

## 9.9) Using second derivatives

## Worked example

Find the interval on which the function is concave:

$$f(x) = x^3 - 2x + 5$$

$$g(x) = 2x^3 - 5x^2 - 6$$

## Your turn

Find the interval on which the function is concave:

$$h(x) = x^3 + 4x + 3$$

$$x \leq 0$$

## Worked example

Find the interval on which the function is convex:

$$f(x) = x^3 - 2x + 5$$

$$g(x) = 2x^3 - 5x^2 - 6$$

## Your turn

Find the interval on which the function is convex:

$$h(x) = x^3 + 4x + 3$$

$$x \geq 0$$

## Worked example

Show that the function is convex for all real values of  $x$ :

$$f(x) = e^{3x} + x^2$$

$$g(x) = e^{4x} + x^4$$

## Your turn

Show that the function is convex for all real values of  $x$ :

$$h(x) = e^{2x} + x^2$$

Shown that  $h''(x) \geq 0$  for  $x \in \mathbb{R}$

## Worked example

Determine if there is a point of inflection on the curve with equation  $y = (x - 3)^4$

## Your turn

Determine if there is a point of inflection on the curve with equation  $y = (x + 5)^4$

No

$$\frac{d^2y}{dx^2} = 12(x + 5)^2 \geq 0$$

Either side of  $x = -5$ ,  $\frac{d^2y}{dx^2}$  always positive.

Therefore local minimum at  $(-5, 0)$

## Worked example

A curve C has equation

$$y = \frac{1}{4}x^2 \ln x - 3x + 7, x > 0$$

Find where C is convex

## Your turn

A curve C has equation

$$y = \frac{1}{5}x^2 \ln x + 4x - 3, x > 0$$

Find where C is convex

$$x \geq e^{-\frac{3}{2}}$$