9.6) Differentiating trigonometric functions

Worked example	Your turn
Differentiate with respect to x: $y = \cot x$	Differentiate with respect to x: $y = \tan x$ $\frac{dy}{dx} = \sec^2 x$

Worked example	Your turn
Differentiate with respect to x: $y = \operatorname{cosec} x$	Differentiate with respect to x: $y = \sec x$ $\frac{dy}{dx} = \sec x \tan x$

Worked example

Differentiate with respect to x:

$$y = \tan 2x$$

$$f(x) = \tan(-\frac{x}{3})$$

Your turn

Differentiate with respect to x:

$$y = \tan 4x$$

$$\frac{dy}{dx} = 4\sec^2 4x$$

Worked example

Differentiate with respect to x:

$$y = \cot 2x$$

$$f(x) = \cot(-\frac{x}{3})$$

Your turn

Differentiate with respect to x:

$$y = \cot 4x$$

$$\frac{dy}{dx} = -4\ cosec^2 4x$$

Worked example

Differentiate with respect to x:

$$y = \sec 2x$$

$$f(x) = \sec(-\frac{x}{3})$$

Your turn

Differentiate with respect to x:

$$y = \sec 4x$$

$$\frac{dy}{dx} = 4 \sec 4x \tan 4x$$

Worked example

Differentiate with respect to x:

$$y = \text{cosec}\, 2x$$

$$f(x) = \text{cosec}(-\frac{x}{3})$$

Your turn

Differentiate with respect to x:

$$y = \text{cosec}\, 4x$$

$$\frac{dy}{dx} = -4\, cosec\, 4x \cot 4x$$

Worked example

Differentiate with respect to x:

$$y = \tan^4 2x$$

$$f(x) = \tan^3(-\frac{x}{3})$$

Your turn

Differentiate with respect to x:

$$y = \tan^2 4x$$

$$\frac{dy}{dx} = 8 \tan 4x \sec^2 4x$$

Worked example

Differentiate with respect to x:

$$y = \cot^4 2x$$

$$f(x) = \cot^3(-\frac{x}{3})$$

Your turn

Differentiate with respect to x:

$$y = \cot^2 4x$$

$$\frac{dy}{dx} = -8 \cot 4x \, cosec^2 4x$$

Worked example

Differentiate with respect to x:

$$y = \sec^4 2x$$

$$f(x) = \sec^3(-\frac{x}{3})$$

Your turn

Differentiate with respect to x:

$$y = \sec^2 4x$$

$$\frac{dy}{dx} = 8\sec^2 4x \tan 4x$$

Worked example

Differentiate with respect to x:

$$y = \text{cosec}^4 2x$$

$$f(x) = \text{cosec}^3(-\frac{x}{3})$$

Your turn

Differentiate with respect to x:

$$y = \text{cosec}^2 4x$$

$$\frac{dy}{dx} = -8\,cosec^2 4x \cot 4x$$

Worked example

Differentiate with respect to x:

$$y = \frac{cosec\ 3x}{x^3}$$

Your turn

Differentiate with respect to x:

$$y = \frac{cosec\ 2x}{x^2}$$

$$\frac{dy}{dx} = -\frac{2cosec\ 2x(x\cot 2x + 1)}{x^3}$$

Worked example

Given that $x = \cot y$, express $\frac{dy}{dx}$ in terms of x.

Your turn

Given that $x = \tan y$, express $\frac{dy}{dx}$ in terms of x.

$$\frac{dy}{dx} = \frac{1}{1 + x^2}$$

Worked example

Differentiate with respect to x:

$$y = \arccos x$$

$$y = \arctan x$$

Your turn

Differentiate with respect to x:

$$y = \arcsin x$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

Worked example

Differentiate with respect to x:

$$y = \arccos x^4$$

$$y = \arctan x^3$$

Your turn

Differentiate with respect to x:

$$y = \arcsin x^2$$

$$\frac{dy}{dx} = \frac{2x}{\sqrt{1 - x^4}}$$

Worked example

Given that $x = \operatorname{cosec} 3y$, find $\frac{dy}{dx}$ in terms of x

Your turn

Given that $x = \sec 2y$, find $\frac{dy}{dx}$ in terms of x

$$\frac{dy}{dx} = \frac{1}{2x\sqrt{x^2 - 1}}$$

Worked example

Given that $y = \arctan\left(\frac{1+x}{1-x}\right)$, find $\frac{dy}{dx}$

Your turn

Given that $y = \arctan\left(\frac{1-x}{1+x}\right)$, find $\frac{dy}{dx}$

$$\frac{dy}{dx} = -\frac{1}{1+x^2}$$