Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number	Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2A: Further Pure Mathematics 1 and Further Pure Mathematics 2

Sample Assessment Material for first teaching September 2017
Time: 1 hour 40 minutes

Paper Reference
8FM0/2A

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

S58538A

1/1/1/1/

S 5 8 5 3 8 A 0 1 2 4

Pearson

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\sec x - \tan x \equiv \frac{1-t}{1+t} \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

(b) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2 \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

(Total for Question 1 is 6 marks)

2. The value, V hundred pounds, of a particular stock t hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V^2 - t}{t^2 + tV} \qquad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate, to the nearest £, the value of the trader's stock half an hour after it was purchased.

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

(Total for Question 2 is 6 marks)

3. Use algebra to find the set of values of x for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 6 marks)

4.

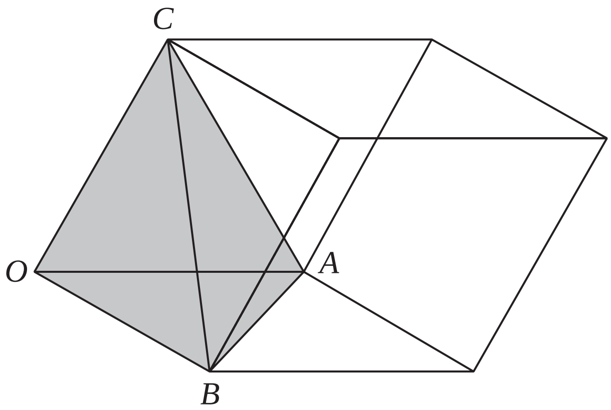

Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 1)$ and $C(1, 1, 2)$, where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

(a) Find the surface area of the glued face of the tetrahedron.

(4)

(b) Find the volume of glass contained in this parallelepiped.

(5)

(c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture.

(1)

Question 4 continued

(Total for Question 4 is 10 marks)

5.

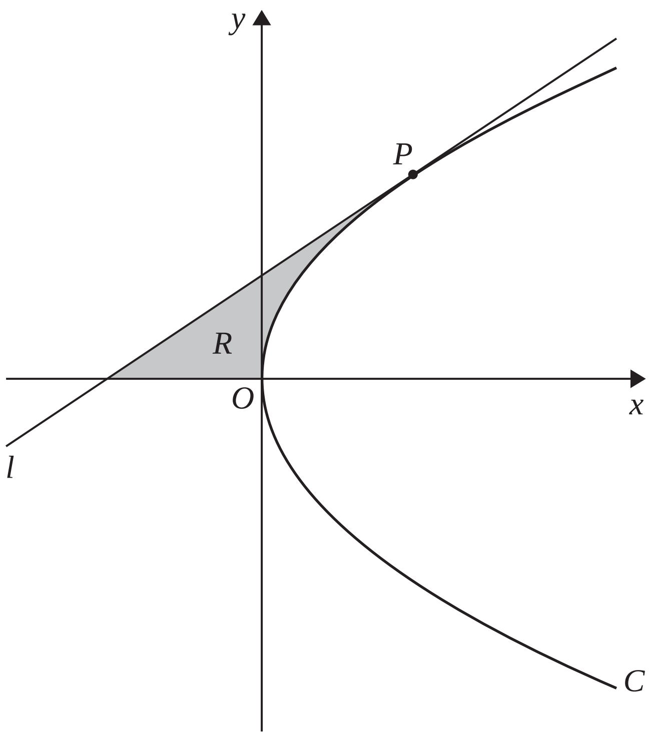

Figure 2

[You may quote without proof that for the general parabola $y^2 = 4ax$, $\dfrac{dy}{dx} = \dfrac{2a}{y}$]

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

The line l is the tangent to C at the point P.

(b) Show that an equation for l is

$$py = x + 4p^2$$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\dfrac{10}{3}$

Given that $p > 0$

(c) show that the area of R is 36

(8)

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

TOTAL FOR SECTION A IS 40 MARKS

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

6. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix}$$

(a) find the characteristic equation of the matrix **A**.

(2)

(b) Hence show that $\mathbf{A}^3 = 43\mathbf{A} - 42\mathbf{I}$.

(3)

Question 6 continued

(Total for Question 6 is 5 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

7. (i) Without performing any division, explain why 8184 is divisible by 6

(2)

(ii) Use the Euclidean algorithm to find integers *a* and *b* such that

$$27a + 31b = 1$$

(4)

Question 7 continued

(Total for Question 7 is 6 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

8. A curve C is described by the equation

$$|z - 9 + 12\mathrm{i}| = 2|z|$$

(a) Show that C is a circle, and find its centre and radius.

(4)

(b) Sketch C on an Argand diagram.

(2)

Given that w lies on C,

(c) find the largest value of a and the smallest value of b that must satisfy

$$a \leqslant \mathrm{Re}(w) \leqslant b$$

(2)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 8 continued

(Total for Question 8 is 8 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

9. The operation * is defined on the set $S = \{0, 2, 3, 4, 5, 6\}$ by $x*y = x + y = xy \pmod 7$

*	0	2	3	4	5	6
0						
2		0				
3						5
4						
5		4				
6						

(a) (i) Complete the Cayley table shown above

(ii) Show that S is a group under the operation *

(You may assume the associative law is satisfied.)

(6)

(b) Show that the element 4 has order 3

(2)

(c) Find an element which generates the group and express each of the elements in terms of this generator.

(3)

Question 9 continued

(Total for Question 9 is 11 marks)

10. A population of deer on a large estate is assumed to increase by 10% during each year due to natural causes.

The population is controlled by removing a constant number, Q, of the deer from the estate at the end of each year.

At the start of the first year there are 5000 deer on the estate.

Let P_n be the population of deer at the end of year n.

(a) Explain, in the context of the problem, the reason that the deer population is modelled by the recurrence relation

$$P_n = 1.1P_{n-1} - Q, \qquad P_0 = 5000, \qquad n \in \mathbb{Z}^+$$

(3)

(b) Prove by induction that $P_n = (1.1)^n (5000 - 10Q) + 10Q, \qquad n \geqslant 0$

(5)

(c) Explain how the long term behaviour of this population varies for different values of Q.

(2)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS