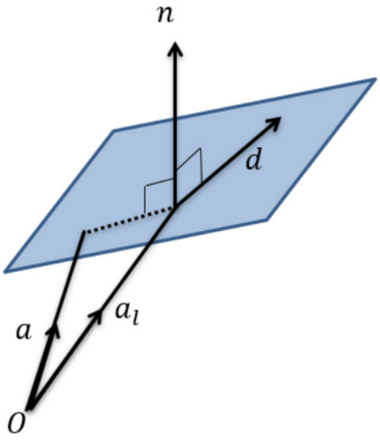
Plane using vectors/lines on plane

Find, in the form $\boldsymbol{r} \cdot \boldsymbol{n} = p$, an equation of the plane which contains the line l and the point with position vector a where l has the equation $\boldsymbol{r} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$ and $\boldsymbol{a} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$
Find also the Cartesian form of the equation.



Key strategy:

A. Identify two vectors on the plane.
B. n is cross product of these (as perpendicular to both)
C. Use $\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$ as usual.

Find, in the form $\boldsymbol{r} \cdot \boldsymbol{n} = p$, an equation of the plane which contains the line l and the point with position vector a where l has the equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$ and $\boldsymbol{a} = \begin{pmatrix} 9 \\ 8 \\ 5 \end{pmatrix}$
Find also the Cartesian form of the equation.

Vectors in direction of plane:

- $\boldsymbol{d} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$
- $\begin{pmatrix} 9 \\ 8 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \\ 2 \end{pmatrix}$

$$\boldsymbol{n} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \times \begin{pmatrix} 8 \\ 6 \\ 2 \end{pmatrix} = \begin{pmatrix} -26 \\ 40 \\ -16 \end{pmatrix}$$

$\boldsymbol{a} \cdot \boldsymbol{n} = 6$
$\boldsymbol{r} \cdot \boldsymbol{n} = 6 \rightarrow -26x + 40y - 16z = 6$

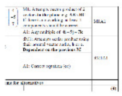
A plane Π passes through the points $A(2,2-1), B(3,2,-1), C(4,3,5)$
Find the equation of the plane Π in the form $\boldsymbol{a}+\lambda\boldsymbol{b}+\mu\boldsymbol{c}$

Find the Cartesian equation of the same plane Π.

[June 2015 Q5] The points A, B and C have position vectors $\begin{pmatrix}1\\3\\2\end{pmatrix}$, $\begin{pmatrix}-1\\0\\1\end{pmatrix}$ and $\begin{pmatrix}2\\1\\0\end{pmatrix}$ respectively.

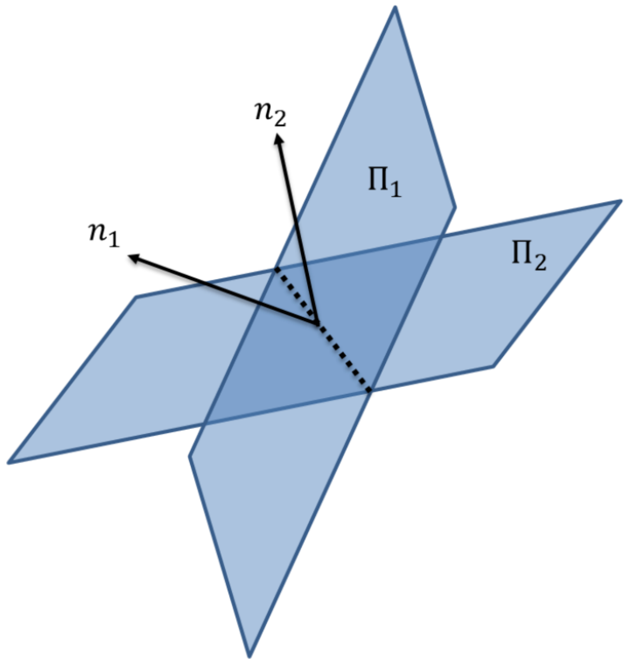
The plane Π contains the points A, B and C.

(c) Find a vector equation of Π in the form $\mathbf{r}.\mathbf{n} = p$. **(4)**



Intersecting Planes

Find the equation of the line of intersection of the planes Π_1 and Π_2 where Π_1 has equation $\boldsymbol{r} \cdot (2\boldsymbol{i} - 2\boldsymbol{j} - \boldsymbol{k}) = 2$ and Π_2 has equation $\boldsymbol{r} \cdot (\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}) = 5$.



Shortest Distance between Skew Lines

Find the shortest distance between the two skew lines with equations $\boldsymbol{r} = \boldsymbol{i} + \lambda(\boldsymbol{j} + \boldsymbol{k})$ and $\boldsymbol{r} = -\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k} + \mu(2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k})$, where λ and μ are scalars.

Shortest distance between 2 skew lines:

$$\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$$
$$\boldsymbol{r} = \boldsymbol{c} + \mu\boldsymbol{d}$$

$$= \left|\frac{(\boldsymbol{a} - \boldsymbol{c}) \cdot (\boldsymbol{b} \times \boldsymbol{d})}{|\boldsymbol{b} \times \boldsymbol{d}|}\right|$$

Test Your Understanding

[June 2009 Q7] The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} \alpha \\ -4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}.$$

...

For other values of α, the lines l_1 and l_2 do not intersect and are skew lines.

Given that $\alpha = 2$,

(*c*) find the shortest distance between the lines l_1 and l_2.

(3)