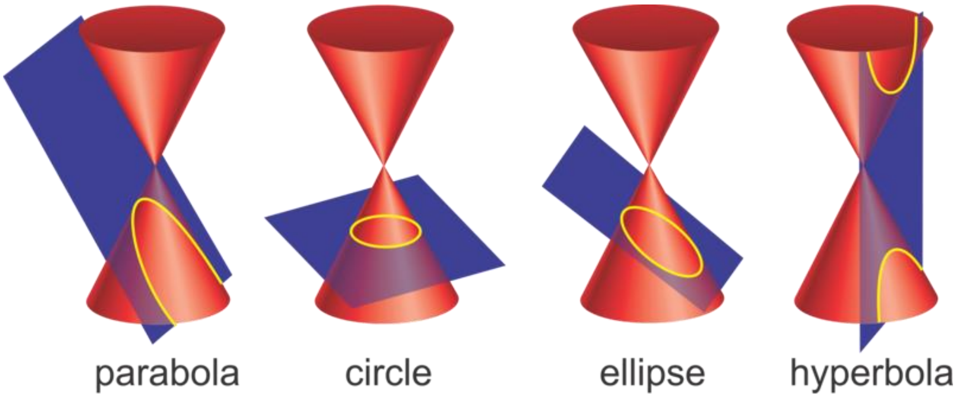
2A Parametrics Revisited



1. A curve has parametric equations

$$x = at^2, \qquad y = 2at, \qquad t \in \mathbb{R},$$

where a is a positive constant. Find the Cartesian equation of the curve

2.

A curve has parametric equations

$$x = ct, y = \frac{c}{t}, t \neq 0,$$

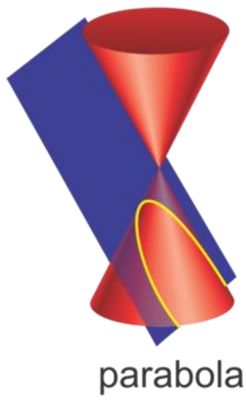
where c is a positive constant.

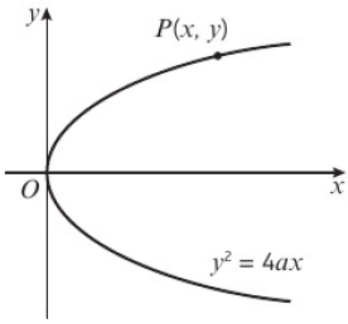
a) Find the Cartesian equation of the curve.

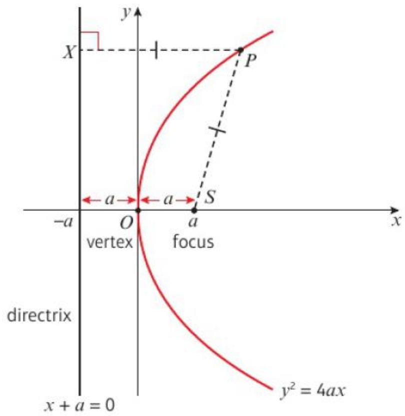
b) Hence sketch the curve

Note: Alternative approach (multiplying to cancel t)

2B Parabolas







1. Find the equation of the parabola with:

a) focus: $(7,0)$ and directrix $x + 7 = 0$

b) focus $\left(\frac{\sqrt{3}}{4}, 0\right)$ and directrix $x = -\frac{\sqrt{3}}{4}$

2. Find the coordinates of the focus and an equation of the directrix of a parabola with equation:

a) $y^2 = 24x$

b) $y^2 = \sqrt{32}x$

2C Parabolas & Chords

1. A point $P(8,\ -8)$ lies on the parabola C with equation $y^2 = 8x$. The point S is the focus of the parabola. The line l passes through S and P.

a) Find the coordinates of S.

b) Find an equation for l, giving your answers in the form $ax + by + c = 0$, where a, b, c are integers.

c) The line l meets the parabola C again at the point Q. The point M is the mid-point of PQ. Find the coordinates of Q.

