9.8) Implicit differentiation

Worked example	Your turn
Find: $\frac{d}{dx}(y^4)$	Find: $\frac{d}{dx}(2y^3)$
	$6y^2\frac{dy}{dx}$
$\frac{d}{dx}(3y^5)$	

Worked example	Your turn
Find: $\frac{d}{dx}(\cos y)$	Find: $\frac{d}{dx}(\sin 3y)$ $3\cos 3y\frac{dy}{dx}$
$\frac{d}{dx}(\tan 2y)$	

Worked example	Your turn
Find: $\frac{d}{dx}(e^{y})$	Find: $\frac{d}{dx}(e^{3y})$
	$3e^{3y}\frac{dy}{dx}$
$\frac{d}{dx}(e^{2y})$	

Worked example	Your turn
Find: $\frac{d}{dx}(xy)$	Find: $\frac{d}{dx}(x^3y)$
	$x^3\frac{dy}{dx} + 3x^2y$
$\frac{d}{dx}(x^2y)$	

Worked example

Find:

$$\frac{d}{dx}(e^{xy})$$

$$\frac{d}{dx}(e^{x^2y})$$

Your turn

Find:

$$\frac{d}{dx}(e^{x^3y})$$

$$\left(x^3\frac{dy}{dx}+3x^2y\right)e^{x^3y}$$

Worked example	Your turn
Find: $\frac{d}{dx}(\cos(x+y))$	Find: $\frac{d}{dx}(\sin(x^3+5y))$
	$\left(3x^2+5\frac{dy}{dx}\right)(\cos(x^3+5y))$
$\frac{d}{dx}(\tan(x^2-4y))$	

Worked example

Find $\frac{dy}{dx}$ where:

$$x^4 - x + y^2 - 3y = 5$$

Your turn

Find $\frac{dy}{dx}$ where:

$$x^3 + x + y^3 + 3y = 6$$

$$\frac{dy}{dx} = \frac{-3x^2 - 1}{3y^2 + 3}$$

Worked example

Find $\frac{dy}{dx}$ at the point $(1, 1)$, given that:

$$6x^2y - \frac{4x}{y^2} = 2$$

Your turn

Find $\frac{dy}{dx}$ at the point $(1, 1)$, given that:

$$4xy^2 + \frac{6x^2}{y} = 10$$

$$\frac{dy}{dx} = -8$$

Worked example

A curve is described by:

$$x^3 + 4y^2 = -12xy$$

Find the gradient of the curve at the points where $x = 8$

Your turn

A curve is described by:

$$x^3 - 4y^2 = 12xy$$

Find the gradient of the curve at the points where $x = -8$

$$\frac{dy}{dx} = -3 \text{ at } (-8, 16)$$

$$\frac{dy}{dx} = 0 \text{ at } (-8, 8)$$

Worked example

$$x^2 + y^2 + 20x + 4y - 8xy = -75$$

Find the values of y for which $\frac{dy}{dx} = 0$

Your turn

$$x^2 + y^2 + 10x + 2y - 4xy = 10$$

Find the values of y for which $\frac{dy}{dx} = 0$

$$y = \frac{7}{3}, 5$$

Worked example

A curve has equation

$$x^2 + 4xy + y^2 - x = 35$$

Find the equation of the tangent to the curve at the point $(2, 3)$.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers

Your turn

A curve has equation

$$x^2 + 2xy - y^2 + x = 20$$

Find the equation of the tangent to the curve at the point $(3, 2)$.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers

$$11x + 2y - 37 = 0$$

Worked example

The curve $ye^{-4x} = 4x - y^2$.
Find the equation of the normal(s) to the curve at the point where $x = 0$.
Give your answer in the form $ax + by + c = 0$

Your turn

The curve $ye^{-2x} = 2x + y^2$.
Find the equation of the normal(s) to the curve at the point where $x = 0$.
Give your answer in the form $ax + by + c = 0$

$2x + y = 0$ at $(0,0)$
$x - 4y + 4 = 0$ at $(0, 1)$