**2A Parametrics Revisited**



1. A curve has parametic equations

$$x=at^{2}, y=2at, t\in R,$$

 where$ a$ is a positive constant. Find the Cartesian equation of the curve

A curve has parametric equations

$$x=ct, y=\frac{c}{t}, t\ne 0,$$

where $c$ is a positive constant.

1. Find the Cartesian equation of the curve.
2. Hence sketch the curve

Note: Alternative approach (multiplying to cancel t)

**2B Parabolas**







1. Find the equation of the parabola with:
2. focus: $(7,0)$ and directrix $x+7=0$
3. focus $\left(\frac{\sqrt{3}}{4},0\right)$ and directrix $x=-\frac{\sqrt{3}}{4}$
4. Find the coordinates of the focus and an equation of the directrix of a parabola with equation:
5. $y^{2}=24x$
6. $y^{2}=\sqrt{32}x$

**2C Parabolas & Chords**

1. A point $P(8, -8)$ lies on the parabola C with equation $y^{2}=8x$. The point $S$ is the focus of the parabola. The line l passes through S and P.
2. Find the coordinates of $S$.
3. Find an equation for $l$, giving your answers in the form $ax+by+c=0$, where $a$, $b$, $c$ are integers.
4. The line $l$ meets the parabola $C$ again at the point $Q$. The point $M$ is the mid-point of $PQ$. Find the coordinates of $Q$.
5. Find the coordinates of $M$.
6. Draw a sketch showing parabola $C$, the line $l$
and the points $P$, $Q$, $S$ and $M$.
7. The parabola $C$ has general point $\left(at^{2},2at\right)$. The line $x=k$ intersects $C$ at the points $P$ and $Q$. Find, in terms of $a$ and $k$, the length of the chord $PQ$.

A quick note on integration:

**2D Rectangular Hyperbolas**





Eccentricity

What makes it rectangular?

1. The rectangular hyperbola $H$ has Cartesian equation $xy=64$. The line $l$ with equation $x+2y-36=0$ intersects the curve at the points $P$ and $Q$.
2. Find the coordinates of $P$ and $Q$.
3. Find the equation of the perpendicular bisector of $PQ$ in the form $y=mx+c$.

**2E Tangents & Normals**

1. The point $P$, where $x=2$, lies on the rectangular hyperbola $H$ with equation $xy=8$. Find:
2. The equation of the tangent $T$.
3. The equation of the normal $N $to $H$ at the point $P$, giving your answer in the form $ax+by+c=0$.
4. The distinct points A and B, where $x=3$ lie on the parabola C with equation $y^{2}=27x$.
5. The line $l\_{1}$ is the tangent to C at A and the line $l\_{2}$ is the tangent to C at B.

Given that at A, $y>0$, find the coordinates of A and B.

1. Draw a sketch showing the parabola C. Indicate A, B, $l\_{1}$ and $l\_{2}$.
2. Find equations for $l\_{1}$ and $l\_{2}$, giving your answer in the form $ax+by+c=0$.
3. The point $P$ with coordinates $\left(75,30\right)$ lies on the parabola $C$ with equation $y^{2}=12x$.

Find the equation of the tangent to $C$ at $P$, giving your answer in the form $y=mx+c$

1. The point $P(4,8)$ lies on the parabola $C$ with equation $y^{2}=4ax$. Find:
2. The value of $a$
3. An equation of the normal to $C$ at $P$

The normal to $C$ at $P$ cuts the parabola again at the point $Q$. Find:

1. The coordinates of $Q$
2. The length $PQ$, giving your answer as a simplified surd

**2F Problem Solving with Tangents & Normals**

1. The point $P(at^{2},2at)$ lies on the parabola $C$ with equation $y^{2}=4ax$ where $a$ is a positive constant. Show that an equation of the normal to $C$ at $P$ is $y+tx=2at+at^{3}$
2. The point $\left(ct,\frac{c}{t}\right), t\ne 0,$ lies on the rectangular hyperbola $H$ with equation $xy=c^{2}$ where $c$ is a positive constant.
3. Show that an equation of the tangent to $H$ at $P$ is $x+t^{2}y=2ct.$

A rectangular hyperbola $G$ has equation $xy=9$. The tangent to $G$ at the point $A$ and the tangent to $G$ at the point $B$ meet at the point $\left(-1,7\right).$

1. Find the coordinates of $A$ and $B$.
2. The parabola $C$ has equation $y^{2}=20x$. The point $P\left(5p^{2},10p\right)$ is a general point on $C$. The line $l$ is normal to $C$ at the point $P$.
3. Show that an equation for $l$ is $px+y=10p+5p^{3}$

The point $P$ lies on $C$. The normal to $C$ at $P$ passes through the point $\left(30,0\right)$ as shown on the diagram. The region $R$ is bounded by this line, the curve $C$ and the $x$-axis.

1. Given that $P$ lies in the first quadrant, show that the area of the shaded region $R$ is $\frac{1100}{3}$

**2G Loci**

1. The curve $C$ is the locus of points that are equidistant from the line with equation $x+6=0$ and the point $(6,0)$. Prove that $C$ has Cartesian equation $y^{2}=4ax,$ stating the value of $a$.
2. The point $P$ lies on a parabola with equation $y^{2}=4ax$. Show that the locus of the midpoints of $OP$ is a parabola.