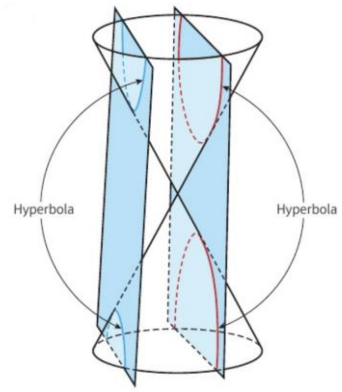
d) Find the coordinates of M.

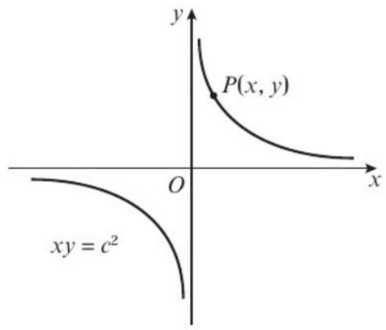
e) Draw a sketch showing parabola C, the line l and the points P, Q, S and M.

2. The parabola C has general point $(at^2, 2at)$. The line $x = k$ intersects C at the points P and Q. Find, in terms of a and k, the length of the chord PQ.

A quick note on integration:

2D Rectangular Hyperbolas





Eccentricity

What makes it rectangular?

1. The rectangular hyperbola H has Cartesian equation $xy = 64$. The line l with equation $x + 2y - 36 = 0$ intersects the curve at the points P and Q.

a) Find the coordinates of P and Q.

b) Find the equation of the perpendicular bisector of PQ in the form $y = mx + c$.

2E Tangents & Normals

1. The point P, where $x = 2$, lies on the rectangular hyperbola H with equation $xy = 8$. Find:

a) The equation of the tangent T.

b) The equation of the normal N to H at the point P, giving your answer in the form $ax + by + c = 0$.

2. The distinct points A and B, where $x = 3$ lie on the parabola C with equation $y^2 = 27x$.

a) The line l_1 is the tangent to C at A and the line l_2 is the tangent to C at B.
Given that at A, $y > 0$, find the coordinates of A and B.

b) Draw a sketch showing the parabola C. Indicate A, B, l_1 and l_2.

c) Find equations for l_1 and l_2, giving your answer in the form $ax + by + c = 0$.

3. The point P with coordinates $(75,30)$ lies on the parabola C with equation $y^2 = 12x$. Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$

4. The point $P(4,8)$ lies on the parabola C with equation $y^2 = 4ax$. Find:

a) The value of a

b) An equation of the normal to C at P

The normal to C at P cuts the parabola again at the point Q. Find:

c) The coordinates of Q

d) The length PQ, giving your answer as a simplified surd

2F Problem Solving with Tangents & Normals

1. The point $P(at^2, 2at)$ lies on the parabola C with equation $y^2 = 4ax$ where a is a positive constant. Show that an equation of the normal to C at P is $y + tx = 2at + at^3$

2. The point $\left(ct, \frac{c}{t}\right)$, $t \neq 0$, lies on the rectangular hyperbola H with equation $xy = c^2$ where c is a positive constant.

a) Show that an equation of the tangent to H at P is $x + t^2 y = 2ct$.

A rectangular hyperbola G has equation $xy = 9$. The tangent to G at the point A and the tangent to G at the point B meet at the point $(-1,7)$.

b) Find the coordinates of A and B.

3. The parabola C has equation $y^2 = 20x$. The point $P(5p^2, 10p)$ is a general point on C. The line l is normal to C at the point P.

a) Show that an equation for l is $px + y = 10p + 5p^3$

The point P lies on C. The normal to C at P passes through the point $(30,0)$ as shown on the diagram. The region R is bounded by this line, the curve C and the x-axis.

b) Given that P lies in the first quadrant, show that the area of the shaded region R is $\frac{1100}{3}$

2G Loci

1. The curve C is the locus of points that are equidistant from the line with equation $x + 6 = 0$ and the point $(6,0)$. Prove that C has Cartesian equation $y^2 = 4ax$, stating the value of a.

2. The point P lies on a parabola with equation $y^2 = 4ax$. Show that the locus of the midpoints of OP is a parabola.