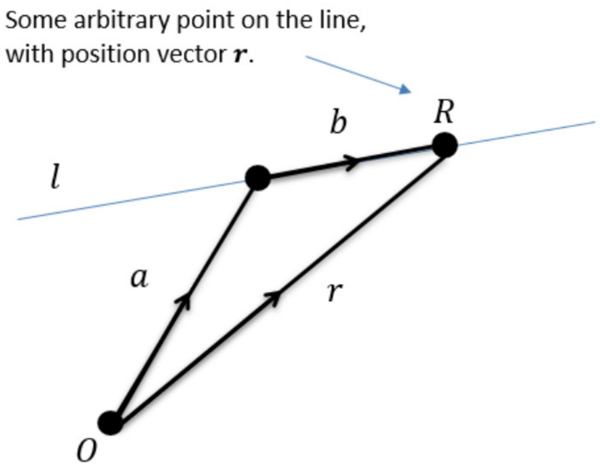
Vector equations of lines



✏ Equation of straight line:

$$(\boldsymbol{r}-\boldsymbol{a})\times\boldsymbol{b}=0$$
$$or\ \boldsymbol{r}\times\boldsymbol{b}=\boldsymbol{a}\times\boldsymbol{b}$$

Find the vector equation of the line through the points $(1,2,-1)$ and $(3,-2,2)$ in the form $(\boldsymbol{r}-\boldsymbol{a})\times\boldsymbol{b}=0$ and in the form $\boldsymbol{r}\times\boldsymbol{b}=\boldsymbol{a}\times\boldsymbol{b}$.

4. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

The point A has coordinates $(4, -3, c)$

(a) Find, in terms of c, an equation for the line that passes through the point A and is perpendicular to the plane Π. Give your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$

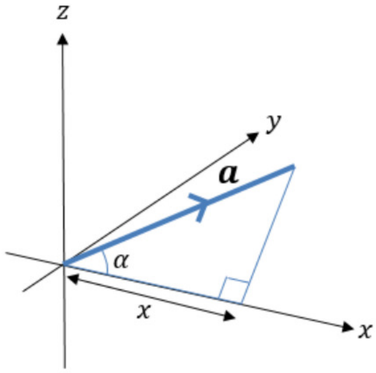
(3)

The point A' is a reflection of the point A in the plane Π

(b) Find, in terms of c, the coordinates of the point A'

(5)

Ratio of cosines of angles with each axis



Recall from Pure Year 2 that if a vector $\boldsymbol{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, then from the diagram we can see that

$$\cos\alpha = \frac{x}{|\boldsymbol{a}|}$$

where α is the angle between $\boldsymbol{a}$ and the x-axis.

Similarly $\cos\beta = \frac{y}{|\boldsymbol{a}|}$ and $\cos\gamma = \frac{z}{|\boldsymbol{a}|}$

Thus $\hat{\boldsymbol{a}} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|} = \begin{pmatrix} \frac{x}{|\boldsymbol{a}|} \\ \frac{y}{|\boldsymbol{a}|} \\ \frac{z}{|\boldsymbol{a}|} \end{pmatrix} = \begin{pmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{pmatrix}$

Recall that $\hat{\boldsymbol{a}}$ is the unit vector scaled version of $\boldsymbol{a}$.

Since $\hat{\boldsymbol{a}}$ is a unit vector, $|\hat{\boldsymbol{a}}| = \sqrt{\cos^2\alpha + \cos^2\beta + \cos^2\gamma} = 1$

✎ If α, β and γ are the angles between a vector $\boldsymbol{a}$ and each of the coordinate axes, then

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

This is sometimes written as

$$l^2 + m^2 + n^2 = 1$$

Where l, m and n are the cosines of angles with each axis

A line L makes angles of α, β and γ with the x, y and z-axes respectively.
Show that $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$.