QQQ – Core Pure Yr1 - Chapter 2 – Complex Numbers & Argand Diagrams

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 16 = Bronze)

1. Given that $z = 1 + \sqrt{3}\mathrm{i}$ and that $\frac{w}{z} = 2 + 2\mathrm{i}$, find

(*a*) w in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$,

(3)

(*b*) the argument of w,

(2)

(*c*) the exact value for the modulus of w.

(2)

On an Argand diagram, the point A represents z and the point B represents w.

(*d*) Draw the Argand diagram, showing the points A and B.

(2)

(*e*) Find the distance AB, giving your answer as a simplified surd.

(2)

2. $$z = 4\left(\cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}\right), \text{ and } w = 3\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right).$$

Express zw in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, $r > 0$, $-\pi < \theta < \pi$.

(3)

3. (a) Shade on an Argand diagram the set of points

$$\left\{z \in \mathbb{C} : |z - 4\mathrm{i}| \leqslant 3\right\} \cap \left\{z \in \mathbb{C} : -\frac{\pi}{2} < \arg(z + 3 - 4\mathrm{i}) \leqslant \frac{\pi}{4}\right\}$$

(6)

The complex number w satisfies

$$|w - 4\mathrm{i}| = 3$$

(b) Find the maximum value of $\arg w$ in the interval $(-\pi, \pi]$.
Give your answer in radians correct to 2 decimal places.

(2)