2F Problem Solving with Tangents & Normals

1. The point $P(at^2, 2at)$ lies on the parabola C with equation $y^2 = 4ax$ where a is a positive constant. Show that an equation of the normal to C at P is $y + tx = 2at + at^3$

2. The point $\left(ct, \frac{c}{t}\right), t \neq 0$, lies on the rectangular hyperbola H with equation $xy = c^2$ where c is a positive constant.

a) Show that an equation of the tangent to H at P is $x + t^2y = 2ct$.

A rectangular hyperbola G has equation $xy = 9$. The tangent to G at the point A and the tangent to G at the point B meet at the point $(-1,7)$.

b) Find the coordinates of A and B.

3. The parabola C has equation $y^2 = 20x$. The point $P(5p^2, 10p)$ is a general point on C. The line l is normal to C at the point P.

a) Show that an equation for l is $px + y = 10p + 5p^3$

The point P lies on C. The normal to C at P passes through the point $(30,0)$ as shown on the diagram. The region R is bounded by this line, the curve C and the x-axis.

b) Given that P lies in the first quadrant, show that the area of the shaded region R is $\frac{1100}{3}$