QQQ – Core Pure Yr1 - Chapter 2 – Complex Numbers & Argand Diagrams

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 16 = Bronze)

1. Given that $z = 1 + \sqrt{3}\text{i}$ and that $\frac{w}{z} = 2 + 2\text{i}$, find

(*a*) w in the form $a + \text{i}b$, where $a, b \in \mathbb{R}$,

(3)

(*b*) the argument of w,

(2)

(*c*) the exact value for the modulus of w.

(2)

On an Argand diagram, the point A represents z and the point B represents w.

(*d*) Draw the Argand diagram, showing the points A and B.

(2)

(*e*) Find the distance AB, giving your answer as a simplified surd.

(2)

2.

$$z = 4\left(\cos\frac{\pi}{4} + \text{i}\sin\frac{\pi}{4}\right), \text{ and } w = 3\left(\cos\frac{2\pi}{3} + \text{i}\sin\frac{2\pi}{3}\right).$$

Express zw in the form $r(\cos\theta + \text{i}\sin\theta)$, $r > 0$, $-\pi < \theta < \pi$.

(3)

3. (a) Shade on an Argand diagram the set of points

$$\left\{z \in \mathbb{C} : |z - 4\text{i}| \leqslant 3\right\} \cap \left\{z \in \mathbb{C} : -\frac{\pi}{2} < \arg(z + 3 - 4\text{i}) \leqslant \frac{\pi}{4}\right\}$$

(6)

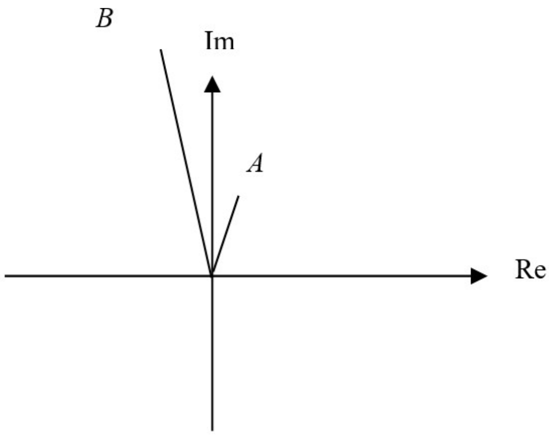
The complex number w satisfies

$$|w - 4\text{i}| = 3$$

(b) Find the maximum value of $\arg w$ in the interval $(-\pi, \pi]$.
Give your answer in radians correct to 2 decimal places.

(2)

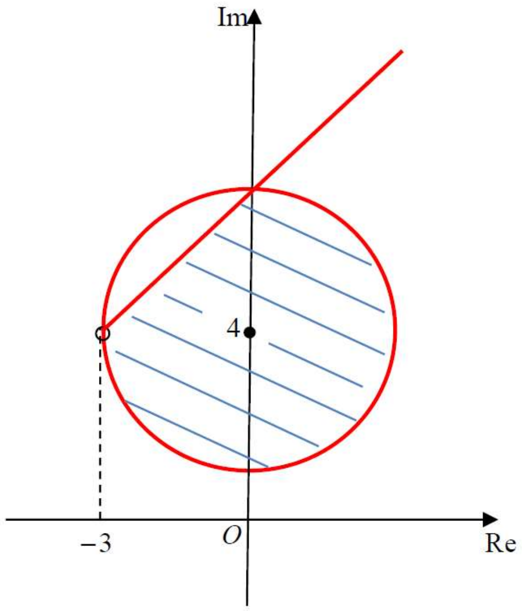
Solutions (

1.	**(a)**	$w = (1+\sqrt{3}i)(2+2i)$ $= (2-2\sqrt{3}), +(2\sqrt{3}+2)\,\mathrm{i}$	**M1** **A1, A1** **(3)**
	(b)	$\arg w = \arctan\left(\dfrac{2\sqrt{3}+2}{2-2\sqrt{3}}\right)$ or adds two args e.g. $60^{\circ}+45^{\circ}$ $= \dfrac{7\pi}{12}$ *or* 105^{o} or 1.83 radians	**M1** **A1** **(2)**
	(c)	$\|w\| = \sqrt{32} = 4\sqrt{2}$	**M1 A1** **(2)**
	(d)	 f.t. w in quadrant other than first	**B1** **B1** **(2)**
	(e)	$\|AB\|^2 = 4+32-16\sqrt{2}\cos 45$ (=20), then square root AB = $2\sqrt{5}$	**M1** **A1** **(2)**
		Or $w-z = 1-2\sqrt{3}+i(2+\sqrt{3})$ $\therefore AB = \|w-z\| = \sqrt{(1-2\sqrt{3})^2+(2+\sqrt{3})^2}$ $= \sqrt{20} = 2\sqrt{5}$	**M1** **A1c.a.o** **(2)**

2.

$zw =$	
$12\left(\cos\frac{\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{\pi}{4}\sin\frac{2\pi}{3}\right) + 12\mathrm{i}\left(\sin\frac{\pi}{4}\cos\frac{2\pi}{3} + \cos\frac{\pi}{4}\sin\frac{2\pi}{3}\right)$	B1 for 12
$= 12\left[\cos\frac{11\pi}{12} + \mathrm{i}\sin\frac{11\pi}{12}\right]$	M1 A1
	(3 marks)

3. **(a)**



	M1	1.1b
	A1	1.1b
	M1	1.1b
	A1	2.2a
	M1	3.1a
	A1	1.1b
	(6)	

(b)

$(\arg w)_{\max} = \frac{\pi}{2} + \arcsin\left(\frac{3}{4}\right)$	M1	3.1a
$= 2.42$ (2 dp) **cao**	A1	1.1b
	(2)	
	(8 marks)	