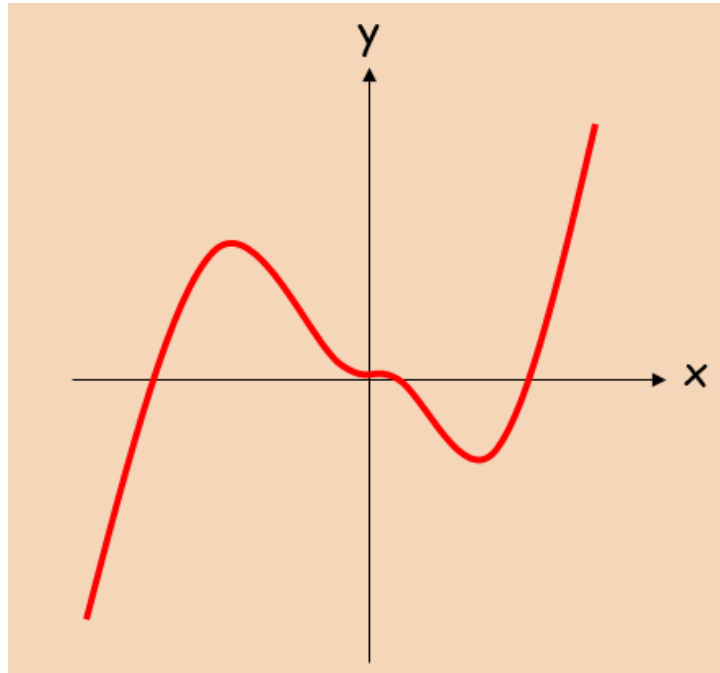
12H Second Derivatives

1. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ of the following:

$$y = 3x^5 + \frac{4}{x^2}$$

2. Find $f'(x)$ and $f''(x)$ of the following:

$$f(x) = 3\sqrt{x} + \frac{1}{2x}$$

12I Stationary Points

Note on f''(x) = 0

1. Find the coordinates of the turning point on the curve $y = x^4 - 32x$, and state whether it is a minimum or maximum.

2. Find the stationary points on the curve: $y = 2x^3 - 15x^2 + 24x + 6$, and state whether they are minima, maxima or points of inflexion

12K Differentiation in Context

1. Given that the volume, $V\ cm^3$, of an expanding sphere is related to its radius, $r\ cm$, by the formula $V = \frac{4}{3}\pi r^3$, find the rate of change of volume with respect to radius at the instant when the radius is 5cm.

2. A large tank (shown) is to be made from 54m² of sheet metal. It has no top.

a) Show that the Volume of the tank will be given by:

$$V = 18x - \frac{2}{3}x^3$$

b) Find the Maximum volume of the tank